A note on ligthly compact spaces

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Lightly compact spaces were introduced in [3] by BAGLEY, CONNEL and MCKNIGHT. These spaces or equivalent forms of these spaces have been investigated in [3], [6], [7], [9], [11], [12], [17], [18], [19], [20]. Lightly compact spaces happen to be important tools in the study of minimal first countable and minimal-E_1 spaces, [cf, respectively [19], [15]]. A space is said to be lightly compact if every locally finite family of open sets is finite or equivalently, every countable open cover admits of a finite subfamily, the closures of whose members cover the space or equivalently, every countable open filter base has an adherent point. In the present paper, we have improved on of the results of Bagley, Connel and McKnight viz: A completely regular space is pseudo-compact if and only if it is lightly compact by replacing complete regularity by a weaker property. Several other results regarding these spaces have been obtained.

Definitions. A space X is pseudo-compact [4] if every real valued continuous mapping defined on X is bounded. A set is regularly closed if it is the closure of some open set or equivalently closure of its own interior. A space X is almost completely regular [14] if for every regularly closed set A and a point x not belonging to A, there exists a continuous mapping f from X into [0, 1] such that $f(x)=0$ and $f(A)=\{1\}$.

Theorem 1. *Let X be an almost completely regular space. Then the following are equivalent:*

(a) *X is pseudo-compact.*

(b) *If $V_1 \supseteqq V_2 \supseteqq \dots$ is any decreasing sequence of non-empty open sets, then $\bigcap \{\mathrm{cl}\, V_n : n \in N\} \neq \emptyset$.*

(c) *X is lightly compact.*

PROOF. (a)$\Rightarrow$(b). Let $\{V_n\}$ be a decreasing sequence of non-empty open sets. Suppose, if possible, that $\bigcap \{\mathrm{cl}\, V_n : n \in N\} = \emptyset$. Then, $\{\mathrm{int\,cl}\, V_n : n \in N\}$ is a locally finite family of open sets, because if $x \in X$ and every neighbourhood of x intersects infinitely many sets of the family $\{\mathrm{int\,cl}\, V_n : n \in N\}$, then since $\{V_n : n \in N\}$ is a decreasing sequence, every neighbourhood of x will intersect every int cl V_n and hence x will belong to $\mathrm{cl\,int\,cl}\, V_n$, that is, $\mathrm{cl}\, V_n$ for all n, which will be a contradiction. Since each V_n is non-empty, there exists a point $x_n \in X$ such that $x_n \in \mathrm{int\,cl}\, V_n$. Since X is almost completely regular, for each n there exists a real valued continuous mapping f_n on X such that $f_n \geqq 0$, $f_n(x_n)=n$, and $f_n(X \sim \operatorname{int} \operatorname{cl} V_n)=\{0\}$. Define a mapping $g: X \to R$ such that $g(x)=\sum_{n=1}^{\infty} f_n(x)$ for all $x \in X$. Then, g is a well-defined real valued continuous mapping on X which is not bounded. This contradicts the pseudo-compactness of the space. Hence (a)$\Rightarrow$(b).

(b)$\Rightarrow$(c). Let $\mathscr{D}=\{D_n : n \in N\}$ be a countable open cover of X. Suppose, if possible, that there does not exist a finite subfamily of $\mathscr{D}$, the closures of whose members cover the space. Let $V_n = X \sim \bigcup \{\operatorname{cl} D_i : i=1, 2, \ldots, n\}$. Then $\{V_n : n \in N\}$ is a decreasing sequence of non-empty open sets and hence by (b), $\bigcap \{\operatorname{cl} V_n : n \in N\} \neq \emptyset$, that is, $\bigcup \{X \sim \operatorname{cl} V_n : n \in N\} \neq X$, that is, $\bigcup \{D_i : i \in N\} \neq X$, which is a contradiction. Hence (b)$\Rightarrow$(c).

(c)$\Rightarrow$(a). Let f be a real valued continuous mapping defined on X. Let $V_n = \{x : |f(x)| < n\}$. Then, $\{V_n : n \in N\}$ is a countable open cover of X and hence has a finite subfamily $\{V_{n_i} : i=1, 2, \ldots, m\}$ such that $X = \bigcup \{\operatorname{cl} V_{n_i} : i=1, 2, \ldots, m\}$. Let $p = \max(n_1, n_2, \ldots, n_m)$. Then $|f(x)| \leqq p$ for all $x \in X$. Hence f is bounded and thus X is pseudo-compact.

Corollary 1 [4]. In a completely regular space X, conditions (a), (b) and (c) of the above theorem are equivalent.

Definitions. A mapping $f: X \to Y$ is *almost continuous* [13] if for each $x \in X$ and each open set V containing $f(x)$, there exists an open set $\mathscr{U}$ containing x such that $f(\mathscr{U}) \subseteqq \operatorname{int} \operatorname{cl} V$ or equivalently, if the inverse image of every regularly open set is open. A set is regularly open if it is equal to the interior of some closed set or equivalently to the interior of its own closure or equivalently, if its complement is regularly closed. A mapping f is almost closed [13] if the image of every regularly closed set is closed. A space X is an E_1-space [1] if every point of X is expressible as a countable intersection of closed neighbourhoods.

Theorem 2. *An almost continuous mapping from a lightly compact space into an E_1-space is almost closed.*

Proof. Let f be an almost continuous mapping from a lightly compact space X into an E_1-space Y. Let A be a regularly closed set. Then, A is lightly compact [15]. Since almost continuous mappings preserve light-compactness [15] and lightly compact subsets of E_1-spaces are closed, [15], $f(A)$ is closed. Thus, f is almost closed and hence the result.

Definition [10]. A mapping $f: X \to Y$ is said to be a *P_0-mapping* if for each $y \in Y$ and each open set $\mathscr{U}$ containing $f^{-1}(y)$, $y \in \operatorname{int}[f(\operatorname{cl} \mathscr{U})]$.

Theorem 3. *Let X be a lightly compact space. Then, the following are true.*

(a) *Every almost continuous mapping from X into an E_1-space is a P_0-mapping.*

(b) *If f is an almost continuous mapping from X into an E_1-space Y, then* $\operatorname{cl}[f(\mathscr{U})] \sim f(\mathscr{U}) \subseteqq f(\partial \mathscr{U})$ *for every open subset $\mathscr{U}$ of X, where $\partial \mathscr{U}$ denotes the boundary of $\mathscr{U}$.*

(c) *Under the same assumption as in* (b), $\operatorname{cl}[f(\mathscr{U})] \subseteqq f(\operatorname{cl}(\mathscr{U}))$ *for every open subset $\mathscr{U}$ of X.*

(d) *Under the same assumption as in* (b), *if a regularly open subset $\mathscr{U}$ of X contains $f^{-1}(y)$, then $y \in \operatorname{int} f(\mathscr{U})$.*

PROOF.

Proof of (a). Let f be an almost continuous mapping from X onto an E_1-space Y. Let $y\in Y$ and $\mathscr{U}$ be an open set containing $f^{-1}(y)$. Suppose, if possible, that $y\notin \operatorname{int} f[\operatorname{cl}(\mathscr{U})]$, that is, $y\in Y\sim \operatorname{int} f[\operatorname{cl}\mathscr{U}]$, that is, $y\in \operatorname{cl}[Y\sim f(\operatorname{cl}\mathscr{U})]$. Since Y is an E_1-space, there exists a countable family $\{G_i: i\in N\}$ of open sets containing y such that $\{y\}=\bigcap\{\operatorname{cl} G_i: i\in N\}$. Let $V_n=\bigcap\{\operatorname{int}\operatorname{cl} G_i: i=1, 2, \dots, n\}$. Then $V_n\cap \cap(Y\sim f(\operatorname{cl}\mathscr{U}))\neq\emptyset$ for all n and therefore $f^{-1}(V_n)\cap f^{-1}(Y\sim f(\operatorname{cl}\mathscr{U}))\neq\emptyset$ for all n, that is, $f^{-1}(V_n)\cap(X\sim \operatorname{cl}\mathscr{U})\neq\emptyset$. Let $\mathscr{B}=\{f^{-1}(V_n)\cap(X\sim\operatorname{cl}\mathscr{U}): n\in N\}$. Then $\mathscr{B}$ is a countable open filter base in the lihtly compact space X and hence has an adherent point, say x. Then $x\in\operatorname{cl}(X\sim\operatorname{cl}\mathscr{U})$. Also $x\in\operatorname{cl}(f^{-1}(V_n))$ for all n. Since f is almost continuous, $\operatorname{cl}[f^{-1}(V_n)]\subseteq f^{-1}(\operatorname{cl} V_n)$. Therefore, $x\in f^{-1}(\operatorname{cl} V_n)$, that is, $f(x)\in\operatorname{cl} V_n$ which implies $f(x)\in\operatorname{cl} G_i$ for all i and hence $y=f(x)$, that is, $x\in f^{-1}(y)$. Since $\mathscr{U}$ is an open set containing $f^{-1}(y)$ which contains x and $x\in\operatorname{cl}(X\sim\operatorname{cl}\mathscr{U})$, $\mathscr{U}\cap(X\sim\operatorname{cl}\mathscr{U})\neq\emptyset$ which is a contradiction. Hence $y\in\operatorname{int}(f(\operatorname{cl}\mathscr{U}))$ and thus f is a P_0-mapping.

Proof of (b). Let $y\in[\operatorname{cl} f(\mathscr{U})]\sim f(\mathscr{U})$, that is, $y\in\operatorname{cl} f(\mathscr{U})$ and $y\notin f(\mathscr{U})$. Since Y is an E_1-space, choose a countable family $\{G_i: i\in N\}$ of open sets containing y such that $\{y\}=\bigcap\{\operatorname{cl} G_i: i\in N\}$. Let $V_n=\bigcap\{\operatorname{int}\operatorname{cl} G_i: i=1, 2, \dots, n\}$. Since $y\in\operatorname{cl} f(\mathscr{U})$, $V_n\cap f(\mathscr{U})\neq\emptyset$ for all n. Hence $\emptyset\neq f^{-1}(V_n)\cap\mathscr{U}$, which is open as f is almost continuous. Therefore, $\mathscr{B}=\{f^{-1}(V_n)\cap\mathscr{U}: n\in N\}$ is a countable open filter base in the lightly compact space X and hence has an adherent point, say x. Then $x\in\operatorname{cl}\mathscr{U}$. Since f is almost continuous, $\operatorname{cl}(f^{-1}(V_n))\subseteq f^{-1}(\operatorname{cl} V_n)$. Also, $x\in\operatorname{cl}(f^{-1}(V_n))$. Therefore, $x\in f^{-1}(\operatorname{cl} V_n)$, that is, $f(x)\in\operatorname{cl} V_n$, that is, $f(x)\in\operatorname{cl} G_i$ for all i and hence $f(x)=y$ and $f(x)\notin f(\mathscr{U})$, that is, $x\notin\mathscr{U}$. Thus $x\in\operatorname{cl}\mathscr{U}\sim\mathscr{U}$, that is, $f(x)\in f[\operatorname{cl}\mathscr{U}\sim\mathscr{U}]= =f(\partial\mathscr{U})$. Hence, $\operatorname{cl} f(\mathscr{U})\sim f(\mathscr{U})\subseteq f(\partial\mathscr{U})$.

Proof of (c). Let $\mathscr{U}$ be an open subset of X. Then $\operatorname{cl}\mathscr{U}$ is a regularly closed subset of the lightly compact space X and hence is lightly compact. Since light-compactness is preserved under almost continuous mappings and lightly compact subsets of E_1-spaces are closed, $f(\operatorname{cl}\mathscr{U})$ is a closed set, therefore $\operatorname{cl} f(\mathscr{U})\subseteq f(\operatorname{cl}\mathscr{U})$.

Proof of (d). Follows immediately from a result of Singal and Singal [13].

Corollary 2. The following are equivalent in a completely regular T_1 space X:

(1) X is lightly compact.

(1′) X is pseudo-compact.

(2) Every continuous mapping from X onto an E_1-space is a P_0-mapping.

(2′) Every continuous mapping from X onto a first countable Hausdorff space is a P_0-mapping.

(3) If f is a continuous mapping from X onto an E_1-space, then $\operatorname{cl}(f(\mathscr{U}))\sim \sim f(\mathscr{U})\subseteq f(\partial\mathscr{U})$ for any open subset $\mathscr{U}$ of X.

(3′) If f is a continuous mapping of X onto a first countable Hausdorff space, then $\operatorname{cl} f(\mathscr{U})\sim f(\mathscr{U})\subseteq f(\partial\mathscr{U})$ for any open subset $\mathscr{U}$ of X.

(4) Under the same assumption as in (3), $\operatorname{cl} f(\mathscr{U})=f(\operatorname{cl}\mathscr{U})$ for any open subset $\mathscr{U}$ of X.

(4′) Under the same assumptions as in (3′), $\operatorname{cl} f(\mathscr{U})=f(\operatorname{cl}\mathscr{U})$ for any open subset $\mathscr{U}$ of X.

(5) Under the same assumption as in (3), if a regularly open subset $\mathscr{U}$ of X contains $f^{-1}(y)$, then $y\in\operatorname{int}(f(\mathscr{U}))$.

(5′) Under the same assumption as in (3′), if a regularly open subset $\mathscr{U}$ of X contains $f^{-1}(y)$, then $y \in \operatorname{int} f(\mathscr{U})$.

PROOF. Since every first countable Hausdorff space is an E_1-space, (2)⇒(2′), (3)⇒(3′), (4)⇒(4′) and (5)⇒(5′). Also, from corollary 1, (1)⇔(1′). From the previous theorem, (1) implies (2), (3), (4) and (5). In [8], ISIWATA has shown that (1′), (2′), (3′), (4′) and (5′) are equivalent. Hence the result.

Theorem 4. *Let f be a continuous mapping from a lightly compact space X onto an E_1-space Y. Then f is quasi-compact if and only if $f(\partial\mathscr{U}) = \partial(f(\mathscr{U}))$ for any inverse open subset $\mathscr{U}$ of X.* (A mapping is said to be quasi-compact if it is onto and $f(\mathscr{U})$ is open whenever $\mathscr{U}$ is an inverse open set.)

PROOF. Let f be quasi-compact. Let $\mathscr{U}$ be an inverse open subset of X. Then $f(\mathscr{U})$ is an open set, therefore $\partial[f(\mathscr{U})] = [\operatorname{cl} f(\mathscr{U})] \sim f(\mathscr{U})$. By theorem 3 (b) $[\operatorname{cl} f(\mathscr{U}) \sim \sim f(\mathscr{U})] \subseteq f(\partial\mathscr{U})$. Therefore, $\partial(f(\mathscr{U})) \subseteq f(\partial\mathscr{U})$. Since $\mathscr{U}$ is an inverse open set, $f(\partial\mathscr{U}) \cap \cap f(\mathscr{U}) = \emptyset$. Since f is continuous and quasi-compact, $f(\operatorname{cl} \mathscr{U}) \subseteq \operatorname{cl} f(\mathscr{U})$ and therefore $f(\operatorname{cl} \mathscr{U}) \sim f(\mathscr{U}) \subseteq \operatorname{cl} f(\mathscr{U}) \sim f(U)$, that is, $f(\operatorname{cl} U) \sim f(U) \subseteq \partial f(U)$. Now $f(\partial U) \subseteq f(\operatorname{cl} U)$ and $f(\partial U) \cap f(U) = \emptyset$, therefore $f(\partial U) \subseteq f(\operatorname{cl} U) \sim f(U) \subseteq \partial f(U)$. Hence $\partial(f(U)) = = f(\partial U)$.

Conversely, let U be an open inverse set. Now $f(\partial U) = \partial f(U)$. Also $\operatorname{cl} U = = U \cup \partial U$, $U \cap \partial U = \emptyset$ and $f(U) \cap f(\partial U) = \emptyset$. Thus $f(\operatorname{cl} U)$ is a union of two disjoint sets $f(U)$ and $f(\partial U)$. On the other hand $\operatorname{cl} f(U) = \partial(f(U)) \cup \operatorname{int} f(U)$, $\partial(f(U)) \cap \cap \operatorname{int} f(U) = \emptyset$. By theorem 3 (c), $f(\operatorname{cl} U) = \operatorname{cl} f(U)$ as f is continuous. This implies, $f(U) = f(\operatorname{cl} U) \sim f(\partial\mathscr{U}) = \operatorname{cl} f(U) \sim f(\partial U) = \operatorname{cl} f(U) \sim \partial f(U) = \operatorname{int} f(U)$. Thus $f(U)$ is open.

Corollary 3 [8]. Let f be a continuous mapping from a completely regular pseudo-compact space X onto a first countable Hausdorff space or an E_1-space Y. Then f is quasi-compact if and only if $f(\partial U) = \partial f(U)$ for every inverse open subset $\mathscr{U}$ of X.

PROOF. Obvious.

Definition. A mapping $f\colon X \to Y$ is said to be *almost* quasi-compact [13] if it is onto and if A is open whenever $f^{-1}(A)$ is regularly open or equivalently, if the image of every regularly open inverse set is open.

Theorem 5. *Let f be a continuous mapping from a lightly compact space X onto an E_1-space Y. Then f is almost quasi-compact if and only if $f(\partial U) = \partial f(U)$ for any regularly open inverse subset $\mathscr{U}$ of X.*

PROOF. The proof is omitted as it is essentially the same as that of theorem 4.

Theorem 6. *Let X be a lightly compact space. Let A be a closed set such that the boundary ∂ A is lightly compact, then A is lightly compact.*

PROOF. Let $\mathscr{D} = \{D_i \colon i \in N\}$ be a countable relatively open cover of A. Then, for each $i \in N$, there exists an open set C_i such that $D_i = C_i \cap A$. Now $\{C_i \colon i \in N\} \cup \cup \{X \sim A\}$ is a countable open cover of the lightly compact space X and hence there exists a finite subfamily $\{C_{i_j} \colon j = 1, 2, \dots, m\}$ of $\{C_i \colon i \in N\}$ such that $X \subseteq \cup$

$\{\text{cl } C_{i_j}: j=1, 2, \ldots, m\} \cup \text{cl } (X \sim A)$. Therefore, int $A = X \sim \text{cl } (X \sim A) \subseteq \cup \{\text{cl } C_{i_j}: j=1, 2, \ldots, m\}$. This implies, int $A = \cup \{\text{cl } C_{i_j} \cap \text{int } A: j=1, 2, \ldots, m\} \subseteq \cup \{\text{cl } (C_{i_j} \cap \text{int } A): j=1, 2, \ldots, m\} \subseteq \cup \{\text{cl } (C_{i_j} \cap A): j=1, 2, \ldots, m\} = \cup \{\text{cl } (C_{i_j} \cap A) \cap A: j=1, 2, \ldots, m\}$ (as A is closed) $= \cup \{(\text{cl } D_{i_j}) \cap A: j=1, 2, \ldots, m\} = \cup \{\text{cl}_{I_A} D_{i_j}: j=1, 2, \ldots, m\}$. Also $\{C_i \cap \partial A: i \in N\}$ is a countable relatively open cover of the lightly compact subspace ∂A and hence there exists a finite subfamily $\{C_{i_j} \cap \partial A: j=1, 2, \ldots, p\}$ of $\{C_i \cap \partial A: i \in N\}$ such that $\partial A = \cup \{\text{cl}_{I_{\partial A}} (C_{i_j} \cap \partial A): j=1, 2, \ldots, p\} \subseteq \cup \{\text{cl } (C_{i_j} \cap A) \cap A: j=1, 2, \ldots, p\}$ (as $\partial A \subseteq A$, A being closed) $= \cup \{\text{cl}_{I_A} D_{i_j}: j=1, 2, \ldots, p\}$. Now $A = \partial A \cup \text{int } A \subseteq \{\cup \{\text{cl}_{I_A} D_{i_j}: j=1, 2, \ldots, p\}\} \cup \{\cup \{\text{cl}_{I_A} D_{i_j}: j=1, 2, \ldots, m\}\}$. Thus A is lightly compact and hence the result.

Definition. A space $(X, \mathcal{I})$ is countably C-compact [17] if given a closed set A and a countable $\mathcal{I}$-open cover $\mathcal{C}$ of A, there exists a finite subfamily of $\mathcal{C}$, say $\{C\ldots: i=1, 2, \ldots, n\}$ such that $A \subseteq \cup \{\text{cl } C_i: i=1, 2, \ldots, n\}$.

Corollary 4. Every lightly compact space in which the boundary ∂A of every closed set A is lightly compact is countably C-compact.

PROOF. Let $(X, \mathcal{I})$ be a lightly compact space with the given property. Let A be a closed set. Then ∂A is lightly compact and hence by Theorem 6, A is lightly compact. Let $\mathcal{C}$ be a countable open cover of A. Then $\{C \cap A: C \in \mathcal{C}\}$ is a countable relatively open cover of A and hence there exists a finite subfamily $\{C_i \cap A: i=1, 2, \ldots, n\}$ of $\{C \cap A: C \in \mathcal{C}\}$ such that $A = \cup \{\text{cl}_{\mathcal{I}_A}(C_i \cap A): i=1, 2, \ldots, n\} \subseteq \cup \{\text{cl}_{\mathcal{I}} C_i: i=1, 2, \ldots, n\}$. Hence X is countably C-compact.

Definition. A space X is said to satisfy the axiom pT_sA [5] if for each point p in X and a closed set A not containing p, there exists a real valued continuous mapping from X such that $f(p) \notin f(A)$.

Theorem 7. *Every lightly compact space satisfying the axiom pT_sA is almost completely regular.*

PROOF. Let X be a lightly compact space with the property pT_sA. Let $x \in X$ and A be a regularly closed set not containing the point x. Then, since X has the property pT_sA, there exists a real valued continuous mapping f on X such that $f(x) \in f(A)$. Since X is lightly compact, A is lightly compact, being a regularly closed subset. Therefore, $f(A)$ is a lightly compact subset of the reals R with the usual topology and hence closed. Since R is completely regular, there exists a continuous mapping $g: R \to [0, 1]$ such that $g(f(x)) = 0$ and $g(f(A)) = \{1\}$. Now $h = g \circ f$ is a continuous mapping from X into $[0, 1]$ such that $h(x) = 0$ and $h(A) = \{1\}$. Hence X is almost completely regular.

Corollary 5 [2]. Every lightly compact semi-regular space having the property pT_sA is completely regular.

PROOF. Obvious, since every semi-regular almost completely regular space is completely regular [cf. [16]].

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