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On curvatures of the Weyl—Otsuki spaces

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T. OTSUKI starting out from an affine connection Γ and from a tensor P^i_j of type (1.1), has developed a so-called *regular general connection*. For covariant derivation of contravariant resp. covariant vectors two different affine connections $'\Gamma$ resp. $''\Gamma$ are used in this theory. They are derived from Γ and P by taking certain requirements. Then A. MOÓR [2] linked Otsuki's connection theory with the metric of a Weyl space and thus constructed the *Weyl—Otsuki spaces* denoted by $W—O_n$.

In this paper we find some relations between curvature tensors of this $W—O_n$ space. First we complete the most important results and formulae from the theory of the Otsuki, resp. Weyl—Otsuki spaces. In § 1. we consider the second covariant derivative of a vector field in an Otsuki space and we give the alternation formulae and the Ricci formula. These formulae are necessary to investigate the skew-symmetry in the first two indices of the curvature tensor of $''\Gamma$.

In § 2. we find the Ricci and Bianchi identities for the curvature tensors of the covariant and contravariant part of the regular general connection Γ.

In § 3.—5. Weyl—Otsuki spaces are studied. For these spaces we find the torsion of the curvature tensor, and different Ricci as well as Bianchi identities (§ 3.). In § 4. we find conditions for the symmetry and skew-symmetry of the curvature tensor of the connection $''\Gamma$. Also some more curvature tensors having the above symmetry properties are constructed. Finally in § 5 $W—O_n$ space of scalar curvature and the analogue of Schur's theorem are investigated.

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Basic formulae of Weyl—Otsuki spaces[1])

Let $\mathfrak{X}$ be a differentiable manifold. A regular general Otsuki connection on it (called simply Otsuki connection in this paper) is given by an affine connection Γ and a tensor field P of type (1, 1), so that in local coordinate $\det |P^i_j| \neq 0$. From the above assumptions it follows the existence of the tensor Q^i_j of type (1, 1) so that

$$P^i_j Q^j_s = P^j_s Q^i_j = \delta^i_s. \tag{0.1}$$

Γ and P determine two affine connections $'\Gamma$ and $''\Gamma$ in the following way

$$'\Gamma_j{}^i{}_k = \Gamma_j{}^l{}_k Q^i_l; \quad ''\Gamma_j{}^i{}_k = (\Gamma_l{}^i{}_k - \partial_k P^i_l) Q^l_j.$$

[1]) Formulae of this paragraph one can find in [1] or [2].

$'\Gamma$ and $''\Gamma$ are classical affine connections. For $'\Gamma_j{}^i{}_k$ and $''\Gamma_j{}^i{}_k$ the following relations hold

$$(0.2)\qquad '\Gamma_j{}^i{}_k = Q^i_s(''\Gamma_r{}^s{}_k P^r_j + \partial_k P^s_j);\quad ''\Gamma_i{}^j{}_k = (P^j_s{}'\Gamma_l{}^s{}_k - \partial_k P^j_l)Q^l_i.$$

An often used relation is

$$(0.3)\qquad \partial_k P^i_j + ''\Gamma_s{}^i{}_k P^s_j - '\Gamma_j{}^s{}_k P^i_s = 0.$$

The operation of covariant and basic covariant differentiation in an Otsuki space, and the Ricci formula with respect to this basic covariant derivation are:

$$(0.4)\qquad DV^{i_1\ldots i_p}_{j_1\ldots j_q} = V^{i_1\ldots i_p}_{j_1\ldots j_q,k}\,dx^k$$

$$(0.5)\qquad V^{i_1\ldots i_p}_{j_1\ldots j_q,k} = P^{i_1}_{s_1}\ldots P^{i_p}_{s_p} V^{s_1\ldots s_p}_{r_1\ldots r_q|k} P^{r_1}_{j_1}\ldots P^{r_q}_{j_q}$$

$$(0.6)\qquad V^{s_1\ldots s_p}_{r_1\ldots r_q|k} = \partial_k V^{s_1\ldots s_p}_{r_1\ldots r_q} + \sum_{t=1}^{p} {'\Gamma_l{}^{s_t}{}_k} V^{s_1\ldots s_{t-1} l s_{t+1}\ldots s_p}_{r_1\ldots r_q} - \sum_{t=1}^{q} {''\Gamma_{r_t}{}^l{}_k} V^{s_1\ldots s_p}_{r_1\ldots r_{t-1} l r_{t+1} r_q}$$

$$(0.7)\qquad 2V^{i_1\ldots i_p}_{j_1\ldots j_q|[h|k]} = -\sum_{t=1}^{p} {'R_s{}^{i_t}{}_{hk}} V^{i_1\ldots i_{t-1} s i_{t+1}\ldots i_p}_{j_1\ldots j_q} + \sum_{t=1}^{q} {''R_{j_t}{}^s{}_{hk}} V^{i_1\ldots i_p}_{j_1\ldots j_{t-1} s j_{t+1}\ldots j_q} - V^{i_1\ldots i_p}_{j_1\ldots j_q|s}\, {''T_h{}^s{}_k},$$

where $V^{i_1\ldots i_p}_{j_1\ldots j_q}$ are the components of a tensor field of type (p, q), and

$$'T_j{}^i{}_k = '\Gamma_j{}^i{}_k - '\Gamma_k{}^i{}_j;\quad ''T_j{}^i{}_k = ''\Gamma_j{}^i{}_k - ''\Gamma_k{}^i{}_j$$

are torsion tensors of $'\Gamma$ and $''\Gamma$ respectively (c.f. [1] — (2.14), (3.8), (3.7), (7.15)). A consequence of the above definitions is

$$(0.8)\qquad \delta^i_{j|k} = '\Gamma_j{}^i{}_k - ''\Gamma_j{}^i{}_k.$$

For covariant derivatives formed with respect to the classical affine connections $'\Gamma$ and $''\Gamma$ we use the notation $'\nabla, ''\nabla$ respectively. The curvature tensor $'R_i{}^j{}_{kl}$ of the connection $'\Gamma$ is given by

$$(0.9)\qquad 'R_i{}^j{}_{kl} = \partial_k{}'\Gamma_i{}^j{}_l - \partial_l{}'\Gamma_i{}^j{}_k + '\Gamma_p{}^j{}_k{}'\Gamma_i{}^p{}_l - '\Gamma_p{}^j{}_l{}'\Gamma_i{}^p{}_k.$$

The curvature tensor $''R_i{}^j{}_{kl}$ of the connection $''\Gamma$ is defined on the same way.

On the other hand, a Weyl space W_n is determined by a positive definite metric tensor $g_{ij}(x)$ and a recurrence vector field $\gamma_k(x)$ connected by the relation

$$(0.10)\qquad \nabla_k g_{ij} = \gamma_k g_{ij}.$$

A. Moór could construct from the connection of an Otsuki space O_n and the metric of a Weyl space a *metric general connection* $W-O_n$. The construction of this space starts from a Weyl space given by g_{ij} and γ_k; and a tensor field P^i_j of an Otsuki space. A symmetric $''\Gamma$ is then determined from g_{ij}, γ_k and P^i_j by the requirement that ∇_k in (0.10) means the Otsuki covariant derivation defined by (0,5). $''\Gamma_j{}^i{}_k$ can be expressed in the following way

$$(0.11)\qquad ''\Gamma_j{}^i{}_k = \frac{1}{2} g^{is}\{(\partial_j g_{ks} + \partial_k g_{sj} - \partial_s g_{jk}) - (\gamma_j m_{ks} + \gamma_k m_{sj} - \gamma_s m_{jk})\},$$

where

$$(0.12)\qquad m_{ij} = g_{sr} Q^s_i Q^r_j.$$

Also the relation

$$(0.13)\qquad P_{ij} = P^s_i g_{sj} = P^s_j g_{si} = P_{ji}$$

is required (cf. [2] — (1.7), (1.8)). $'\Gamma$ is then determined by $''\Gamma$ and P, in a symple form.

§ 1. *The alternation formulae*

We find the alternation formulae of an Otsuki space. Since $Dx^i = dx^i$ we get from (0.4), (0.5) and (0.6) the relation

$$\Delta Dx^i = P^i_b(\delta\, dx^b + '\Gamma_l{}^b{}_k\, dx^l \delta x^k),$$

where Δ and D denote covariant differentiations. If δ and d are commutable differential symbols, we have

$$(1.1)\qquad (\Delta D - D\Delta) x^i = \frac{1}{2} P^i_b {'T_k}{}^b{}_l\, dx^k \wedge \delta x^l.$$

We denote

$$(1.2)\qquad '\Omega^i \overset{\text{def}}{=} \frac{1}{2} P^i_b {'T_k}{}^b{}_l\, dx^k \wedge \delta x^l.$$

From (0.4), (0.5) and (0.6) we also get

$$D\xi^i = P^i_s(\partial_k \xi^s + '\Gamma_l{}^s{}_k \xi^l)\, dx^k$$

for the contravariant vector field ξ^i, and using (0.8) and (0.3) we obtain

$$(1.3)\qquad (\Delta D - D\Delta)\xi^i = \left[\frac{1}{2} P^i_b P^b_a {'R_p}{}^a{}_{lk} \xi^p\, dx^k \wedge \delta x^l\right] - [(\Delta \xi^s) \wedge (D\delta^i_m) Q^m_s].$$

The first term of the right side contains the components of the curvature tensor of $'\Gamma$, and we denote it by

$$(1.4)\qquad '\Omega^i_p \overset{\text{def}}{=} \frac{1}{2} P^i_b P^b_a {'R_p}{}^a{}_{lk}\, dx^k \wedge \delta x^l.$$

The second term contains the covariant differential of the vector field ξ^i and of the Kronecker δ. This term vanishes if ξ^i is a parallel vector field, or if $D\delta^i_m = 0$.

Using the notation $M_j^i = P_r^i P_j^r$ (cf. [1] (2.20)) we get for the tensor field $V_{j_1 \dots j_q}^{i_1 \dots i_p}$

$$(1.5)\quad (\Delta D - D\Delta) V_{j_1 \dots j_q}^{i_1 \dots i_p} = \frac{1}{2} M_{k_1}^{i_1} \dots M_{k_p}^{i_p} M_{j_1}^{h_1} \dots M_{j_q}^{h_q} \Big(\sum_{a=1}^{p} {}''R_{h_a}{}^{s}{}_{tr} V_{h_1 \dots h_{a-1} s h_{a+1} \dots h_q}^{k_1 \dots k_p} -$$

$$- \sum_{b=1}^{q} {}'R_s{}^{k_b}{}_{tr} V_{h_1 \dots h_q}^{k_1 \dots k_{b-1} s k_{b+1} \dots k_p} \Big) dx^t \wedge \delta x^r -$$

$$- \sum_{a=1}^{p} P_{k_1}^{i_1} \dots P_{k_{a-1}}^{i_{a-1}} P_{k_{a+1}}^{i_{a+1}} \dots P_{k_p}^{i_p} P_{j_1}^{h_1} \dots P_{j_q}^{h_q} (\Delta V_{h_1 \dots k_q}^{k_1 \dots k_{a-1} s k_{a+1} \dots k_p}) \wedge (D\delta_m^{i_a}) Q_s^m -$$

$$- \sum_{b=1}^{q} P_{k_1}^{i_1} \dots P_{k_p}^{i_p} P_{j_1}^{h_1} \dots P_{j_{b-1}}^{h_{b-1}} P_{j_{b+1}}^{h_{b+1}} \dots P_{j_q}^{h_q} (\Delta V_{h_1 \dots h_{b-1} s h_{b+1} \dots h_q}^{k_1 \dots k_p}) \wedge (D\delta_{j_b}^m) Q_m^s,$$

similarly to (1.3). This is an analogon of the well known alternation formula of Riemannian geometry (see [3] (105.16)). This proves the following

Theorem 1. *The alternation formula in spaces with regular general connection* Γ *has the form* (1.5).

In case of vanishing of the covariant differential of the Kronecker δ (1.5) reduces to the known formula of the classical affine connection if $M_j^i = \delta_j^i$.

We shall use the Pfaffian forms defined by

$$(1.6)\qquad {}'\omega_j^i(d) \stackrel{\text{def}}{=} {}'\Gamma_j{}^i{}_k\, dx^k; \quad {}''\omega_j^i(d) \stackrel{\text{def}}{=} {}''\Gamma_j{}^i{}_k\, dx^k$$

and also the exterior derivatives in the usual way. The exterior derivative of a 1-form ${}'\omega_j^i$ is defined by

$$(1.7)\qquad {}^*d\,{}'\omega_p^a(d, \delta) \stackrel{\text{def}}{=} d\,{}'\omega_p^a(\delta) - \delta\,{}'\omega_p^a(d).$$

It is easy to prove that

$$(1.8)\qquad \text{a) } {}'\Omega^i = P_b^i (dx^k \wedge {}'\omega_k^b); \quad \text{b) } {}'\Omega_p^i = P_b^i P_a^b ({}'\omega_p^r \wedge {}'\omega_r^a - {}^*d\,{}'\omega_p^a).$$

Now we use covariant derivatives in order to get the Ricci formulae of a covariant tensor field $V_{i_1 \dots i_p}$ with respect to the covariant differentiation of a regular general connection Γ.

Let us form the covariant derivative of the vector ξ_i according to (0.5)

$$\xi_{i,k} = P_i^a (\partial_k \xi_a - {}''\Gamma_a{}^p{}_k \xi_p).$$

By repeated derivation and alternation we get

(1.9 $\xi_{i,k} = P_i^a (\partial_k \xi_a - {}''\Gamma_a{}^p{}_k \xi_p).$

Her By repeated derivation and alternation we get

$$\text{con}(1.9)\qquad \xi_{i,k,l} - \xi_{i,l,k} = 2\xi_{s|c} \delta_{i,[l}^s P_{k]}^c + P_i^b P_b^s (P_k^a \xi_{s|a|l} - P_l^a \xi_{s|a|k}).$$

Let Here we applied (0.6). We want to know how this expression depends on the components of the curvature tensor. For this aim we calculate

$$P_k^a \xi_{s|a|l} - P_l^a \xi_{s|a|k} = (P_k^a \delta_l^r - P_l^a \delta_k^r) \xi_{s|a|r}.$$

Let us consider

$$\xi_{s|a|r} = \xi_{s|(a|r)} + \xi_{s|[a|r]}.$$

Applying the Ricci formula (0.7) to $\xi_{s|[a|r]}$, and using (1.9) we have

$$(1.10)\ \xi_{i,[k,l]} = \xi_{s|a}\delta^s_{i,[l}P^a_{k]} + \frac{1}{2} P^b_i P^s_b \{2P^{(a}_{[k}\delta^{r)}_{l]}\xi_{s|(a|r)} + P^{[a}_{[k}\delta^{r]}_{l]}({}''R_s{}^f{}_{ar}\xi_f - {}''T_a{}^m{}_r\xi_{s|m})\}.$$

This is the Ricci formula for the tensor field ξ_i with respect to the covariant derivatives for a regular general connection Γ. With an analogous calculation for the tensor field $V_{i_1\dots i_p}$ we get the following formula

$$(1.11)\ V_{i_1\dots i_p,[k,l]} = Q^r_m \sum_{h=1}^{p} P^{b_1}_{i_1}\dots P^{b_{h-1}}_{i_{h-1}} P^{b_{h+1}}_{i_{h+1}}\dots P^{b_p}_{i_p} V_{b_1\dots b_{h-1}rb_{h+1}\dots b_p,c}\delta^m_{i_h,[l}P^c_{k]} +$$

$$+ \frac{1}{2} M^{b_1}_{i_1}\dots M^{b_p}_{i_p}\{2P^{(a}_{[k}\delta^{r)}_{l]}V_{b_1\dots b_p|(a|r)} +$$

$$+ P^{[a}_{[k}\delta^{s]}_{l]}\sum_{h=1}^{p} {}''R_{b_h}{}^b{}_{ar}V_{b_1\dots b_{h-1}bb_{h+1}\dots b_p} - P^{[a}_{[k}\delta^{s]}_{l]}{}''T_a{}^r{}_s V_{b_1\dots b_p|r}\}.$$

Thus we have

Theorem 2. *The Ricci formula with respect to covariant derivation defined by* (0.5) *for a covariant tensor field* $V_{i_1\dots i_p}$ *has the form* (1.11).

If we suppose that $P^i_j = \varrho\delta^i_j$, we have $\delta^i_{j,k} = 0$, $P^{(a}_{[k}\delta^{r)}_{l]} = 0$ and $P^{[a}_{[k}\delta^{r]}_{l]} = \varrho\delta^a_{[k}\delta^r_{l]}$. Thus (1.11) gets the form

$$V_{i_1\dots i_p,[k,l]} = \frac{1}{2}\varrho^{2p+1}\left(\sum_{h=1}^{p} {}''R_{i_h}{}^a{}_{kl}V_{i_1\dots i_{h-1}ai_{k+1}\dots i_p} - {}''T_k{}^m{}_l V_{i_1\dots i_p|m}\right).$$

For $\varrho = 1$ this is the Ricci formula (0.7). In this case the covariant derivatives defined by (0.5) and (0.6) coincide.

§ 2. *Ricci and Bianchi identities for Otsuki spaces*

In view of (1.8a) and the definition (1.7) the exterior derivative of the 1-form $'\Omega^i$ is

$${}^*d'\Omega^i = (\partial_m P^i_b)(dx^m \wedge {}'\Omega^a Q^b_a) - P^i_b(dx^p \wedge {}^*d'\omega^b_p).$$

Substituting $\partial_m P^i_b$ from (0.3), ${}^*d'\omega^b_p$ from (1.8b), using relations (0.1), (1.8a) and definitions (1.6) we get from the above identity

$$(2.1)\qquad {}^*d'\Omega^i + {}''\omega^i_a \wedge {}'\Omega^a - (dx^p \wedge {}'\Omega^t_p)Q^i_t = 0.$$

This relation gives the following

Theorem 3. (2.1) *is the generalization of the Ricci identity for Otsuki spaces.*

Using definitions (1.2), (1.4), (1.6), identity (2.1), and the skew-symmetry of the torsion tensors we get the following identity

$$P^i_s\{{}'T_p{}^s{}_{k|m} - {}'T_b{}^s{}_p{}''T_k{}^b{}_m + {}'R_p{}^s{}_{km}\}dx^m \wedge dx^p \wedge dx^k = 0.$$

This identity is true for all values of dx^m, dx^p, dx^k, and the skew-symmetry of the term in the bracket yealds

$$'T_{[p\ k|m]}^{\ s} - 'T_{b\ [p}^{\ s}\, ''T_{k\ m]}^{\ b} + 'R_{[p\ km]}^{\ s} = 0, \tag{2.2}$$

where $[pkm]$ denote the usual alternation for p, k, m (see [5]). In the following we shall use this symbolic of Schouten.

Corollary 1. (2.2) *is the Ricci identity for the curvature tensor of* $'\Gamma$ *for a regular general connection* Γ *expressed in components.*

Let us now apply the Ricci formula (0.7) to the Kronecker δ. It follows that

$$'R_{p\ km}^{\ s} = ''R_{p\ km}^{\ s} - 2\delta^s_{p|[k|m]} - \delta^s_{p|r}\, ''T_{k\ m}^{\ r}. \tag{2.3}$$

From (0.8) we obviously obtain

$$'T_{p\ k}^{\ b} = 2\delta^b_{[p|k]} + ''T_{p\ k}^{\ b}.$$

Substituting the above relation and (2.3) into (2.2), we obtain the identity

$$''T_{[p\ k|m]}^{\ b} - ''T_{b\ [p}^{\ s}\, ''T_{k\ m]}^{\ b} + ''R_{[p\ km]}^{\ s} - \delta^s_{b|[p}\, ''T_{k\ m]}^{\ b} = 0. \tag{2.4}$$

Thus we have another consequence of (2.1):

Corollary 2. *The identity* (2.4) *is the Ricci identity for the curvature tensor of* $''\Gamma$.

If the affine connections $'\Gamma$ and $''\Gamma$ are symmetric, from (2.2) and (2.4) we have the known Ricci identities for symmetric affine connections.

Let us now start from the definition (1.8b). By virtue of a calculation analogous to that of identity (2.1) we obtain the identity

$$^*d\,'\Omega^i_p - P^i_s Q^b_a \delta^s_{b|m}(dx^m \wedge '\Omega^a_p) - '\omega^r_p \wedge '\Omega^i_r + ''\omega^i_a \wedge '\Omega^a_p = 0 \tag{2.5}$$

which proves the following

Theorem 4. (2.5) *is the Bianchi identity for the curvature tensor of* $'\Gamma$.

Starting from (2.5), using definitions (1.4), (1.6), (0.6), and taking into consideration (0.1), (0.8) we get the relation

$$P^i_b P^b_a\{'R_{p\ kl|m}^{\ a} - \delta^s_{p|m}\, 'R_{s\ kl}^{\ a} + ''T_{k\ l}^{\ s}\, 'R_{p\ ms}^{\ a}\}\, dx^m \wedge dx^k \wedge dx^l = 0. \tag{2.6}$$

Now it is possible to express the following

Corollary 3. *The Bianchi identity for the curvature tensor of* $'\Gamma$ *with respect to the basic covariant derivation is*

$$'R_{p\ kl|m}^{\ a} - \delta^s_{p|m}\, 'R_{s\ kl}^{\ a} + ''T_{k\ l}^{\ s}\, 'R_{p\ ms}^{\ a} + \{klm\} = 0. \tag{2.7}$$

In (2.7) $\{klm\}$ denotes the sum of expressions which we get by cyclic permutation of the indices in the foregoing expression. Substituting (2.3) into (2.7) we get the Bianchi identity for the curvature tensor formed with respect to the part $''\Gamma$ of a regular connection Γ. That is

$$(2.8)\qquad {}''R_p{}^a{}_{kl|m}-2\delta^a_{p|[k|l]|m}-(\delta^a_{p|s}{}''T_k{}^s{}_l)_{|m}-\delta^s_{p|m}({}''R_s{}^a{}_{kl}-2\delta^a_{s|[k|l]}-\delta^a_{s|t}{}''T_k{}^t{}_l)+$$
$$+{}''T_k{}^s{}_l({}''R_p{}^a{}_{ms}-2\delta^a_{p|[m|s]}-\delta^a_{p|t}{}''T_m{}^t{}_s)+\{klm\}=0.$$

If $P^i_j=\varrho\delta^i_j$, $\varrho=$const., then from (0.6) and (0.8) follows $\delta^i_{j|k}={}'\Gamma_j{}^i{}_k-{}''\Gamma_j{}^i{}_k=0$, and the identities (2.7) and (2.8) coincide. If we suppose that the affine connections ${}'\Gamma$ and ${}''\Gamma$ are symmetric, then we have the well known Bianchi identities.

§ 3. *Ricci and Bianchi identities in* W—O$_n$

Now we shall investigate the W—O$_n$ spaces. The basic formulae of the following investigations are (0.10)—(0.13).

Since ${}''\Gamma_j{}^i{}_k$ are components of a classical symmetric affine connection, we get

$$(3.1)\qquad {}''T_j{}^i{}_k={}''\Gamma_j{}^i{}_k-\Gamma_k{}^i{}_j=0.$$

Using the notation

$$(3.2)\qquad \{{}^{\,i}_{j\,k}\}=\frac{1}{2}g^{is}(\partial_j g_{ks}+\partial_k g_{sj}-\partial_s g_{jk})\quad (\textit{Crhistoffel's symbols})$$

and

$$(3.3)\qquad K_j{}^i{}_k=\frac{1}{2}g^{is}(\gamma_j m_{ks}+\gamma_k m_{sj}-\gamma_s m_{jk})$$

from (0.11) we get

$$(3.4)\qquad {}''\Gamma_j{}^i{}_k=\{{}^{\,i}_{j\,k}\}-K_j{}^i{}_k.$$

$K_j{}^i{}_k$ is symmetric in j and k. Thus symmetry follows from symmetry in covariant indices of Christoffel symbols of second kind and from (3.1). From the identity (2.4) and the relation (3.1) we obtain the classical Ricci identity

$$(3.5)\qquad {}''R_{[j}{}^i{}_{kl]}=0.$$

Applying the Ricci formula (1.10) on the recurrence vector $\gamma_k(x)$, we get

$$(3.6)\qquad (\nabla_l\nabla_k-\nabla_k\nabla_l)\gamma_i=2\gamma_{s|c}P^c_{[k}\nabla_{l]}\delta^i_s+2P^b_i P^s_b P^{(a}_{[k}\delta^{r)}_{l]}\gamma_{s|(a|r)}-$$
$$-P^b_i P^s_b P^{[a}_{[k}\delta^{r]}_{l]}({}''R_s{}^p{}_{ar}\gamma_p-{}''T_a{}^p{}_r\gamma_{s|p}).$$

If γ_k is a parallel vector field independent from the direction of the displacement, then we have from (0.5)

$$(3.7)\qquad \gamma_{k||l}=0;\quad \nabla_l\gamma_k=0.$$

From this we obtain

Theorem 5. *In a* W—O$_n$ *space which admits a recurrence vector* γ_k *satisfying* (3.7), *we have*

$${}''R_s{}^p{}_{t[l}P^t_{k]}\gamma_p=0.$$

If $P^i_j=\varrho\delta^i_j$, $\varrho=$const.$\neq 0$, we get

$${}''R_r{}^p{}_{kl}\gamma_p=0.$$

This is the condition for γ_k to be a parallel vector field of a space with affine connection.

The relation between the covariant derivation with respect to the connection $''\Gamma$ and the basic covariant derivation with respect to the regular general connection Γ is given by

$$V^{i_1\dots i_p}_{j_1\dots j_q|k} = {''\nabla_k} V^{i_1\dots i_p}_{j_1\dots j_q} + \sum_{t=1}^{p} \delta^{i_t}_{s|k} V^{i_1\dots i_{t-1} s i_{t+1}\dots i_p}_{j_1\dots j_q}. \tag{3.8}$$

Applying this relation and (3.1) to the identity (2.8), we get the identity

$$''\nabla_m {''R_p{}^a{}_{kl}} + \delta^a_{s|m} {''R_p{}^s{}_{kl}} - 2\delta^a_{p|m|[k|l]} - \delta^s_{p|m} {''R_s{}^a{}_{kl}} + $$
$$+ 2\delta^s_{p|m}\delta^a_{s|[k|l]} + \{klm\} = 0.$$

It is easy to prove that

$$2\delta^a_{p|[k|l]|m} + \{klm\} = 2\delta^a_{p|m|[k|l]} + \{klm\}. \tag{3.9}$$

From the Ricci formula given by (0.7) and from the equation which we get by applying (0.7) on $\delta^a_{p|m}$ and by using (3.1) we obtain

$$2\delta^a_{p|m|[k|l]} = -{'R_s{}^a{}_{kl}}\delta^s_{p|m} + {''R_p{}^s{}_{kl}}\delta^a_{s|m} + {''R_m{}^s{}_{kl}}\delta^a_{p|s}. \tag{3.10}$$

Substituting (3.10) into (3.9), using (3.5), (2.3) and (3.1), we get

$$''\nabla_m {''R_p{}^a{}_{kl}} + \{klm\} = 0. \tag{3.11}$$

This is the *Bianchi identity for the curvature tensor of* $''\Gamma$ with respect to the*s ymmetric affine connection* $''\Gamma$ *itself.*

Let us now consider the definition (0.9) of the components of the curvature tensor formed from $''\Gamma$. Substituting the relation (3.8) into (0.9) we obtain

$$''R_j{}^i{}_{kl} = \partial_k\{{}_j{}^i{}_l\} + \{{}_p{}^i{}_k\}\{{}_j{}^p{}_l\} - \partial_k K_j{}^i{}_l + K_p{}^i{}_k K_j{}^p{}_l - \{{}_p{}^i{}_k\}K_j{}^p{}_l - K_p{}^i{}_k\{{}_j{}^p{}_l\} - k/l \tag{3.12}$$

where k/l denotes the same expression on the right hand side but k and l changed. Let us take the notations

$$R_j{}^i{}_{kl} = \partial_k\{{}_j{}^i{}_l\} + \{{}_p{}^i{}_k\}\{{}_j{}^p{}_l\} - k/l$$

and

$$M_j{}^i{}_{kl} = \partial_k K_j{}^i{}_l - K_p{}^i{}_k K_j{}^p{}_l + \{{}_p{}^i{}_k\}K_j{}^p{}_l + \{{}_j{}^p{}_l\}K_p{}^i{}_k - k/l.$$

Then (3.12) gets the form

$$''R_j{}^i{}_{kl} = R_j{}^i{}_{kl} - M_j{}^i{}_{kl}. \tag{3.13}$$

Here $R_j{}^i{}_{kl}$ denotes the curvature tensor of a Riemannian space, and $M_j{}^i{}_{kl}$ is called the *torsion of the curvature tensor of* $''\Gamma$.

Let $\overset{c}{\nabla}$ denote the covariant derivation with respect to the $\{{}_j{}^i{}_k\}$. Then

$$\overset{c}{\nabla}_k K_j{}^i{}_l = \partial_k K_j{}^i{}_l + \{{}_p{}^i{}_k\}K_j{}^p{}_l - \{{}_j{}^p{}_k\}K_p{}^i{}_l - \{{}_l{}^p{}_k\}K_j{}^i{}_p.$$

If we substitute this relation in the above definition of $M_j{}^i{}_{kl}$, we have

$$M_j{}^i{}_{kl} = \overset{c}{\nabla}_k K_j{}^i{}_l - K_p{}^i{}_k K_j{}^p{}_l - k/l.$$

From the properties of $R_j{}^i{}_{kl}$ and $''R_j{}^i{}_{kl}$ it follows that $M_j{}^i{}_{kl}$ is a skew-symmetric tensor in k and l, and the following theorem holds

Theorem 6. *In* W—O$_n$ *spaces the torsion of the curvature tensor of* $''\Gamma$ *satisfies the identity*

$$M_{[j}{}^{i}{}_{kl]} = 0. \tag{3.14}$$

We can say that (3.12) *is the Ricci identity of the tensor* $M_j{}^i{}_{kl}$.

Now we want to find the Bianchi identities for W—O$_n$ spaces. (3.8) and (3.11) give

$$''R_p{}^a{}_{kl|m} - \delta^a_{s|m}\, ''R_p{}^s{}_{kl} + \{klm\} = 0.$$

From (3.13) and the above identity we get

$$R_p{}^a{}_{kl|m} - M_p{}^a{}_{kl|m} - \delta^a_{s|m} R_p{}^s{}_{kl} + \delta^a_{s|m} M_p{}^s{}_{kl} + \{klm\} = 0. \tag{3.15}$$

We want to describe the relation between the covariant derivation with respect to the regular general connection Γ and the Riemannian connection. This is given by

$$V^{i_1\dots i_p}_{j_1\dots j_q|m} = \overset{c}{\nabla}_m V^{i_1\dots i_p}_{j_1\dots j_q} + \sum_{t=1}^{p} (\delta^{i_t}_{s|m} - K_s{}^{i_t}{}_m) V^{i_1\dots i_{t-1} s i_{t+1}\dots i_p}_{j_1\dots j_q} + \tag{3.16}$$

$$+ \sum_{t=1}^{q} K_{j_t}{}^s{}_m V^{i_1\dots i_p}_{j_1\dots j_{t-1} s j_{t+1}\dots j_q}.$$

$R_j{}^i{}_{kl}$ satisfies the Bianchi identity with respect to the Riemannian connection. So, if we apply (3.16) on (3.15), we get

$$M_p{}^a{}_{kl|m} - \delta^a_{s|m} M_p{}^s{}_{kl} - K_p{}^s{}_m R_s{}^a{}_{kl} + K_s{}^a{}_m R_p{}^s{}_{kl} + \{klm\} = 0. \tag{3.17}$$

Theorem 7. *The Bianchi identity of the tensor* $M_j{}^i{}_{kl}$ *with respect to the covariant derivation is* (3.17).

From (3.8), resp. (3.16), and (3.17) it follows, that the following holds

$$''\nabla_m M_p{}^a{}_{kl} + 2\overset{c}{\nabla}_{[k}\overset{c}{\nabla}_{l]} K_p{}^a{}_m + \{klm\} = 0 \tag{3.18}$$

or

$$\overset{c}{\nabla}_m M_p{}^a{}_{kl} + 2\, ''\nabla_{[k}\, ''\nabla_{l]} K_p{}^a{}_m + \{klm\} = 0. \tag{3.19}$$

If we denote the torsion of the curvature tensor of $''\Gamma$ with respect to the operation $''\nabla_k$ by

$$''M_j{}^i{}_{kl} = ''\nabla_k K_j{}^i{}_l + K_p{}^i{}_k K_j{}^p{}_l - k/l \tag{3.20}$$

then we have

$$''R_j{}^i{}_{kl} = R_j{}^i{}_{kl} - ''M_j{}^i{}_{kl}.$$

It is obvious, that also the tensor $''M_j{}^i{}_{kl}$ satisfies the identities (3.14), (3.18) and (3.19).

§ 4. *Some properties of the curvature tensors of* $''\Gamma$

We determine conditions for the skew-symmetry in the first two indices of the curvature tensor of $''\Gamma$.

From (0.11) and (0.7) it follows that

$$(4.1) \qquad (\Delta D - D\Delta) g_{ij} = P_i^a P_j^b g_{ab} \partial_{[l} \gamma_{k]} dx^k \wedge \delta x^l.$$

Applying (1.5) on the metric tensor g_{ij}, we have

$$(4.2) \quad (\Delta D - D\Delta) g_{ij} = \left\{ 2\gamma_{[l} \nabla_{k]} \delta^b_{(j} g_{i)b} + \frac{1}{2} P_i^a P_a^b P_j^c P_c^d (''R_d{}^s{}_{kl} g_{sb} + ''R_b{}^s{}_{kl} g_{sd}) \right\} dx^k \wedge \delta x^l.$$

Let us substitute (4.1) in (4.2) and express the part which contains the components of the curvature tensor of $''\Gamma$ with respect to (0.1) and (0.12). Using the relation

$$''R_i{}^a{}_{kl} g_{aj} = ''R_{ijkl}$$

we obtain

$$(4.3) \qquad ''R_{abkl} + ''R_{bakl} = 4Q_r^j Q_{(a}^r m_{b)s} \gamma_{[k} \nabla_{l]} \delta_j^s + 2m_{ab} \partial_{[k} \gamma_{l]}.$$

This gives

Theorem 8. *In order that the tensor* $''R_{ijkl}$ *be skew-symmetric in its first two indices, it is necessary and sufficient that*

$$(4.4) \qquad 2Q_r^t Q_{(i}^r m_{j)s} \gamma_{[k} \nabla_{l]} \delta_t^s + m_{ij} \partial_{[k} \gamma_{l]} = 0.$$

If $\delta^s_{j|l} = 0$, that is $'\Gamma \equiv ''\Gamma$, then from (4.4) we have

$$\partial_{[l} \gamma_{k]} = 0.$$

This means that the recurrence vector γ_k is a gradient vector. In W—O_n spaces we can express the above condition also in the form $\overset{c}{\nabla}_l \gamma_k - \overset{c}{\nabla}_k \gamma_l = 0$.

We define the tensor

$$(4.5) \qquad \hat{R}_{ijkl} \overset{\text{def}}{=} ''R_{ijkl} - T_{ijkl}$$

where

$$(4.6) \qquad T_{ijkl} \overset{\text{def}}{=} 2Q_r^t Q_{(i}^r m_{j)s} \gamma_{[k} \nabla_{l]} \delta_t^s + m_{ij} \partial_{[k} \gamma_{l]}.$$

T_{ijkl} is symmetric in i, j and skew-symmetric in k and l. The just defined tensor $\hat{R}_{ijkl}$ is skew-symmetric in its first two indices, i.e.

$$(4.7) \qquad \hat{R}_{ijkl} = -\hat{R}_{jikl}.$$

This will be proved by (4.3).

We shall now determine the conditions for

$$(4.8) \qquad \hat{R}_{ijkl} = \hat{R}_{klij}.$$

The Ricci identity holds for the tensor $''R_i{}^j{}_{kl}$, and since

$$''R_i{}^a{}_{kl} g_{aj} = ''R_{ijkl}$$

from (4.5) we have

$$\hat{R}_{ijkl} + \hat{R}_{kjli} + \hat{R}_{ljik} + (T_{ijkl} + \{ikl\}) = 0.$$

By adding to the above identity a similar one which we get from the former changing the indices $i \to j$ and $k \to l$, we obtain

$$2\hat{R}_{ijkl}-\hat{R}_{kjil}+\hat{R}_{ljik}-\hat{R}_{lijk}+\hat{R}_{kijl}+(T_{ijkl}+\{ikl\})+(T_{jilk}+\{jlk\})=0.$$

If we now change in this identity the indices i, j to k, l, and substract the new identity from the above, we have

$$\hat{R}_{ijkl}-\hat{R}_{klij}=2T_{i[kl]j}-i|j. \tag{4.9}$$

This means that (4.8) is true if and only if

$$T_{i[kl]j}-i|j=0. \tag{4.10}$$

Let us construct the tensor

$$^{*}R_{ijkl} \overset{\text{def}}{=\!=} \hat{R}_{ijkl}-(T_{i[kl]j}-i|j). \tag{4.11}$$

According to the relation

$$T_{i[kl]j}-i|j=T_{k[ij]l}-k/l$$

$^{*}R_{ijkl}$ satisfies the equations

$$^{*}R_{ijkl}={}^{*}R_{klij};\quad {}^{*}R_{ijkl}=-{}^{*}R_{jikl}.$$

We want to show some special cases in which (4.10) is satisfies.

Theorem 9. T_{ijkl} *satisfies* (4.10) *if:*

a) $\gamma_k=0$, *or* b) $P^i_j=\varrho\delta^i_j$ *and* γ_k *is a gradient vector.*

Indeed, in the case a) we have $''\Gamma_j{}^i{}_k=\{{}_j{}^i{}_k\}$, and by the covariant part of the regular general connection Γ a Riemannian space is determined, and W—O$_n$ reduces to a *Riemann—Otsuki* space R—O$_n$.

In the case b) we have $\delta^i_{j|k}=0$, and from (4.6) it follows that (4.10) is satisfied. In this case W—O$_n$ reduces to a *generalised Weyl* space. From (4.5) and (4.11) we get

$$''R_{ijkl}=-''R_{jikl}=''R_{klij}.$$

From this relation, (3.13), and from the fact that $M_i{}^s{}_{kl}g_{sj}=M_{ijkl}$ is the torsion of the curvature tensor of $''\Gamma$, we obtain

$$M_{ijkl}=-M_{jikl}=M_{klij}.$$

From this and (3.14) it is obvious that the tensor M_{ijkl} satisfies the Ricci identity for every three of the four indices $ijkl$.

§ 5. W—O$_n$ *spaces of scalar curvature*

Let us consider a fixed point P of the space W—O$_n$ and two arbitrary linearly independent vectors X, Y in the same space. These vectors span up a two-dimensional linear vector manifold $T_2(P)$ at P. The scalar defined by

$$\hat{R}(x, X, Y)=\frac{\hat{R}_{ijkl}X^iX^kY^jY^l}{(g_{ik}g_{jl}-g_{il}g_{jk})X^iX^kY^jY^l} \tag{5.1}$$

is called the *sectional curvature* of W—O_n at P and $T_2(P)$ determined by X, Y.

We prove that $\hat{R}(x, X, Y)$ depends only on $T_2(P)$ and not on the vectors which span it (cf. [4] chap. 7, § 3. resp. (3.41)). Let us introduce the notations:

$$Z^A = X^{[i} Y^{j]}; \quad \hat{R}_{AB} = \hat{R}_{ijkl}; \quad G_{AB} = g_{ik} g_{jl} - g_{il} g_{jk} \tag{5.2}$$

$$A, B = 1, 2, \ldots, \frac{1}{2} n(n-1); \quad i, j = 1, 2, \ldots, n.$$

Then we have

$$\hat{R}(x, Z) = \frac{\hat{R}_{(AB)} Z^A Z^B}{G_{AB} Z^A Z^B}. \tag{5.3}$$

Definition 1. The W—O_n space is a space of scalar curvature of first order if, $\hat{R}(x, Z)$ is independent of Z^A.

We shall calculate $\hat{R}_{(AB)}(x, Z)$ in W—O_n spaces of scalar curvature of first order. From the definition we have $\hat{R}(x, Z) = \hat{R}(x)$ and from (5.3) it follows that

$$(\hat{R}(x) G_{AB} - \hat{R}_{(AB)}) Z^A Z^B = 0 \tag{5.4}$$

for all pairs of vectors of $T_2(P)$. Using (5.2) we get

$$\hat{R}_{ijkl} - \hat{R}_{klij} = 2\hat{R}(x)(g_{ik} g_{jl} - g_{il} g_{jk}). \tag{5.5}$$

Let us replace in the above consideration the tensor $\hat{R}_{ijkl}$ with $^*R_{ijkl}$ which has the same symmetry and skew-symmetry properties as the curvature tensor of a Riemannian metric, i.e.

$$^*R_{(ij)kl} = 0; \quad ^*R_{ij(kl)} = 0; \quad ^*R_{ijkl} = {}^*R_{klij}. \tag{5.6}$$

Then in place of $\hat{R}(x, X, Y)$ we have the scalar

$$^*R(x, X, Y) = \frac{^*R_{ijkl} X^i X^k Y^j Y^l}{(g_{ik} g_{jl} - g_{il} g_{jk}) X^i X^k Y^j Y^l} \tag{5.7}$$

and we give the following definition:

Definition 2. A W—O_n space is of second order scalar curvature if $^*R(x, X, Y)$ given by (5.7) is independent of the vectors X and Y.

Using (5.6) and the above definition one can prove in the same way, as in the Riemannian geometry, that if the W—O_n is of scalar curvature of second order, then (cf. [3] § 118)

$$^*R_{ijkl} = {}^*R(x)(g_{ik} g_{jl} - g_{il} g_{jk}) \tag{5.8}$$

or

$$(5.9)\qquad {}^*R_i{}^j{}_{kl} = {}^*R(x)(g_{ik}\delta^j_l - g_{il}\delta^j_k).$$

The scalar ${}^*R(x)$ is given by

$$(5.10)\qquad {}^*R(x) = \frac{1}{n(n-1)}\,{}^*R^{kl}{}_{kl}.$$

From the relations (5.7) and (5.8) follows that

Theorem 10. (5.8) *or* (5.9) *is a necessary and sufficient condition for a* W—O$_n$ *to be a space of scalar curvature of second order.*

In the following we shall give the relation between the tensors ${}^*R_{ijkl}$ and R_{ijkl}. From (4.11), (4.5) and (3.13) it follows that

$$(5.11)\qquad {}^*R_{ijkl} = R_{ijkl} - M_{ijkl} + T_{ijkl} - (T_{i[kl]j} - i\,|\,j).$$

From (5.11) and (3.13) follows

$$''R_{ijkl} = {}^*R_{ijkl} - T_{ijkl} + (T_{i[kl]j} - i\,|\,j)$$

or in other form

$$(5.12)\qquad ''R_i{}^j{}_{kl} = {}^*R_i{}^j{}_{kl} - V_i{}^j{}_{kl}$$

where

$$(5.13)\qquad V_i{}^j{}_{kl} \overset{\text{def}}{=} g^{js}(T_{iskl} - (T_{i[kl]s} - i|s)).$$

Let us now derive (5.12) with respect to the regular general connection Γ, and apply (3.8) in the obtained derivative. Using the fact that the tensor $''R_i{}^j{}_{kl}$ satisfies the classical Bianchi identity, we get

$$(5.14)\qquad {}^*R_i{}^j{}_{kl|r} - \delta^j_{t|r}\,''R_i{}^t{}_{kl} - V_i{}^j{}_{kl|r} + \{klr\} = 0.$$

Substituting $''R_i{}^j{}_{kl}$ from (5.12) into (5.14) we arrive to the relation

$$(5.15)\qquad {}^*R_i{}^j{}_{kl|r} - \delta^j_{t|r}\,{}^*R_i{}^t{}_{kl} - \delta^j_{t|r}V_i{}^t{}_{kl} + V_i{}^j{}_{kl|r} = 0.$$

(5.15) *is the Bianchi identity of the curvature tensor with respect to the basic covariant derivation.*

As the well known Schur's theorem tells us, a Riemannian space is of constant curvature iff it is of scalar curvature. In the remaining part of this paper we investigate W—O$_n$ spaces of second order scalar curvature. We show that the analogon of Schur's theorem does not hold in general in these spaces, and we find those spaces in which the analogon of Schur's theorem holds.

Since our space now is of second order scalar curvature, so we have (5.9). Substituting (5.12) into (5.15) we obtain

$$\begin{aligned}{}^*R_{|r}(g_{ik}\delta^j_l - g_{il}\delta^j_k) + {}^*R(g_{ik|r}\delta^j_l - g_{il|r}\delta^j_k - \\ - \delta^j_{t|r}V_i{}^t{}_{kl} + V_i{}^j{}_{kl|r} + \{klr\} = 0.\end{aligned}$$

For $j=l$ we have

$$^*R_{|r}\,g_{ik}(n-2)+{}^*Rg_{ik|r}(n-2)+{}^*R_{|k}\,g_{ir}(2-n)+{}^*Rg_{ir|k}(2-n)-$$
$$-\delta^j_{t|r}V^t_{i\,kj}-\delta^j_{t|k}V^t_{i\,jr}-\delta^j_{t|j}V^t_{i\,rk}+V_i{}^j{}_{[kj|r]}=0.$$

Multiplication with g^{ik} gives

$$(5.16)\qquad {}^*R_{|r}(n-1)(n-2)+{}^*R(g_{ik|r}-g_{ir|k})g^{ik}(n-2)-$$
$$-\delta^j_{t|r}V^{kt}{}_{kj}-\delta^j_{t|k}V^{kt}{}_{jr}-\delta^j_{t|j}V^{kt}{}_{rk}+V^{kj}{}_{[kj|r]}=0.$$

From the relation $P^s_iP^r_jg_{sr|k}=\nabla_kg_{ij}=\gamma_kg_{ij}$ it follows that

$$(5.17)\qquad g_{sr|k}=\gamma_kg_{ij}Q^i_sQ^j_r=\gamma_km_{sr}.$$

Substituting (5.17) into (5.16) we obviously get

$$(5.18)\qquad (n-2)(n-1)\,{}^*R_{|r}+{}^*R(\gamma_rQ^s_tQ^t_s-\gamma_kQ^s_rQ^k_s)(n-2)-$$
$$-\delta^j_{t|r}V^{kt}{}_{kj}-\delta^j_{t|k}V^{kt}{}_{jr}-\delta^j_{t|j}V^{kt}{}_{rk}+V^{kj}{}_{[kj|r]}=0.$$

And from this

$$(5.19)\qquad (n-2)(n-1)^*R_{|r}={}^*R(\gamma_rQ^s_tQ^t_s-\gamma_kQ^s_rQ^k_s)(2-n)+\delta^j_{t|r}V^{kt}{}_{kj}+$$
$$+\delta^j_{t|k}V^{kt}{}_{jr}+\delta^j_{t|j}V^{kt}{}_{rk}-V^{kj}{}_{[kj|r]}.$$

Thus ${}^*R_{|r}=\dfrac{\partial R}{\partial x^r}$ does not vanish in general. This shows that the investigated W—O$_n$ with second order scalar curvature may be of scalar but not of constant curvature, and so Schur's theorem does not hold in general. Thus we have

Theorem 11. *The vanishing of the right hand side of* (5.19) *is the condition of the validity of Schur's theorem.*

It means, that Schur's theorem holds in those W—O$_n$ with second order scalar curvature in which the right side of (5.19) vanishes. We show that in Riemann—Otsuki spaces with second order scalar curvature Schur's theorem holds. These R—O$_n$ spaces are those W—O$_n$ in which $\gamma_k=0$ (see theorem 9). From (5.19) it follows immediately

Corollary 5. *If* $\gamma_k=0$ *and* $n>2$, *then*

$${}^*R_{|r}=0,$$

i.e, ${}^*R(x)=$const., *and the* R—O$_n$ *space with second order scalar curvature is of constant curvature.*

Another consequence of (5.19) is .

Theorem 12. *If* $P^i_j=\varrho\delta^i_j$, $\varrho=$const$\neq0$, γ_k *is a gradient vector and* $n>2$, *then the scalar* ${}^*R(x)$ *is the solution of the differential equation*

$${}^*R_{|r}=-{}^*R(x)\gamma_r(x)\varrho^{-2}.$$

PROOF. From case b) of Theorem 9 it follows that $T_{ijkl}=0$, and then (5.18) with respect to (5.13) has the form

$$^*R_{|r}+{}^*R\gamma_r\varrho^{-2}=0.$$

It is easy to see the

Corollary 6. *If in theorem* 12. $\varrho=1$, *then the space* W—O$_n$ *reduces to the classical Weyl space and* $\gamma_r=\partial_r \ln {}^*R^{-1}$.

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