Correction and remark to my paper "Characterizations of the Baer radical class by almost nilpotent rings"

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I shall use the terminology and notations of [2].

1. Almost nilpotent rings may be defined in three non-equivalent ways.

Definition A (G. A. P. HEYMAN): A ring A is almost nilpotent, if every nonzero ideal of A *strictly contains a power of* A.

Definition B [1]: A ring A is almost nilpotent, if every proper homomorphic image of A is nilpotent. *The prime simple rings are assumed not to be almost nilpotent.*

Definition C [2]: A ring is almost nilpotent, if every proper homomorphic image of A is nilpotent. *The prime rings are assumed to be almost nilpotent.*

Denoting the classes of all almost nilpotent rings given by Definitions A, B and C by $\mathbf{L}_A, \mathbf{L}_B$ and $\mathbf{L}_C$, respectively, we have obviously the relations $\mathbf{L}_A \subsetneqq \mathbf{L}_B$ and $\mathbf{L}_B \subsetneqq \mathbf{L}_C$

2. Using *Definition A* the subdirectly irreducible almost nilpotent rings are always nilpotent. Hence *all the results of* [2] *are valid.* (The proofs are analogous to those in [2] but somewhat simpler.)

3. Using *Definition B* we can say the followings:

i) *Theorem 1 of* [2] *is valid.* From line 14 on p. 16 of [2] the proof should be completed as follows:

$A \in \mathcal{S}\mathbf{R}$, contradicting $0 \neq A \in \mathbf{R}$. Hence $H \in \mathbf{R}$ holds. Since $A \in \mathbf{L}$, the ring A/H is nilpotent and so $A/H \in \mathcal{S}\mathbf{R}$. Further, $A \in \mathbf{R}$ implies $A/H \in \mathcal{S}\mathbf{R} \cap \mathbf{R} = \mathbf{O}$. Hence $H = A \in \mathbf{L}$ which is a contradiction because H, as a prime simple ring, is not almost nilpotent. Thus $\mathbf{Z} \subseteq \mathbf{R}$ is proved.

ii) Instead of [2] Theorem 2 one can easily prove

Theorem 2*. *Let* $\mathbf{R}$ *be a radical class such that* $\mathbf{R} \cap \mathbf{Z} \neq \mathbf{O}$. $\mathbf{R}$ *satisfies condition* (L) *iff* $\mathbf{Z} \subseteq \mathbf{R}$.

Correspondingly, in Corollaries 2 and 3 of [2] the adjective "hereditary" must be replaced by condition $\mathbf{R} \cap \mathbf{Z} \neq \mathbf{O}$.

4. Using *Definition C*, as *I* did in [2], we remark the followings:

i) *Theorem 1 of* [2] *is false* (the ring B constructed in the proof is not associative). In fact, let $\mathbf{R}$ be the lower radical determined by a single simple ring A with unity. It is easy to see that $\mathbf{R} \cap \mathbf{L} \neq \mathbf{O}$ and $\mathbf{Z} \nsubseteq \mathbf{R}$ hold and in addition $\mathbf{R}$ satisfies condition (L).

ii) I was not able to prove or disprove the assertion of [2] Theorem 2. To prove it, it is sufficient to have an affirmative answer for the following

Problem (T. L. JENKINS): *Let H be a prime simple ring without unity. Does there exist a subdirectly irreducible ring B with heart H such that* $B/H \neq 0$ *is nilpotent?*

Nevertheless, Theorem 2* holds also in the terms of Definition *C*.

References

[1] L. C. A. VAN LEEUWEN and G. A. P. HEYMAN, A radical determined by a class of almost nilpotent rings, *Acta Math. Acad. Sci. Hungar.* **26** (1975), 259—262.

[2] R. WIEGANDT, Characterizations of the Baer lower radical class by almost nilpotent rings, *Publ. Math. (Debrecen)* **23** (1976), 15—17.

(Received September 13, 1977.)