On logarithmic property of a degree of finite group

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№ 1. A minimal degree $n(\Gamma)$ of a permutation group which is isomorphic to a given finite group Γ is called the degree of Γ. The question of finding $n(\Gamma)$ was posed in the most general setting by 0. YU. SCHMIDT (1, p. 89). For a long time there existed a hypothesis that the degree $n(\Gamma)$ has *"a logarithmic property" ("l-property"): $n(A\times B)=n(A)+n(B)$*. This property was proved in the following cases: 1) A and B are Abelian (2—5); 2) The orders of the groups A and B are doprime (5); 3) A and B are completely reducible (6).

In the present paper the above-mentioned hypothesis is disproved for a class of solvable groups. In our counterexample A is solvable and B is Abelian. Besides we prove the "l-property" for a class of nilpotent groups answering a question posed by JOHNSON (5). This result follows from some sufficient condition for the validity of the "l-property" which is derived here and includes all above-mentioned and some new cases.

№ 2. *Counterexample.* Let us consider the regular representation of the cyclic groups $\mathbf{Z}_2$ and $\mathbf{Z}_5$. Let $\Gamma=\mathbf{Z}_2 \wr \mathbf{Z}_5$ be a wreath product of the corresponding permutation groups. The group Γ is injected in the symmetric group σ_{10} and hence $n(\Gamma)\leqq 10$. Besides it is a semi-direct product of its normal subgroups $(\mathbf{Z}_2)^5$ and $\mathbf{Z}_5$ (every element of Γ is uniquely representable as $\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5\beta$ where $\alpha_i\in\mathbf{Z}_2$, $\beta\in\mathbf{Z}_5$; the set of elements with $\beta=1$ is a normal subgroup isomorphic to $(\mathbf{Z}_2)^5$; the set of elements with $\alpha_i=1$ $(i=1, 2, \ldots, 5)$ is a subgroup which is isomorphic to $\mathbf{Z}_5$). Let B be the center of the group Γ. The group B is isomorphic to $\mathbf{Z}_2$ and consists of a unit element e and a "diagonal" element $z=\alpha_1\cdot\ldots\cdot\alpha_5$ where $\alpha_i\in\mathbf{Z}_2$ are not equal to e. Let A be the intersection of Γ with an alternating group. The group A consists of such products which contain even number of elements $\alpha_i\neq e$. Hence A is isomorphic to the semi-direct product of the normal subgroup $N\approx(\mathbf{Z}_2)^4$ and of $\mathbf{Z}_5$. It is obvious that A and B are normal subgroups in Γ. As z is an odd permutation, $A\cap B=E$. Besides, as $\Gamma/A\approx\mathbf{Z}_2\approx B$, we have $AB=\Gamma$. Thus Γ is a direct product of A and B.

Let us show that $n(\Gamma)<n(A)+n(B)$. If this is not true, then $n(A)\leqq n(\Gamma)-n(B)\leqq 8$. Hence A permits a faithful representation T in the group σ_8. But $n(N)=n(\mathbf{Z}_2^4)=8$. Thus the representation $T|N$ is minimal and has no fixed points. It follows that a number of orbits of this representation is less than 4. Furthermore, as N is a normal subgroup of A, the representation T is inducing an action of $\mathbf{Z}_5\subset A$ on the orbit set of the respresentation $T|N$. As $n(\mathbf{Z}_5)=5$ and $\mathbf{Z}_5$ is a simple group, this action is trivial, i.e. $\mathbf{Z}_5$ is invariant on all orbits.

The representation $T|N$ is intransitive (the only faithful transitive representation of an Abelian group is a regular one). As the number of orbit elements divides the order of N which is equal to 16, every orbit contains not more than 4 elements. If follows from this that $\mathbf{Z}_5$ acts trivially on every orbit and hence $\mathbf{Z}_5 \subset \operatorname{Ker} T$ in contradiction with the faithfulness of T.

№ 3. *Notation:* $\Gamma = A \times B$; $p_A: \Gamma \to A$, $p_B: \Gamma \to B$ are natural projections; $T: \Gamma \to \sigma(\Omega)$ is a representation of Γ in the permutation group on Ω; O is an arbitrary orbit for T. T_0 is a restriction of T on O; T' is a restriction of T on $\Omega \setminus O$; $S_A = \operatorname{Ker}(T_0|A)$, $S_B = \operatorname{Ker}(T_0|B)$.

A faithful representation T is said to be minimal if $|\Omega| = n(\Gamma)$. We call a minimal representation completely minimal if the number of its orbits is maximal. The number of orbits of a completely minimal representation will be denoted by $m(\Gamma)$. In what follows we shall assume T to be completely minimal.

Proposition 1. *At least one of the representations $T_0|A$, $T_0|B$ is transitive.*

PROOF. Let proposition 1 be not true and let O_A be an orbit of $T_0|A$, O_B be an orbit of $T_0|A$. Let us consider the representation $T_1: \Gamma \to \sigma(O_A)$; $T_1 = R_A p_A$, R_A being the restriction of $T_0|A$ on O_A. In the same way $T_2 = R_B p_B$. Consider the direct sum $\tilde{T} = T_1 \dotplus T_2 \dotplus T'$ (i.e. $\tilde{T}$ acts on the disjoint union V of the sets O_A, O_B and $\Omega \setminus O$ where the restriction of $\tilde{T}$ on O_A coincides with T_1 the restriction of $\tilde{T}$ on O_B coincides with T_2 and the restriction of $\tilde{T}$ on $\Omega \setminus O$ coincides with T').

We shall show that $\operatorname{Ker} R_A = S_A$. Obviously, $S_A \subset \operatorname{Ker} R_A$. On the contrary, as $\operatorname{Ker} R_A$ is a normal subgroup of Γ, a set of the fixed points of $\operatorname{Ker} R_A$ is invariant with respect to the representation T. This set contains O_A. But O is the smallest invariant set containing O_A. Hence all points of O are fixed under the action of $\operatorname{Ker} R_A$ and it follows that $\operatorname{Ker} R_A \subset S_A$. Thus $\operatorname{Ker} T_1 = p_A^{-1} \operatorname{Ker} R_A = \operatorname{Ker} R_A \times B = S_A \times B$. In the same way $\operatorname{Ker} T_2 = A \times S_B$. Now $\operatorname{Ker} \tilde{T} = \operatorname{Ker} T_1 \cap \operatorname{Ker} T_2 \cap \operatorname{Ker} T' = (S_A \times S_B) \cap \operatorname{Ker} T' \subset \operatorname{Ker} T_0 \cap \operatorname{Ker} T' = \operatorname{Ker} T$. As the representation T is faithful, $\tilde{T}$ is faithful as well.

Now we shall show that $|V| \leqq |\Omega|$. To prove this let us consider the set W of orbits of Γ contained in O. As A is a normal subgroup in Γ, T is inducing a transitive action of Γ on W. Hence $|O_A||W| = |O|$. As $O_A \neq O$, we have $|O_A| \leqq \frac{|O|}{2}$. Analogously $|O_B| \leqq \frac{|O|}{2}$. Therefore, $|V| = |O_A| + |O_B + |\Omega \setminus O| \leqq |O| + |\Omega \setminus O| = |\Omega|$.

We have already proved that $\tilde{T}$ is a minimal representation of Γ. But the number of orbits of $\tilde{T}$ exceeds that of T by one.

Remark. We have assumed above that if T is transitive, then $T_1 \dotplus T_2 \dotplus T' \equiv T_1 \dotplus T_2$; $\operatorname{Ker} T' \equiv \Gamma$. In what follows this agreement will be observed without comment.

From now on we assume $T_0|A$ to be transitive. Let us consider an action of B/S_B on O induced by $T_0|B$. It is obvious that the orbits of this action coincide with the orbits of B which lay in O.

Proposition 2. *The action of B/S_B is semiregular, i.e. regular on each orbit.*

PROOF. Let $x \in O$ be a fixed point for $b \in B$. As b commutes with A, the set of fixed points for b is invariant with respect to the action of A. As A acts transitively on O, b acts trivially on O, i.e. $b\in S_B$. So the stabilizer of any point $x\in O$ in B is equal to S_B and hence the stabilizer of $x\in O$ in B/S_B is trivial.

Now note that T induces the action of A/S_A on the set W of orbits of B which lay in O. Let K be the number of such orbits and let N be a Kernel of the induced action (i.e. N consists of such elements of A/S_A which are invariant on all above--mentioned orbits of B).

Proposition 3. *The group N is injectable in the group $(B/S_B)^K$.*

PROOF. Let O_j $(j=1, \dots, K)$ be orbits of B on O. Choose $x_j\in O_j$ and suppose $\alpha\in N$. As α is invariant on O_j, there exists $\beta_j\in B/S_B$ such that $\alpha x_j=\beta_j^{-1}x_j$ $(j=1, \dots, K)$. The elements β_j are uniquely defined as the action of B/S_B is semiregular. Let us consider a homomorphism $i\colon N\to(B/S_B)^K$ such that $\alpha\to(\beta_1, \dots, \beta_K)$ and show that i is an injection. To prove this let $\beta_j=1$ $(j=1, \dots, K)$, y be any element from the orbit O_j and γ be an element from B/S_B mapping x_j into y. Then $\alpha y=\alpha\gamma x_j=\gamma\beta_j^{-1}x_j=\gamma x_j=y$. As j is arbitrary, all elements from O are fixed under α. The action of N on O being faithful, it implies $\alpha=1$.

Let $j\colon \Gamma\to\Gamma/\text{Ker } T_0$ be a natural homomorphism. The group A/S_A permutes a natural injection into $j\Gamma$: $A/S_A\approx jA\approx j\Gamma$.

Proposition 4. *Let Z be the center of A/S_A. Then $N\cap Z\subset jB$.*

PROOF. Let $\zeta\in N\cap Z$. Using notation from proposition 2 we have $\zeta x_1=\beta x_1$ (where $x_1\in O_1$, $\beta\in B/S_B$). Let $y\in 0$. As A/S_A acts on O transitively, there exists $\alpha\in A/S_A$ such that $y=\alpha x_1$. Hence $\zeta y=\zeta\alpha x_1=\alpha\zeta x_1=\alpha\beta x_1=\beta\alpha x_1=\beta y$ (we used commutativeness of α with ζ as $\zeta\in Z$ and with β as jA commutes with jB). Thus ζ acts on 0 in the same way as $\beta=jb$ $(b\in B)$, which is equivalent to $\zeta=jb\in jB$.

Proposition 5. *Let $T_0|B$ be intransitive and nontrivial. Then $j(\text{Ker } T')\cap\cap N\neq E$.*

PROOF. Let us consider the following representations: $T_1=\varrho_A p_A$, ϱ_A being the induced action of A on the set W of orbits of B contained in O; $T_2=R_B p_B$, R_B being the action of B on one of these orbits O_B (T_2 has been already considered in Proposition 1). Put $\tilde{T}=T_1\dotplus T_2\dotplus T'$; $\tilde{T}$ acts on the set $V=W\cup O\cup(\Omega\setminus O)$.

Now, assuming that $j(\text{Ker } T')\cap N=E$, we shall obtain a contradiction with the complete minimality of T. Indeed, $\text{Ker } T_1=p_A^{-1}(\text{Ker } \varrho_A)=\text{Ker } \varrho_A\times B=(\eta^{-1}N)\times B$ where $\eta\colon A\to A/S_A$ is a natural homomorphism. While proving Proposition 1 we have shown that $\text{Ker } T_2=A\times S_B$. Hence, $\text{Ker } \tilde{T}=\text{Ker } T_1\cap\text{Ker } T_2\cap\text{Ker } T'=$ $=[(\eta^{-1}N)\times S_B]\cap\text{Ker } T'$. Let L be the group $(\eta^{-1}N)\times S_B]$. We have $j(\text{Ker } \tilde{T})\subset$ $\subset jL\cap j(\text{Ker } T')=N\cap j(\text{Ker } T')=E$ by supposition. It follows now that $\text{Ker } \tilde{T}\subset$ $\subset\text{Ker } T_0\cap\text{Ker } T'=E$, and $\tilde{T}$ is a faithful representation.

Now we shall prove that $|V|\leqq|\Omega|$. First we have $|W|\cdot|O_B|=|O|$. As B acts on O nontrivially, $|W|\leqq\frac{|O|}{2}$. As B acts on O intransitively, $|O_B|\leqq\frac{|O|}{2}$. Finally, $|V|=|W|+|O_B|+|\Omega\setminus O|\leqq|O|+|\Omega\setminus O|^2=|\Omega|$.

We proved that $\tilde{T}$ is minimal. But $\tilde{T}$ has one orbit more than T which leads to a contradiction.

Corollary. *If B acts nontrivially on O, then $N\neq E$.*

PROOF. If B acts intransitively on O, then even $N \cap j(\mathrm{Ker}\, T') \neq E$. If B is transitive, then $N = A/S_A \neq E$ (as A acts transitively on O too and a minimal representation has no one-element orbits).

The following proposition is of a key significance.

Proposition 6. *If $T_0|B$ is non-trivial, then B/S_B is isomorphic to the cyclic group $\mathbf{Z}_{p^s}$ or to the generalized group of quaternious Q_{2^r}.*

PROOF. It is sufficient to prove that a minimal subgroup of B/S_B is unique. If this is not true, let us consider two cases.

1) Let all minimal subgroups of B/S_B be invariant; H_1 and H_2 are two such subgroups. As H_i is a normal subgroup in the whole group $j\Gamma$, T induces a representation T_i of Γ on the set W_i where W_i is the set of orbits of H_i contained in O (by the way, H_i acts only on O). Let us show that the representation $\tilde{T} = T_1 \dotplus T_2 \dotplus T'$ on the set $V = W_1 \cup W_2 \cup (\Omega \setminus O)$ is minimal.

Let O_1 and O_2 be some orbits of H_1 and H_2. Then the intersection $O_1 \cap O_2$ contains no more than one point. Really, if $x \in O$ then $H_1 x \cap H_2 x = x$ as the action of B/S_B is semiregular and $H_1 \cap H_2 = E$ (H_1 and H_2 are different minimal subgroups in B/S_B).

Furthermore, $\mathrm{Ker}\, T_1 \cap \mathrm{Ker}\, T_2 = \mathrm{Ker}\, T_0$. An inclusion $\mathrm{Ker}\, T_0 \subset \mathrm{Ker}\, T_1 \cap \cap \mathrm{Ker}\, T_2$ is obvious. On the other hand, if $\gamma \in \mathrm{Ker}\, T_1 \cap \mathrm{Ker}\, T_2$, then γ is invariant on all orbits of H_1 and H_2 and hence on all pairwise intersections of these orbits. But, as it was proved above, these pairwise intersections divide 0 in one element sets. Hence $\gamma \in \mathrm{Ker}\, T_0$.

Now the faithfulness of $\tilde{T}$ is obvious: $\mathrm{Ker}\, \tilde{T} = \mathrm{Ker}\, T_1 \cap \mathrm{Ker}\, T_2 \cap \mathrm{Ker} T' = = \mathrm{Ker}\, T_0 \cap \mathrm{Ker}\, T' = \mathrm{Ker}\, T = E$.

The inequality $|V| \leqq |W|$ follows from $|O| = |W_i| \cdot |H_i|$ (as B/S_B acts semiregularly). As $\tilde{T}$ contains more orbits than T (one orbit more), the case 1 is ruled out.

2) Now let B/S_B contain a noninvariant subgroup H. Put $T_1 = r_A p_A$, r_A being an action of A on the space W of orbits of H (contained in O); $T_2 = r_B p_B$, r_B being an action of the group B on the homogeneous space $U = (B/S_B)/H$ (r_B is not induced by T as against all preceding developments). Construct the representation $\tilde{T} = T_1 \dotplus T_2 \dotplus T'$ as previously. Let us prove the faithfulness of $\tilde{T}$.

We shall prove that $\mathrm{Ker}\, r_A = S_A$. Really, let $a \in \mathrm{Ker}\, r_A$. It means that a is invariant on all orbits of H. So there exists $\beta \in H$ such that $ax_0 = \beta x_0$, $x_0 \in O$. Let now x be an arbitrary point from the orbit Bx_0. Then $x = \gamma_x x_0$ where γ_x is a uniquely defined element of B/S_B. But then $ax = a(\gamma_x x_0) = \gamma_x (ax_0) = \gamma_x \beta x_0 = (\gamma_x \beta \gamma_x^{-1}) x$. As a is invariant on Hx, there exists $\beta_1 \in H$ such that $\beta_1 x = ax = (\gamma_x \beta \gamma_x^{-1}) x$. At last as the action of B/S_B is semiregular, $\gamma_x \beta \gamma_x^{-1} = \beta_1 \in H$. If x runs through the orbit Bx_0, then γ_x runs through the whole group B/S_B. It follows now that all elements conjugated with $\beta \in H$ belong to H. But as H is the minimal and noninvariant subgroup, $\beta = 1$. Hence $ax_0 = \beta x_0$. As x_0 is an arbitrary element of O, we have $a \in S_A$. This proves that $\mathrm{Ker}\, R_A \subset S_A$. The inverse inclusion is obvious.

Furthermore, $\mathrm{Ker}\, r_B = S_B$ as H does not contain the normal subgroups of B/S_B and hence the action of B/S_B on the homogeneous space $(B/S_B)/H$ is faithful. Now the proof of faithfulness of $\tilde{T}$ is the same as in Proposition 1.

Minimality of $\tilde{T}$ follows from $|U|\cdot|H|=|B/S_B|\leqq|O|$ and $|W||H|=|O|$. Besides, the number of orbits of $\tilde{T}$ is greater by one than that of T. This gives a contradiction, q.e.d.

№ 4. A representation of the group $\Gamma=A\times B$ will be said to have a canonical structure if either A or B acts trivially on every orbit of Γ. If a completely minimal representation of Γ is canonical, then $n(\Gamma)=n(A)+n(B)$, $m(\Gamma)=m(A)+m(B)$.

The principal lemma. *Let the following condition be satisfied for any pair of homomorphic images $\bar{A}$, $\bar{B}$ of the groups A and B:*

(c) *If* 1) *either $\bar{A}$ or $\bar{B}$ (for example $\bar{B}$) is isomorphic to $\mathbf{Z}_{p^s}$ or Q_{2^r} and* 2) *$N\neq E$ is an arbitrary normal subgroup of $\bar{A}$ permitting injection into $\bar{B}^k$ then every normal subgroup $L\neq E$ contained in N has a nontrivial intersection with the center Z of $\bar{A}$.*

Then the "l-property" is true for the degree and the number of orbits: $n(A\times B)=$ $=n(A)+n(B)$; $m(A\times B)=m(A)+m(B)$.

If there are no $\bar{A}$, $\bar{B}$ satisfying the suppositions of condition (c), *then any completely minimal representation T of $A\times B$ has the canonical structure.*

Proof. We shall construct the faithful representation $\hat{T}\colon \Gamma\to\sigma(\Omega)$ with the properties 1) $\hat{T}$ is invariant on O and $\Omega\setminus O$ 2) $\hat{T}_0$ is transitive; 3) $\hat{T}'=T'$; 4) either A or B are in $\operatorname{Ker}\hat{T}_0$.

If B acts on O trivially, let $\hat{T}=T$. If not, $B/S_B\approx\mathbf{Z}_{p^s}$ or Q_{2^r} (Proposition 6) and A/S_A contains a normal subgroup N as described in Proposition 3 which is injectable in $(B/S_B)^k$. By the corollary from Proposition 5, $N\neq E$. Thus the suppositions of condition (c) are satisfied for quotient groups A/S_A, B/S_B.

Hence $N\cap Z\neq E$. But $N\cap B\subset jB$ (Proposition 4) and thus $N\cap Z\subset jB\cap jA$. It follows now that j is non-injective, i.e. T_0 is not a faithful representation. Thus T is non-transitive.

Furthermore, let us consider the commutative diagram

$$\begin{array}{ccc} \Gamma & \xrightarrow{\ j\ } & \Gamma/\operatorname{Ker} T_0 \\ {\scriptstyle\varphi}\downarrow & & \uparrow{\scriptstyle\psi} \\ & (A/S_A)\times(B/S_B) & \end{array}$$

where φ and ψ are the natural homomorphisms. As $S_A=A\cap\operatorname{Ker} j$, $\Theta=\psi|(A/S_A)$ is a monomorphism. Let us denote $\operatorname{Ker} T'$ by G. To avoid unconvenient notations let us introduce identifications by isomorphisms: $N\approx\Theta^{-1}N$, $Z\approx\Theta^{-1}Z$. Let us show that $\varphi G\cap\varphi B=E$ or $\varphi G\cap N=E$. Otherwise $\varphi G\cap\varphi B\supset H_2$, H_2 being the only minimal subgroup in φB. On the other hand, by the condition (c) $L=\varphi G\cap$ $\cap N\cap Z\neq E$. But $\psi L\subset jG\cap N\cap Z\subset jB$ (Proposition 4) and hence $\psi H_2\subset\psi L$. Define $H_1\subset L$ by the condition $\psi H_1=\psi H_2$. Then $\varphi G\supset H_1\times H_2$ in contradiction with the injectivity of $\psi|\varphi G$.

Hence, if $T_0|B$ is not transitive, then $\varphi G\cap\varphi B=E$ (Proposition 5). If $T_0|B$ is transitive, then $N=\varphi A$. In both cases $\varphi G\cap\varphi B=E$ or $\varphi G\cap\varphi A=E$. To be definite, let $\varphi G\cap\varphi B=E$. Then $\hat{T}=(T_0|A)p_A\dotplus T'$ is a faithful representation with the properties 1)—4). These properties are obvious. Let us check the faithfulness. As $\operatorname{Ker}(T_0|A)=S_A$, we have $\operatorname{Ker}\hat{T}=(S_A\times B)\cap G$ and $\varphi(\operatorname{Ker}\hat{T})\subset$ $\subset\varphi B\cap\varphi G=E$. Thus $\operatorname{Ker}\hat{T}\subset\operatorname{Ker}\varphi=S_A\times S_B$. Besides, $\operatorname{Ker}\hat{T}\subset G$. It follows now that $\operatorname{Ker}\hat{T}=E$.

The consecutive application of this construction to all orbits turns T into the representation of canonical structure with the same space Ω and the same space of orbits. This proves the "l-property".

We proved above that if A acts transitively on O and B acts nontrivially on O, then the suppositions of the condition (c) are satisfied for the quotient groups A/S_A and B/S_B. Hence if they are not satisfied for any homomorphic images $\bar{A}$ and $\bar{B}$, then either A or B acts trivially, i.e. T has the canonical structure.

№ 5. The principal lemma implies the following.

Theorem. *The logarithmic property for the degree* $(n(A\times B)=n(A)+n(B))$ *and for the number of orbits of completely minimal representation* $(m(A\times B)=m(A)+m(B))$ *is true in the following cases:*

1) A and B are nilpotent groups; 2) there are no groups $\mathbf{Z}_{p^s}$ *and* Q_{2^r} *among the homomorphic images of A and B; 3) A and B are completely reducible* (cf. (6)); *4) A is completely reducible, B is nilpotent; 5) the orders of A and B are coprime* (cf. (5)).

Every completely minimal representation of $A\times B$ *is of the canonical structure in the cases 2) and 5).*

PROOF. In the case 2) the first supposition of condition (c) is not satisfied. In the case 5) it may be satisfied but the second supposition is ruled out (as any prime number dividing the order of N is the common divisor of the orders of A and B).

In a nilpotent group any nontrivial normal subgroup has non-void intersection with the center. In a completely reducible group any normal subgroup which is injectable in $(\mathbf{Z}_{p^s})^k$ or $(Q_{2^r})^k$ is Abelian and, being complementable, is central. As transition to the homomorphic images preserves nilpotentness and complete reducibility, condition (c) is satisfied for the pair of groups A and B if A and B are independently nilpotent or completely reducible. This proves the theorem in the cases 1), 3), 4).

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