Asymptotic analysis of a complex renewable system operating in Markovian environments

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Dedicated to Professor Zoltán Daróczy on his 50th birthday

Abstract. The present paper is concerned with an asymptotic analysis of a complex renewable system operating in random environments. Supposing "fast repair" it is shown, that the time to the first system failure converges in distribution, under appropriate norming, to an exponentially distributed random variable.

Keywords: operating time, repair time, "fast repair", random environments, system failure, weak convergence.

1. Introduction

The final goal of reliability theory is to give an estimate of the most important characteristics of the given system. The measure of greatest interest is the distribution of the time to the first system failure. In many models of practical interest "small parameters" are usually present e.g. the failure rates of elements are much smaller than their repair rates. (This is termed in reliability theory as "fast repair".) This situation enables us to use approximate methods in reliability calculations. For good reviews and materials the interested reader is referred, among others, to [3—8, 11—15]. It is also well-known that the great majority of problems can be treated by the help of Semi-Markov Processes, Semi-Regenerative Processes or, more generally, processes with an embedded point process [cf. 5]. For those models, mostly stationary reliability measures are obtained, and characteristics like time to the first system failure are difficult to obtain. Since the failure-free operation of the system corresponds to sojourn time problems we can use the results obtained for SMP. It is easy to see that in the case of fast repair the exit from a given subset of the state space of the underlying SMP is a "rare" event, that is, it occurs with a small probability. Thus, it is natural to investigate the asymptotic behavior of sojourn time in a given subset, provided that the probability of exit from it tends to zero [see 1—2, 9—10].

The aim of the present paper is to deal with an asymptotic analysis of a complex renewable system operating in random environments. Supposing "fast repair" it is shown that the time to the first system failure converges in distribution, under appropriate norming, to an exponentially distributed random variable. The main contribution of the paper is the following. The failure and repair intensities of the elements depend on the indices of the failed elements and the state of the given random environment. As a result of this assumption, the corresponding subset of the limiting Markov process — constructed to the problem — is not a simple essential class of states. Hence, the "classical" methods cannot be applied. Using the results of [1—2] the asymptotic exponentiality is proved.

2. Preliminary results

Let $(X_\varepsilon(k),\ k \geqq 0)$ be a Markov chain with state space

$$\sum_{q=0}^{m+1} X_q,\ X_i \cap X_j = 0, \quad i \neq j,$$

defined by the transition matrix $\|p_\varepsilon(i^{(q)}, j^{(z)})\|$ satisfying the following conditions:

1. $p_\varepsilon(i^{(o)}, j^{(o)}) \to p_o(i^{(o)}, j^{(o)}),\ i^{(o)}, j^{(o)} \in X_o$ and $P_o = \|p_o(i^{(o)}, j^{(o)})\|,\ i^{(o)}, j^{(o)} \in X_o$ is irreducible;
2. $p_\varepsilon(i^{(q)}, j^{(q+1)}) = \varepsilon\alpha^{(q)}(i^{(q)}, j^{(q+1)}) + o(\varepsilon),\ i^{(q)} \in X_q, \quad j^{(q+1)} \in X_{q+1}$;
3. $p_\varepsilon(i^{(q)}, f^{(q)}) \to 0,\ i^{(q)}, f^{(q)} \in X_q,\ q \geqq 1$;
4. $p_\varepsilon(i^{(q)}, j^{(z)}) \equiv 0,\ i^{(q)} \in X_q,\ j^{(z)} \in X_z,\ z-q \geqq 2$.

In the sequel the set of states X_q is called the q-th level of the chain, $q=0, \ldots, m+1$.

Let us single out the subset of states

$$\langle \alpha_m \rangle = \bigcup_{q=0}^{m} X_q.$$

Denote by $\{\Pi_\varepsilon(i^{(q)}),\ i^{(q)} \in X_q\},\ q=\overline{0, m}$ the stationary distribution of a chain with transition matrix

$$\left\| \frac{p_\varepsilon(i^{(q)}, j^{(z)})}{1 - \sum\limits_{k^{(m+1)} \in X_{m+1}} p_\varepsilon(i^{(q)}, k^{(m+1)})} \right\|, \quad i^{(q)} \in X_q,\ j^{(z)} \in X_z, \quad q, z \leqq m,$$

furthermore denote by $g_\varepsilon(\langle \alpha_m \rangle)$ the steady-state probability of exit from $\langle \alpha_m \rangle$, that is

$$g_\varepsilon(\langle \alpha_m \rangle) = \sum_{i^{(m)} \in X_m} \Pi_\varepsilon(i^{(m)}) \sum_{j^{(m+1)} \in X_{m+1}} p_\varepsilon(i^{(m)}, j^{(m+1)}).$$

Denote by $\{\Pi_o(i^{(o)}),\ i^{(o)} \in X_o\}$ the steady-state distribution corresponding to P_o and let

$$\bar{\Pi}_o = \{\Pi_o(i^{(o)}),\ i^{(o)} \in X_o\}, \quad \bar{\Pi}_\varepsilon^{(q)} = \{\Pi_\varepsilon(i^{(q)}),\ i^{(q)} \in X_q\}$$

be row-vectors. Finally, let

$$A^{(q)} = \|\alpha^{(q)}(i^{(q)}, j^{(q+1)})\|, \quad i^{(q)} \in X_q, \quad j^{(q+1)} \in X_{q+1}, \quad q = \overline{0, m}.$$

Conditions (1)—(4) enable us to compute the main terms of the asymptotic expression for $\overline{\Pi}_\varepsilon^{(q)}$ and $g_\varepsilon(\langle\alpha_m\rangle)$. Namely, we obtain

$$\begin{aligned}\overline{\Pi}_\varepsilon^{(q)} &= \varepsilon^q \overline{\Pi}_o A^{(o)} A^{(1)} \dots A^{(q-1)} + o(\varepsilon^q), \quad q = \overline{1, m}, \\ g_\varepsilon(\langle\alpha_m\rangle) &= \varepsilon^{m+1} \overline{\Pi}_o A^{(o)} A^{(1)} \dots A^{(m)} \underline{1} + o(\varepsilon^{m+1}),\end{aligned} \tag{1}$$

where $\underline{1}=(1, \dots, 1)$ is a column vector (cf. Anisimov [1—2]).

Let $(\xi_\varepsilon(t), t \geqq 0)$ be an SMP given by the embedded Markov chain $(X_\varepsilon(k), k \geqq 0)$ satisfying conditions (1)—(4). Let the times $\tau_\varepsilon(j^{(s)}, k^{(z)})$-transition time from state $j^{(s)}$ to state $k^{(z)}$-fulfil condition

$$E \exp \{i\theta\beta_\varepsilon \tau_\varepsilon(j^{(s)}, k^{(z)})\} = 1 + a_{jk}(s, z, \theta)\varepsilon^{m+1} + o(\varepsilon^{m+1})$$

where β_ε is some normalizing factor. Denote by $\Omega_\varepsilon(m)$ the instant at which the SMP reaches the $m+1$-th level for the first time, provided $\xi_\varepsilon(0) \in \langle\alpha_m\rangle$. Then we have:

Theorem 1. (Cf. Anisimov [2] pp. 153.) *If the above conditions are satisfied then*

$$\lim_{\varepsilon \to 0} E \exp \{i\theta\beta_\varepsilon \Omega_\varepsilon(m)\} = (1 - A(\theta))^{-1},$$

where

$$A(\theta) = \frac{\sum_{j,k \in X_o} \Pi_o(j) p_o(j, k) a_{jk}(0, 0, \theta)}{\overline{\Pi}_o A^{(o)} \dots A^{(m)} \underline{1}}.$$

In particular, if $a_{jk}(s, z, \theta) = i\theta m_{jk}(s, z)$, $(i = \sqrt{-1})$ then the limit is an exponentially distributed random variable with parameter

$$\overline{\Pi}_o A^{(o)} \dots A^{(m)} \underline{1} / \Big(\sum_{j,k \in X_o} \Pi_o(j) p_o(j, k) m_{jk}(0, 0)\Big).$$

3. The mathematical model

Let us consider a renewable system consisting of N elements and n repair crews. The elements are assumed to operate in a random environment governed by an irreducible, aperiodic Markov chain $(X_1(t), t \geqq 0)$ with state space $\{1, \dots, r_1\}$ and with transition density matrix

$$\Big\{a_{i_1 j_1},\ i_1, j_1 = \overline{1, r_1},\ a_{i_1 i_1} = \sum_{j \neq i_1} a_{i_1 j}\Big\}.$$

Whenever $X_1(t) = i_1$ and at time t there are s, $s = \overline{0, N-1}$ elements with indices $k_1, \dots, k_s$ at the repair facility, the probability of the j-th element failure in the interval $(t, t+h)$ is

$$\lambda_j(i_1 \colon k_1, \dots, k_s) h + o(h), \quad j \in \{1, \dots, N\} \setminus \{k_1, \dots, k_s\}.$$

When the elements fail they enter a repair facility and will be immediately served, unless all the repairmen are busy, otherwise they will wait in a queue in the order of their breakdowns. The repair facility is supposed to be imbedded in a random environment, governed by an irreducible, aperiodic Markov chain $(X_2(t), t \geqq 0)$

with state space $\{1, \ldots, r_2\}$ and with transition density matrix

$$\{b_{i_2 j_2},\ i_2, j_2 = \overline{1, r_2},\ b_{i_2 i_2} = \sum_{j \neq i_2} b_{i_2 j}\}.$$

Similarly, whenever $X_2(t) = i_2$ and at time t there are s, $s = \overline{1, N}$ elements with indices $k_1, \ldots, k_s$ at the service facility, the probability of the j-th element repair in the interval $(t, t+h)$ is

$$\mu_j(i_2\colon k_1, \ldots, k_s, \varepsilon) h + o(h), \quad j \in \{k'_1, \ldots, k'_{\min(s,n)}\}$$

where $\{k'_1, \ldots, k'_{\min(s,n)}\}$ denotes the indices of elements under repair. The environmental processes, operating and repair times are assumed to be independent of each other.

Let us consider the system under the assumption of "fast repair", that is, $\mu_j(i_2\colon k_1, \ldots, k_s, \varepsilon) \to \infty$ as $\varepsilon \to 0$.

For simplicity let

$$\mu_j(i_2\colon k_1, \ldots, k_s\colon \varepsilon) = \mu_j(i_2\colon k_1, \ldots, k_2)/\varepsilon.$$

The system is said to be failed iff the number of failed elements is $m+1$, $1 < m < N$. Our goal is to determine the distribution of the failure-free operation time of the system. Therefore, construct the following multi-dimensional Markov process

$$Z_\varepsilon(t) = \{X_1(t), X_2(t)\colon Y_\varepsilon(t);\ \gamma_1(t), \gamma_2(t), \ldots, \gamma_{Y_\varepsilon(t)}(t)\}$$

with state space

$$\{(i_1, i_2\colon s;\ k_1, \ldots, k_s),\ i_1 = \overline{1, r_1},\ i_2 = \overline{1, r_2},\ s = \overline{0, N},\ (k_1, \ldots, k_s) \in v_N^s,\ k_0 = 0\},$$

where $X_1(t), X_2(t)$: governing Markov chains,
$Y_\varepsilon(t)$: number of failed elements at time t,
$\gamma_1(t), \gamma_{Y_\varepsilon(t)}(t)$: indices of failed elements at time t in the order of their breakdowns,
V_N^s: set of all variations of order s of the integers $1, \ldots, N$. Let us single out the subset of states

$$\langle \alpha_m \rangle = \{(i_1, i_2\colon q;\ k_1, \ldots, k_q),\ i_1 = \overline{1, r_1},\ i_2 = \overline{1, r_2},\ q = \overline{0, m}, (k_1, \ldots, k_q) \in V_N^q\}.$$

Let

$$\Omega_\varepsilon(m) = \inf\big(t\colon Y_\varepsilon(t) = m+1/Y_\varepsilon(0) \leq m\big)$$

that is, the instant at which the system breaks down for the first time. Hence, the problem is to determine the distribution of the first exit of $Z_\varepsilon(t)$ from $\langle \alpha_m \rangle$.
Let

$$a_{i_1 i_1} + b_{i_2 i_2} + \sum_{j \neq k_1, \ldots, k_s} \lambda_j(i_1\colon k_1, \ldots, k_s) + $$
$$+ \sum_{j=1}^{\min(s,n)} \mu_{k_j}(i_2\colon k_1, \ldots, k_s)/\varepsilon = R(i_1, i_2\colon s;\ k_1, \ldots, k_s).$$

It is easy to see, that the sojourn time of $Z_\varepsilon(t)$ $\tau_\varepsilon(i_1, i_2\colon s; k_1, \ldots, k_s)$ in state $(i_1, i_2\colon s; k_1, \ldots, k_s)$ is exponentially distributed with parameter $R(i_1, i_2\colon s; k_1, \ldots, k_s)$. Furthermore, it can be readily verified that the transition probabilities for the embedded Markov chain as $\varepsilon \to 0$ are

$$p_\varepsilon[(i_1, i_2\colon s;\ k_1, \ldots, k_s), (j_1, i_2\colon s;\ k_1, \ldots, k_s)] = o(1), \quad s \geqq 1,$$

$$p_\varepsilon[(i_1, i_2\colon s;\ k_1, \ldots, k_s), (i_1, j_2\colon s;\ k_1, \ldots, k_s)] = o(1), \quad s \geqq 1,$$

$$p_\varepsilon[(i_1, i_2\colon s;\ k_1, \ldots, k_s), (i_1, i_2\colon s+1;\ k_1, \ldots, k_s, k_{s+1})] =$$

$$= \big(\lambda_{k_{s+1}}(i_1\colon k_1, \ldots, k_s)\varepsilon / \sum_{j=1}^{\min(s,n)} \mu_{k_j}(i_2\colon k_1, \ldots, k_s)\big)\big(1+o(1)\big),$$

$$p_\varepsilon[(i_1, i_2\colon 0;\ 0), (j_1, i_2\colon 0;\ 0)] = a_{i_1 j_1}/R(i_1, i_2\colon 0;\ 0),$$

$$p_\varepsilon[(i_1, i_2\colon 0;\ 0), (i_1, j_2\colon 0;\ 0)] = b_{i_2 j_2}/R(i_1, i_2\colon 0;\ 0),$$

$$p_\varepsilon[(i_1, i_2\colon 0;\ 0), (i_1, i_2\colon 1;\ k)] = \lambda_k(i_1\colon 0)/R(i_1, i_2\colon 0;\ 0).$$

This agrees with the conditions (1)—(4), but here the zero level is the set

$$\{(i_1, i_2\colon 0;\ 0), (i_1, i_2\colon 1;\ k), i_1 = \overline{1, r_1},\ i_2 = \overline{1, r_2},\ k = \overline{1, N}\}$$

while the q-th level is the set

$$\{(i_1, i_2\colon q+1;\ k_1, \ldots, k_{q+1}), i_1 = \overline{1, r_1},\ i_2 = \overline{1, r_2},\ (k_1, \ldots, k_{q+1}) \in V_N^{q+1}\}.$$

Since the level 0 is in the limit forms an essential class, the probabilities $\Pi_o(i_1, i_2\colon 0; 0)$, $\Pi_o(i_1, i_2\colon 1; k)$ satisfy the following system of equations

$$\Pi_o(i_1, i_2\colon 0;\ 0) = \sum_{j_1 \neq i_1} \Pi_o(j_1, i_2\colon 0;\ 0) a_{j_1 i_1}/R(j_1, i_2\colon 0;\ 0) + \tag{2}$$

$$+ \sum_{j_2 \neq i_2} \Pi_o(i_1, j_2\colon 0;\ 0) b_{j_2 i_2}/R(i_1, j_2\colon 0;\ 0) + \sum_{k=1}^{N} \Pi_o(i_1, i_2\colon 1;\ k),$$

$$\Pi_o(i_1, i_2\colon 1;\ k) = \Pi_o(i_1, i_2\colon 0;\ 0)\lambda_k(i_1\colon 0)/R(i_1, i_2\colon 0;\ 0). \tag{3}$$

Denote by

$$(\Pi_{i_1}^{(1)},\ i_1 = \overline{1, r_1}),\ (\Pi_{i_2}^{(2)},\ i_2 = \overline{1, r_2})$$

the sationary distribution of the governing Markov chains $(X_1(t), t \geqq 0), (X_2(t), t \geqq 0)$, respectively. Clearly

$$\Pi_{i_1}^{(1)} a_{i_1 i_1} = \sum_{j_1 \neq i_1} \Pi_{j_1}^{(1)} a_{j_1 i_1}, \quad i_1 = \overline{1, r_1}, \tag{4}$$

$$\Pi_{i_2}^{(2)} b_{i_2 i_2} = \sum_{j_2 \neq i_2} \Pi_{j_2}^{(2)} b_{j_2 i_2}, \quad i_2 = \overline{1, r_2}. \tag{5}$$

It is not difficult to verify that the solution of (2), (3) subject to (3), (4) is

$$\Pi_o(i_1, i_2\colon 0;\ 0) = B\Pi_{i_1}^{(1)} \Pi_{i_2}^{(2)} R(i_1, i_2\colon 0;\ 0),$$

$$\Pi_o(i_1, i_2\colon 1;\ k) = B\Pi_{i_1}^{(1)} \Pi_{i_2}^{(2)} \lambda_k(i_1\colon 0),$$

where

$$B = \Big[\sum_{i_1=1}^{r_1} \sum_{i_2=1}^{r_2} \Pi_{i_1}^{(1)} \Pi_{i_2}^{(2)} \big(2 \sum_{j=1}^{N} \lambda_j(i_1\colon 0) + a_{i_1 i_1} + b_{i_2 i_2}\big)\Big]^{-1}.$$

Hence, according to (1) we obtain

$$\Pi_\varepsilon(i_1, i_2\colon q;\ k_1, \dots, k_q) =$$

$$= \varepsilon^{q-1} B \Pi_{i_1}^{(1)} \Pi_{i_2}^{(2)} \frac{\prod_{s=0}^{q-1} \lambda_{k_{s+1}}(i_1\colon k_1, \dots, k_s)}{\prod_{s=1}^{q-1} \sum_{j=1}^{\min(s,n)} \mu_{k_j}(i_2\colon k_1, \dots, k_s)} \times (1+o(1)), \quad q > 1,$$

and

$$g_\varepsilon(\langle \alpha_m \rangle) = \varepsilon^m B \sum_{i_1=1}^{r_1} \sum_{i_2=1}^{r_2} \sum_{(k_1, \dots, k_{m+1}) \in V_N^{m+1}} \Pi_{i_1}^{(1)} \Pi_{i_2}^{(2)} \times$$

$$\times \frac{\prod_{s=0}^{m} \lambda_{k_{s+1}}(i_1\colon k_1, \dots, k_s)}{\prod_{s=1}^{m} \sum_{j=1}^{\min(s,n)} \mu_{k_j}(i_2\colon k_1, \dots, k_s)} \times (1+o(1)),$$

where

$$\Pi_\varepsilon(i_1, i_2\colon s;\ k_1, \dots, k_s)$$

is the steady-state distribution of chain with transition matrix

$$\left\| \frac{p_\varepsilon[(i_1, i_2\colon q;\ k_1, \dots, k_q), (j_1, j_2\colon z;\ k_1, \dots, k_z)]}{1 - \sum_{j_1=1}^{r_1} \sum_{j_2=1}^{r_2} \sum_{(k_1, \dots, k_{m+1})} p_\varepsilon[(i_1, i_2\colon q;\ k_1, \dots, k_q), (j_1, j_2\colon m+1;\ k_1, \dots, k_{m+1})]} \right\|$$

$$i_1, j_1 = \overline{1, r_1}, \quad i_2, j_2 = \overline{1, r_2}, \quad (k_1, \dots, k_p) \in V_N^p, \quad p = q, z, \quad q, z \leqq m.$$

Taking into account the exponentiality of $\tau_\varepsilon(i_1, i_2\colon s; k_1, \dots, k_s)$, for fixed θ we obtain

$$E \exp\{i\varepsilon^m \theta \tau_\varepsilon(i_1, i_2\colon 0;\ 0)\} = 1 + (\varepsilon^m \theta i / R(i_1, i_2\colon 0;\ 0))(1+o(1)),$$

$$E \exp\{i\varepsilon^m \theta \tau_\varepsilon(i_1, i_2\colon s;\ k_1, \dots, k_s)\} = 1 + o(\varepsilon^m), \quad s > 0, \quad (k_1, \dots, k_s) \in V_N^s.$$

Notice that $\beta_\varepsilon = \varepsilon^m$ and therefore by the help of Theorem 1 we immediately get:

Theorem 2. *For the system in question, under the above assumptions, independently of the initial state, the distribution of the normalized random variable* $\varepsilon^m \Omega_\varepsilon(m)$ *converges weakly to an exponentially distributed random variable with parameter*

$$\Lambda = \sum_{i_1=1}^{r_1} \sum_{i_2=1}^{r_2} \sum_{(k_1, \dots, k_{m+1}) \in V_N^{m+1}} \Pi_{i_1}^{(1)} \Pi_{i_2}^{(2)} \frac{\prod_{s=0}^{m} \lambda_{k_{s+1}}(i_1\colon k_1, \dots, k_s)}{\prod_{s=1}^{m} \sum_{j=1}^{\min(s,n)} \mu_{k_j}(i_2\colon k_1, \dots, k_s)}.$$

Thus, for the time to the first system failure we have

$$P(\Omega_\varepsilon(m) > t) \cong \exp(-\varepsilon^m \Lambda t).$$

In particular,

$$\lambda_j(i_1\colon k_1, \dots, k_s) = \lambda, \quad \mu_j(i_2\colon k_1, \dots, k_s) = \mu$$

so we obtain

$$\Lambda = \lambda \left(\frac{\lambda}{\mu}\right)^m \binom{N}{m+1} (m+1)! \prod_{s=1}^{m} \frac{1}{\min(s, n)}. \tag{6}$$

For the variable $\varepsilon^{n+m}\Omega_\varepsilon(n+m)$ (6) assumes the form

$$\Lambda = \lambda \left(\frac{\lambda}{\mu}\right)^{m+n} \binom{N}{n+m+1} \frac{(n+m+1)!}{n!\, n^m}. \tag{7}$$

It is well-known that if $N\to\infty$ and $\lambda\to 0$ such that $N\lambda\to\lambda'$, then the stationary distribution of a finite-source $M/M/n$ system coincides with the steady-state distribution of a $M/M/n$ system with arrival intensity λ' and with service rate μ. In fact, from (7) as $N\to\infty$ and $\lambda\to 0$ such that $N\lambda\to\lambda'$ we have

$$\Lambda = \frac{1}{n!\, n^m} \lambda' \left(\frac{\lambda'}{\mu}\right)^{n+m}$$

which was obtained by ANISIMOV [2] pp. 157.

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