Inverse limits of the Cantor-manifolds

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ABSTRACT. We say that a compact space X is (n, k)-Cantor-manifold if $\dim X=n$, and if for every closed subset $F\subseteqq X$ of the dimension $\dim F\leqq n-k$ the set $X\setminus F$ is connected.

In the present paper we investigate the following question: Under what conditions the limit of the inverse system of the n-dimensional Cantor-manifolds is (n, k)-Cantor-manifold?

The partial answers for the inverse systems with fully closed monotone bonding mappings and for inverse systems of metric Cantor-manifolds with open bonding mappings are given.

0. Introduction

All spaces in this paper are assumed to be Hausdorff, and this assumption will be used without explicit mention.

We say that a compact space X is an ind (Ind, dim)-n-dimensional Cantor-manifold if $\operatorname{ind} X$ ($\operatorname{Ind} X$, $\dim X$)$=n\geqq 1$ such that no closed subset $F\subseteqq X$ satisfying the inequality $\operatorname{ind} F$($\operatorname{Ind} F$, $\dim F$)$\leqq n-2$ separates the space X i.e. for every such set the complement $X\setminus F$ is connected |7; 91|.

If we omit the assumption that X is a compact space, we obtain the definition of a generalized Cantor-manifold.

The cardinality of the set A is denoted by $|A|$. The symbol $\operatorname{cf}(A)$ means the cofinality of the well-ordered set A i.e. the smallest ordinal number which is cofinal in A.

If $f\colon X\to Y$ is a mapping, then $f^{\#} A$ is the set $\{y: f^{-1}(y(\subseteqq A\}$ for $A\subseteqq X$.

1. Inverse limits of k-dimensional cantor-manifolds

A compact space X is (n, k)-Cantor-manifold if $\dim X=n$ and if $X\setminus F$ is connected for each closed $F\subseteqq X$ with $\dim F\leqq n-k$.

A k-dimensional Cantor-manifold is $(k, 2)$-Cantor-manifold.

We start with the next theorem

1.1. Theorem. *Let $\underline{X}=\{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of k-dimensional Cantor-manifolds X_α. If $\underline{X}$ satisfies the property:*

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(P1) For every closed $(n-k)$-dimensional subset $F \subseteq \varprojlim X$ the set $f_\alpha(F)$, $\alpha \in A$, is $\leqq (k-2)$-dimensional,
(P2) For every open connected $Y_\alpha \subseteq X_\alpha$, $\alpha \in A$, the set $f_\alpha^{-1}(Y_\alpha)$ is connected, then $X = \varprojlim X$ is (n, k)-Cantor-manifold.

PROOF. Let F be a closed subset of X of the dimension $\leqq n-k$. By (P1) we have $\dim f_\alpha(F) \leqq k-2$. This means that $Y_\alpha = X_\alpha \setminus f_\alpha(F)$ is open and connected. From (P2) it follows that $f_\alpha^{-1}(Y_\alpha)$ is connected. Since $X \setminus F = \cup \{Y_\alpha : \alpha \in A\}$ it follows that $X \setminus F$ is connected |6: 435.|. Q.E.D.

1.2. Remark. The property (P2) is satisfied if $f_{\alpha\beta}$ are monotone or open mappings. This follows from |6: 6.1.28. Theorem| and from the fact that if X is an inverse system of connected spaces and open-closed projections, then $\lim \underline{X}$ is connected.

We say that a mapping $f: X \to Y$ is fully closed |8| if for every point $y \in Y$ and for each finite cover $\{U_1, U_2, ..., U_n\}$ of $f^{-1}(y)$ the set $\{y\} \cup (f^{\#} U_1 \cup ... \cup f^{\#} U_n)$ is open |8|.

If $f: X \to Y$ is fully closed then f is closed. If Y has no isolated points and $f: X \to Y$ is open fully closed, then f is a homeomorphism |8|.

1.3. Lemma. |8: Lemma 1.|. *If $f: X \to Y$ is compact mapping of regular space X, then f is fully closed iff f is closed and for each pair F_1, F_2 of disjoint closed subset of X the set $f(F_1) \cap f(F_2)$ is discrete.*

From this Lemma it follows

1.4. Lemma. *If $f: X \to Y$ is a fully closed compact mapping of T_3 space X, then $f|F: F \to f(F)$ is fully closed for every closed subset F of X.*

1.5. Lemma. |8| *Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system such that $f_{\alpha\beta}$ are fully closed perfect mappings. The projection $f_\alpha: \varprojlim X \to X_\alpha$ are fully closed iff $f_{\alpha\beta}$ are fully closed.*

1.6. Lemma. |8: Teorema 4.|. *If $f: X \to Y$ is fully closed surjection betwen normal spaces, then* $\dim X \leqq \dim X + 1$.

From the preceding Lemmas it follows

1.7. Theorem. *Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of the n-dimensional Cantor-manifolds X_α. If the mappings $f_{\alpha\beta}$ are montone fully closed, then $X = \varprojlim X$ is a $(n, 3)$-Cantor-manifold.*

PROOF. It is known that $\dim X \leqq n$. From lemmas 1.6. and 1.5. it follows that $\dim X \geqq n-1$. This means that either $\dim X = n$ or $\dim X = n-1$. Consider a closed $F \subseteq X$ with $\dim F \leqq n-3$. From Lemmas 1.6. and 1.4. it follows that $\dim f_\alpha(F) \leqq \dim F + 1 \leqq n-2$. Since X_α is the Cantor-manifold we infer that $X_\alpha \setminus f_\alpha(F)$ is connected. The set $f_\alpha^{-1}(X_\alpha \setminus f_\alpha(F))$ is also connected since f_α are montone mappings. The properties (P1) and (P2) of Theorem 1.1 are satisfied. The proof is completed.

1.8. Remark. If $\dim X = n-1$, then X is the $(n-1)$-dimensional Cantor-manifold.

Now we pas to the inverse systems of a generalized Cantor-manifolds.

1.9. Lemma. *Let* $\underline{X}=\{X_n, f_{nm}, N\}$ *be an inverse sequence of the countably compact spaces* X_n. *If the mappings* f_{nm} *are fully closed, then the projections* f_n: $X=\varprojlim X\to X_n$, $n\in N$, *are fully closed.*

PROOF. Let x_n be a point of X_n and $\{U_1, \ldots, U_K\}$ an open cover of the set $f_n^{-1}(x_n)$. We consider the family $\{f_m^{\#} U_1, \ldots, f_m^{\#} U_K\}$, $m\geqq n$. If the set $Y_m=f_{nm}^{-1}(x_n)\setminus(f_m^{\#} U_1\cup \cup\ldots\cup f_m^{\#} U_K)$ is non-empty for every $m\geqq n$, then we obtain the inverse system $Y=\{Y_m, f_{pm}/Y_m, n\leqq p\leqq m\}$ which has a non-empty limit |18|. This is imposible since $f_n^{-1}(x_n)\subseteqq(U_1\cup\ldots\cup U_K)$ and $U_i=\cup\{f_m^{-1}f_m^{\#} U_i: m\geqq n\}$. Hence, there exist $m_0\geqq n$ such that $Y_{m_0}=\emptyset$ i.e. $f_{nm_0}^{-1}(x_n)\subseteqq f_{m_0}^{\#} U_1\cup\ldots\cup f_{m_0}^{\#} U_K$. Since f_{nm_0} is fully closed, the set $\{x_n\}\cup(f_{nm_0}^{\#} f_{m0}^{\#} U_1\cup\ldots\cup f_{nm_0}^{\#} f_{m0}^{\#} U_K)$ is open. The proof is completed since

$$f_{nm_0}^{\#} f_{m0}^{\#} U_i = f_n^{\#} U_i \quad \text{for every} \quad i\in\{1, \ldots, k\}.$$

By the similar method of proof we have

1.10. Lemma. *Let* $\underline{X}=\{X_\alpha, f_{\alpha\beta}, \omega_\tau\}$ *be an inverse system of* $\aleph_\tau$*-compact spaces* X_α. *If the mappings* $f_{\alpha\beta}$ *are fully closed, the projections* f_α: $\varprojlim X\to X_\alpha$, $\alpha\in A$, *are fully closed.*

1.11. Theorem. *Let* $\underline{X}=\{X_n, f_{nm}, N\}$ *be an inverse sequence of the k-dimensional normal countably compact generalized Cantor-manifolds* X_n. *If the mappings* f_{nm} *are monotone fully closed surjections, then* $X=\varprojlim X$ *is* $(k, 3)$*-Cantor-manifold.*

PROOF. It suffices in the proof of Theorem 1.7. to apply the fact that f_n: $X\to X_n$, $n\in N$, are monotone mappings and X is connected |18|.
Similarly, on can prove

1.12. Theorem. *Let* $\underline{X}=\{X_\alpha, f_{\alpha\beta}, \omega_\tau\}$ *be an inverse system of the k-dimensional normal* $\aleph_\tau$*-compact generalized Cantor-manifolds* X_α. *If the mappings* $f_{\alpha\beta}$ *are monotone fully closed surjections, then* $X=\varprojlim X$ *is* $(k, 3)$*-Cantor-manifold.*

1.13. Remark. If $\dim X=k-1$, then X is $(k-1)$-dimensional generalized Cantor-manifold.

1.14. Remark. The space X in Theorem 1.11. (in Theorem 1.12.) is normal countably compact (normal $\aleph_\tau$-compact) |18|.
The next part of this Section is devoted to inverse systems of metric Cantor-manifold.

1.15. Lemma. *If* f: $X\to Y$ *is an open-closed surjection betwen separable metric spaces such that for every* $y\in Y$ *the fibre* $f^{-1}(y)$ *is a discrete subspace of* X, *then* $\operatorname{ind} Z=\operatorname{ind} f(Z)$ *for every closed subset* $Z\subseteqq X$.

PROOF. Let $B=\{U_i: i\in N\}$ be a base for X and let A_i be definied as in the proof of 1.12.5. Lemma in |7|. We have $X=U\{A_i: i\in N\}$ (see the proof of 1.12.7. Lemma in |7|). The set $A_i\cap Z$ is F_σ relative to Z and $f(A_i\cap Z)$ is F_σ relative to $f(Z)$. Furthemore, Z is separable metric space and $f/A_i\cap Z$: $A_i\cap Z\to f(A_i\cap Z)$ is a homeomorphism. This means that $\operatorname{ind}(A_i\cap Z)=\operatorname{ind} f(A_i\cap Z)\leqq\operatorname{ind} f(Z)$ and $\operatorname{ind} Z\geqq$

$\geqq \operatorname{ind}(A_i \cap Z) = \operatorname{ind} f(A_i \cap Z)$. From this relations and the relations $Z = U\{A_i \cap Z: i \in N\}$, $f(Z) = \cup \{f(A_i \cap Z): i \in N\}$ it follows that $\operatorname{ind} Z = \operatorname{ind} f(Z)$. Q.E.D.

1.16. Theorem. *Let* $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ *be an inverse system of k-dimensional generalized metric Cantor-manifolds* X_α *and open—closed surjections* $f_{\alpha\beta}$. *The space* $X = \varprojlim X$ *is k-dimensional generalized Cantor-manifold if the following conditions are satisfied:*

(C1) *X is a metric space,*

(C2) *For every* $x_\alpha \in X_\alpha$, $\alpha \in A$, *the fiber* $f_\alpha^{-1}(x_\alpha)$ *is a discrete subspace of X.*

PROOF. X is separable metric space of the dimension $\dim X = k$ (Lemma 1.15.). For every $F \subseteq X$ of the dimension $\dim F \leqq k-2$ it follows that $\dim f_\alpha(F) = \dim F \leqq k-2$ (Lemma 1.15.). Hence, condition (P1) of Theorem 1.1. is satisfied. The condition (P2) follows from the fact that the projections $f_\alpha: X \to X_\alpha$, $\alpha \in A$, are open-closed. (See 1.2. Remark.). The proof is completed.

1.17. If the spaces X_α are compact (countably compact), then (C2) means that $f_\alpha^{-1}(x_\alpha)$ is finite.

1.18. Lemma. *Let* $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ *is an inverse system such that the cardinality* $|f_{\alpha\beta}^{-1}(x_\alpha)| \leqq l$ *for each fiber* $f_{\alpha\beta}^{-1}(x_\alpha)$ *and some fixed natural number l. Then* $|f_\alpha^{-1}(x_\alpha)| \leqq l$ *for every* $f_\alpha^{-1}(x_\alpha)$, $\alpha \in A$.

PROOF. Trivial.

From Theorem 1.16. and Lemma 1.18. it follows

1.19. Theorem. *Let* $\underline{X} = \{X_n, f_{nm}, N\}$ *be an inverse sequence of metric k-dimensional Cantor-manifolds. If the mappings* f_{nm} *are open onto mappings with the property that there exists a natural number* $l \geqq 1$ *such that* $|f_{nm}^{-1}(x_n)| \leqq l$ *for every n, m and* x_n, *then* $X = \varprojlim X$ *is a metric k-dimensional Cantor-manifold.*

By the same method of proof we have the next theorem.

1.20. Theorem. *Let* $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ *be an inverse system of k-dimensional metric Cantor-manifolds. If the mappings* $f_{\alpha\beta}$ *are open surjections with the property that for every n and* x_n *there exist a natural numbers m, l such that* $|f_{nm}^{-1}(x_n)| \leqq l$, *then* $X = \varprojlim X$ *is a Cantor-manifold.*

PROOF. The space X is metrizable |3| i.e. separable metric space since X is compact. Theorem 1.16. completes the proof.

1.21. Lemma. *Let* $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ *be an σ-directed inverse system. If the fiber* $f_{\alpha\beta}^{-1}(x_\alpha)$ *are finite, then the fiber* $f_\alpha^{-1}(x_\alpha)$ *are finite.*

PROOF. Suppose that $|f_\alpha^{-1}(x_\alpha)| = \{x^{(1)}, x^{(2)}, \ldots, x^{(n)}, \ldots\}$. For every pair of points $x^{(i)}, x^{(j)}$ there exists $\alpha(i,j)$ such that $f_\beta(x^{(i)}) \neq f_\beta(x^{(j)})$ for each $\beta \geqq \alpha(i,j)$. Since $\underline{X}$ is σ-directed, there exist $\gamma \geqq \alpha(i,j)$, $i \in N$, $j \in N$. This means that the cardinality of the set $f_\gamma f_\alpha^{-1}(x_\alpha) = f_{\alpha\gamma}^{-1}(x_\alpha)$ is $\aleph_0$. A contradiction!

1.21. Theorem. *Let* $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ *be a σ-directed inverse system of k-dimensional locally connected metric Cantor-manifolds X such that* $f_{\alpha\beta}^{-1}(x_\alpha)$ *are finite subsets of* X_β, *then* $X = \varprojlim X$ *is k-dimensional locally connected metric Cantor-manifold.*

Proof. X is locally connected |9|. This means that $w(X)=\aleph_0$ |22: Theorem 1.| i.e. X is separable metric space. Now, apply Theorem 1.16.

1.22. Theorem. *Let* $\underline{X}=\{X_\alpha, f_{\alpha\beta}, A\}$ *be a well-ordered inverse system such that* $\mathrm{cf}(A)>\omega_1$. *If the fibers* $f_{\alpha\beta}^{-1}(x_\alpha)$ *are finite, and if* X_α *are metric k-dimensional Cantor-manifold, then* $X=\varprojlim \underline{X}$ *is k-dimensional metric Cantor-manifold.*

Proof. The space X is metric since $w(X)\leqq\aleph_0$ |28|. As in the preceding theorems we infer that X is a Cantor-manifold.

In the non-metric case, we can prove

1.23. Theorem. *Let* $\underline{X}=\{X_\alpha, f_{\alpha\beta}, A\}$ *be an inverse system of n-dimensional Cantor-manifold* X_α. *If the mappings* $f_\alpha\colon X=\varprojlim \underline{X}\to X_\alpha$, $\alpha\in A$, *are open with finite fibers, then X is n-dimensional Cantor-manifold.*

Proof. From |14: III. 2. Theorem| it follows that $\dim f_\alpha(F)=\dim F$ for every $\alpha\in A$ and every closed $F\subseteq X$. Hence $\dim F=n$. Furthermore, the property (P1) of Theorem 1.1. is satisfied. For (P2) see 1.2. Remark.

1.24. Remark. From |14: III. 2. Theorem| it follows also that Theorem 1.23. holds for inverse system $\underline{X}$ of generalized Cantor-manifolds always when f_α are open-closed mappings with finite fibers and normal limit X. This is the case, for example, for inverse sequence of countable compact normal generalized Cantor-manifolds since X is normal countably compact spaces |18|. From |6: 2.7.15(b)| it follows that Theorem 1.23. holds for inverse sequence of perfectly normal generalized Cantor-manifolds with open-closed onto bonding projections which have finite fiber.

1.25. Corollary. *If* $\underline{X}$ *in Theorem* 1.23. *is σ-directed or* $|f^{-1}(x)|\leqq l$ *for each* α, β, x_α *and some fixed natural number l, then X is n-dimensional Cantor-manifold.*

1.26. Theorem. *Let* $\underline{X}=\{X_\alpha, f_{\alpha\beta}, A\}$ *be an inverse system of n-dimensional Cantor-manifold. If there exists a natural number* $l\geqq 1$ *such that* $|f_{\alpha\beta}^{-1}(x_\alpha)|\leqq l$ *for each* α, β *and* x_α, *then* $X=\varprojlim \underline{X}$ *is* $(\dim X, \dim X-n+l+1)$*-Cantor-manifold.*

Proof. Let F be a closed subset of X such that $\dim F\leqq\dim X-(\dim X-n+l+1)=n-l-1$. By |1: 450| and Lemma 1.18. we have $\dim f_\alpha(F)\leqq\dim F+l-1\leqq n-l-1+l-1=n-2$. The condition (P1) of Theorem 1.1. is satisfied. This means that the set $Y_\alpha=X_\alpha\setminus f_\alpha(F)$ is connected. For every $\beta>\alpha$ the set $Y'_{\alpha\beta}=f_{\alpha\beta}^{-1}f_\alpha(F)$ has the dimension $\leqq n-2$ |1: 452| since $\dim f_\alpha\leqq 0$. It follows that the set $Y_{\alpha\beta}=X_\beta\setminus Y'_{\alpha\beta}=f_{\alpha\beta}^{-1}(Y_\alpha)$ is connected. The inverse system $\underline{Y}=\{Y_{\alpha\beta}, \beta\geqq\alpha\}$ has a connected limit |6: 6.1.18. Theorem|. Since $\lim \underline{Y}=f_\alpha^{-1}(Y_\alpha)$ we infer that X satisfies (P2) of Theorem 1.1. This means that $X\setminus F$ is connected i.e. X is $(\dim X, \dim X-n+l+1)$-Cantor-manifold. Q.E.D.

1.27. Remark. If $\dim X=n-l+1$, then X is $(n-l+1)$-dimensional Cantor-manifold.

Now we consider the inverse system with monotone bonding mappings.

1.28. Theorem. *Let* $\underline{X}=\{X_\alpha, f_{\alpha\beta}, A\}$ *be an inverse system of hereditarily normal Ind-n-dimensional Cantor-manifolds* X_α. *If* $f_{\alpha\beta}$ *are monotone mappings such that there* *exists a natural number* $l \geqq 1$ *such that* $|\mathrm{Fr}\, f_{\alpha\beta}^{-1}(x_\alpha)| \leqq l$ *then* $X = \varprojlim X$ *is* (Ind X, Ind $X-n+l+1$)*-Cantor-manifold.*

PROOF. It is readily seen that $|\mathrm{Fr}\, f_\alpha^{-1}(x_\alpha)| \leqq l$. From |24: Theorem VII. 8| it follows that $\mathrm{Ind}\, f_\alpha(F) \leqq \mathrm{Ind}\, F + l - 1 \leqq (\mathrm{Ind}\, X - \mathrm{Ind}\, X + n - l - 1) + l - 1 = n - 2$ for each closed $F \subseteq X$ and each $\alpha \in A$. This means that $Y_\alpha = Y_\alpha \setminus f_\alpha(F)$ is connected. Furthermore, the set $f_\alpha^{-1}(Y_\alpha)$ is connected since f_α is a monotone mapping. Theorem 1.1. completes the proof.

1.29. Corollary. *Let* $\underline{X} = \{X_\alpha, f_{\alpha\beta}, N\}$ *be an inverse system of metric n-dimensional Cantor-manifolds. If* $f_{\alpha\beta}$ *are monotone mappings such that* $|\mathrm{Fr}\, f_{\alpha\beta}^{-1}(x_\alpha)| \leqq l, l \geqq 1$, *then* $\varprojlim X$ *is* (Ind X, Ind $X-n+l+1$)*-Cantor-manifold.*

1.30. Corollary. *If X in the preceding theorem is an inverse sequence, then* $\varprojlim X$ is (dim X, dim $X-n+l+1$)*-Cantor-manifold.*

1.31. Corollary. *If in the preceding Theorems* $|\mathrm{Fr}\, f_{\alpha\beta}^{-1}(x_\alpha)| = 1$, *then* $\mathrm{Ind}\, X = n$, $\dim X = n$ *respectively. This means that in* 1.30. *X is n-dimensional Cantor-manifold.*

1.32. Remark. The inductively-open mapping were introduced by Arhangelskii |4: 209|. A mapping $f\colon X \to Y$ is said to be inductively-open if there exists a subspace $X_1 \subseteq X$ such that $f(X_1) = Y$ and if $f|X_1\colon X_1 \to Y$ is open.

If $f\colon X \to Y$ an inductively-open and closed mapping betwen metric spaces X and Y such that $|f^{-1}(y)| \leqq \aleph_0$, $y \in Y$, then $\dim Y \leqq \dim X$ |4: 9.1. Theorem|. If we asume that X and Y are Cantor-manifolds and $|f^{-1}(y)| \leqq k$, then $\dim X \leqq \dim Y + \dim f = \dim Y$.

1.33. Problem. Is it true that the assumption that $f_{\alpha\beta}$ are open in Theorem 1.16., 1.19., 1.20—1.27. can be replaced by the assumption "$f_{\alpha\beta}$ are inductively-open"?

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