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Non–convex perturbations of evolution equations with m–dissipative operators in Banach spaces

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Abstract. In this paper we establish the existence of integral solutions for a nonlinear, multivalued evolution equation of the form $\dot{x}(t) \in Ax(t) + F(t, x(t))$, where $A : X \to 2^X$ is an m–dissipative operator and $F(\cdot, \cdot)$ a nonconvex valued perturbation. Our result generalizes a recent existence theorem of Cellina–Marchi (Israel J. Math 46 (1983), pp. 1–11).

Key words and phrases : m–dissiaptive operator, compact semigroup, lower semicontinuous multifunction, Arzela–Ascoli theorem, parabolic equation.

1. Introduction

Evolution equations of the form $-\dot{x}(t) \in Ax(t)+f(t)$ in a Hilbert space, were first studied by BREZIS [4], with A a maximal monotone operator and $f(\cdot)$ an integrable perturbation. The work of BREZIS was extended by ATTOUCH–DAMLAMIAN [1], to systems of the form $-\dot{x}(t) \in Ax(t) + F(t, x(t))$, with $F(\cdot, \cdot)$ being a multivalued perturbation having convex values. ATTOUCH–DAMLAMIAN [1] proved two existence results :
one with A being a general maximal monotone operator, but with the underlying state space being $\mathbf{R}^n$ and the other with A being a subdifferential (i.e. $A = \partial\phi$, with ϕ being a proper, closed, convex function) and the underlying state space being any separable Hilbert space. Recently CELLINA–MARCHI [6] proved an existence theorem for the case where the multivalued perturbation has nonconvex values and the state spaces is $\mathbf{R}^n$. The study of those evolution equations in general Banach space (not necessarily Hilbert), was initiated by PAZY [12], who considered the case of A being a densely defined, linear, m–accretive operator and the perturbation was single valued. A nonlinear version of Pazy's theorem was

proved by VRABIE [14], who also considered the case of multivalued perturbations with convex values, extending this way the work of ATTOUCH–DAMLAMIAN [1]. Other interesting works in these or related issues were done by GUTMAN [8], HARAUX [9] and SCHECHTER [13] (he studied the dependence of the solutions on variations of the initial data).

In this note, we extend the result of CELLINA–MARCHI [6] to arbitrary separable Banach spaces, weakening also the hypotheses on the multivalued perturbation $F(t,x)$. Instead of assuming joint Hausdorff continuity for $F(t,x)$, we only require lower semicontinuity in the variable x, a more natural hypothesis in the context of applications.

2. Preliminaries

Let (Ω,Σ) be a measurable space and X a separable banach space. By $P_f(X)$ we will denote the collection of all nonempty, closed subsets of X. A multifunction $F:\Omega\to P_f(X)$ is said to be graph measurable, if $GrF=\{(\omega,x)\in\Omega\times X : x\in F(\omega)\}\in\Sigma\times B(X)$, where $B(X)$ is the Borel σ –field of X. Now let $\mu(\cdot)$ be a σ–finite measure on Σ. By S_F^1 we will denote the set of integrable selectors of $F(\cdot)$ i.e. $S_F^1=\{f\in L^1(X) : f(\omega)\in F(\omega)\mu$ – a.e. $\}$. Using Aumann's selection theorem, it is easy to check that if $\omega\to|F(\omega)|=\sup\{\|x\| : x\in F(\omega)\}$ is in L_+^1 (in which case we say that $F(\cdot)$ is integrably bounded), then $S_F^1\neq\emptyset$. If Y,Z are Hausdorff topological spaces and $G:Y\to 2^Z\setminus\{\emptyset\}$, then we say that $F(\cdot)$ is lower semicontinuous (l.s.c.), if for all $U\subseteq Z$ open, the set $G^-(U)=\{y\in Y : G(y)\cap U\neq\emptyset\}$ is open in Y. If Y,Z are metric spaces, then the above definition is equivalent to saying that for all $y_n\to y$ we have $G(y)\subseteq \underline{\lim}G(y_n)=\{z\in Z : z=\lim z_n, z_n\in G(y_n)\}$.

Next let X be any Banach space. Let $J:X\to 2^{X^*}$ be the duality map of X i.e. $J(x)=\{x^*\in X^* : (x^*,x)=\|x\|^2=\|x^*\|^2\}$. Clearly the values of $J(\cdot)$ are closed, convex, bounded subsets of X^*, which because of the Hahn–Banach theorem are also nonempty. Recall that if X^* is strictly convex, then $J(\cdot)$ is single valued. Using $J(\cdot)$ we can define the upper semi–inner product (denoted by $(\cdot,\cdot)_+$) and the lower semi–inner product (denoted by $(\cdot,\cdot)_-$) as follows:

$(x,y)_+=\sup\{(x^*,y) : x^*\in J(x)\}$
and $(x,y)_-=\inf\{x^*,y) : x^*\in J(x)\}$

for all $x,y\in X$. An operator $A:X\to 2^X$ is said to be dissipative (see BARBU [2]), if $(x-x',y-y')_-\leq 0$ for any $(x,y)(x',y')\in GrA$. We say that A is m–dissipative, if it is dissipative and in addition $R(I-\lambda A)=X$ for all $\lambda>0$. It is well known that an m–dissipative operator generates a semigroup $\{S(t)\}_{t\geq 0}$ of nonlinear contractions, via the Crandall–Liggett formula

$$S(t)x=\lim_{n\to\infty}\left(I-\tfrac{t}{n}A\right)^{-n}_{x} \qquad t\geq 0, \qquad x\in\overline{D(A)}.$$

Now let A be an m–dissipative operator, $f \in L^1(X)$ and $x_0 \in \overline{D(A)}$. Consider the following Cauchy problem on $T = [0, b]$:

$$\left\{\begin{matrix} \dot{x}(t) \in Ax(t) + f(t) \\ x(0) = x_0 \end{matrix}\right\} \qquad (^*)$$

Following BENILAN [3], we say that a function $x \in C(T, X)$ is an "integral solution" of $(^*)$, if $x(0) = x_0$ and

$$\|x(t) - y\|^2 \leq \|x(s) - y\|^2 + 2\int_s^t (f(r) + z, x(r) - y)_+ dr$$

for all $(y, z) \in GrA$ and all $0 \leq s \leq t \leq b$.

It is well known that under the above hypotheses Cauchy problem $(^*)$ has a unique integral solution. Moreover this unique integral solution depends continuously on the data of the problem. In fact if $x_1(\cdot)$ is the solution of $(^*)$ with data $(x_{01}, f_1) \in \overline{D(A)} \times L^1(X)$ and $x_2(\cdot)$ the solution of $(^*)$ with data $(x_{02}, f_2) \in \overline{D(A)} \times L^1(X)$, then we have

$$\|x_1(t) - x_2(t)\|^2 \leq \|x_{01} - x_{02}\|^2 + 2\int_0^t (f_1(r) - f_2(r), x_1(r) - x_2(r))_+ dr, t \in T,$$

or equivalently

$$\|x_1(t) - x_2(t)\| \leq \|x_{01} - x_{02}\| + \int_0^t \|f_1(r) - f_2(r)\| dr.$$

If A is densely defined, linear, m–accretive, then the notion of integral solution coincides with that of mild solution.

Recall that a "strong solution" of $(^*)$ is a continuous function $x : T \to X$ (i.e. $x(\cdot) \in C(T, X)$), for which we have that $x(t) \in D(A)$, is differentiable a.e. on $(0, b)$ and satisfies $(^*)$ a.e. with $x(0) = x_0 \in D(A)$.

Every strong solution is an integral solution. The converse is true only if we impose additional hypotheses on X, A and f. We are not going to go into the details of that problem. We only mention that if $X = \mathbf{R}^n$ and A is maximal monotone or if X is a Hilbert space and $A = \partial\phi$, with ϕ being a proper, closed, convex function on X, then every integral solution is also strong for any initial condition $x_0 \in \overline{D(A)}$. For further details we refer to BARBU [2], BREZIS [4] and SCHECHTER [13].

3. The theorem

In this section we will establish the existence of an integral solution for the following multivalued evolution equation:

$$\begin{Bmatrix} \dot{x}(t) \in Ax(t) + F(t, x(t)) \\ x(0) = x_0 \end{Bmatrix} \qquad (^{**})$$

By an integral solution of $(^{**})$, we mean a function $x \in C(T, X)$, which is an integral solution (as defined in section 2) of $\dot{x}(t) \in Ax(t) + f(t)$, $x(0) = x_0$ for some $f \in S^1_{F(\cdot, x(\cdot))}$.

Let $T = [0, b]$ and let X be a separable Banach space. We will need the following hypotheses:

H(A): $A : X \to 2^X$ is an m–dissipative operator, which generates a semigroup of compact nonlinear contractions (i.e. $S(t) : \overline{D(A)} \to \overline{D(A)}$ is compact for $t > 0$),

H(F): $F : T \times X \to P_f(X)$ is a multifunction s.t.
(1) $(t, x) \to F(t, x)$ is graph measurable,
(2) for every $t \in T, x \to F(t, x)$ is l.s.c.,
(3) $|F(t, x)| = \sup\{\|y\| : y \in F(t, x)\} \leq a(t) + b(t)\ \|x\|$ a.e. with $a(\cdot), b(\cdot) \in L^1_+$.

H_0 : $x_0 \in \overline{D(A)}$.

We have the following existence result concerning $(^{**})$.

Theorem. *If hypotheses $H(A), H(F)$ and H_0 hold, then $(^{**})$ admits an integral solution.*

PROOF. We will start by determining an a priori bound for the integral solutions of $(^{**})$. So suppose $x(\cdot) \in C(T, X)$ is such a solution of $(^{**})$. Recalling that $S(t)x_0$ is the integral solution of $\dot{y}(t) \in Ay(t), y(0) = x_0$ and using the inequalities of section 2, we have :

$$\|x(t) - S(t)x_0\| \leq \int_0^t \|f(s)\| ds$$

for all $t \in T$ and some $f \in S^1_{F(\cdot, x(\cdot))}$. Since $t \to S(t)x_0$ is continuous on T and using hypothesis $H(F)$ (3), we have

$$\|x(t)\| \leq M_1 + \int_0^t [a(s) + b(s)\|x(s)\|] ds$$

for some $M_1 > 0$. Applying Gronwall's inequality, we get that

$$\|x(t)\| \leq K \exp \|b\|_1 = M_2$$

where $K = M_1 + \|a\|_1$. Then define a new multifunction $\hat{F} : T \times X \to P_f(X)$ as follows:

$$\hat{F}(t, x) = \begin{cases} F(t, x) & \text{if } \|x\| \leq M_2 \\ F\left(t, \frac{M_2 x}{\|x\|}\right) & \text{if } \|x\| > M_2 \end{cases}$$

Observe that $\hat{F}(t,x) = F(t, p_{M_2}(x))$, where $p_{M_2}(\cdot)$ is the M_2 –radial retraction. We have $Gr\hat{F} = \{(t,x,y) \in T \times X \times X : (t, p_{M_2}(x), y) \in GrF\}$. Let $r : T \times X \times X \to T \times X \times X$ be defined by $r(t,x,y) = (t, p_{M_2}(x), y)$. Recalling that $p_{M_2}(\cdot)$ is 2–Lipschitz, we have that $r(\cdot,\cdot,\cdot)$ is continuous, hence measurable. So, since $GrF \in \Sigma \times B(X) \times B(X)$, we have $r^{-1}(GrF) = Gr\hat{F} \in \Sigma \times B(X) \times B(X)$ i.e. $\hat{F}(\cdot,\cdot)$ is graph measurable. Also since $\hat{F}(t,\cdot)$ is the composition of the Lipschitz function $p_{M_2}(\cdot)$ with the l.s.c. multifunction $F(t,\cdot)$, we have that $\hat{F}(t,\cdot)$ is l.s.c.. Finally note that $|F(t,x)| \leq a(t) + M_2 b(t) = \gamma(t)$ a.e. with $\gamma(\cdot) \in L^1_+$.

In the sequel we will consider the following multivalued Cauchy problem:

$$\begin{Bmatrix} \dot{x}(t) \in Ax(t) + \hat{F}(t,x(t)) \\ x(0) = x_0 \end{Bmatrix} \qquad (^{**})'$$

Let $h \in L^1(X)$ and consider the Cauchy problem

$$\begin{Bmatrix} \dot{x}(t) \in Ax(t) + h(t) \\ x(0) = x_0 \end{Bmatrix} \qquad (^{***}).$$

We know (see section 2), that $(^{***})$ has a unique integral solution. Let $r : L^1(X) \to C(T,X)$ be the map that to each $L^1(X)$–perturbation $h(\cdot)$ assigns the corresponding unique integral solution $r(h)(\cdot) \in C(T,X)$ of $(^{***})$. Let $B(\gamma) = \{h \in L^1(X) : \|h(t)\| \leq \gamma(t) \text{ a.e. }\}$. Our claim is that $K = r(B(\gamma))$ is relatively compact in $C(T,X)$.

To this end, first we will show that for every $t \in T, K(t) = r(B(\gamma))(t) = \{x(t) : x(\cdot) = r(h)(\cdot), h \in B(\gamma)\}$ is compact in X. For $t = 0$, we have $K(0) = \{x_0\}$ and so the claim is automatically verified. Hence let $t > 0, t \in T$. Note that $B(\gamma)$ is a uniformly integrable subset of $L^1(X)$. So given $t \in (0,b]$ and $\epsilon > 0$, we can find $\delta(\epsilon) \in (0,t)$ s.t. for $B \subseteq T$ Lebesgue measurable with $\lambda(B) < \delta$, we have:

$$\int_B \|h(s)\| ds < \epsilon$$

for all $h \in B(\gamma)$. Now consider the following Cauchy problem; on $[t-\delta, t]$:

$$\begin{Bmatrix} \dot{x}(\delta)(s) \in Ax(\delta)(s) \\ x(\delta)(t-\delta) = r(h)(t-\delta) \end{Bmatrix}$$

where $h \in B(\gamma)$. From the inequalities of section 2, we have:

$$\|x(\delta)(t) - r(h)(t)\| \leq \int_{t-\delta}^{t} \|h(s)\| ds < \epsilon$$

for all $h \in B(\gamma)$. Also recall that

$x(\delta)(t) = S(\delta)r(h)(t-\delta) \subseteq S(\delta)K(t-\delta)$.
and the latter is relatively compact in X, since $K(t-\delta) = \{y(t-\delta) : y(\cdot) \in K\}$ is bounded and $S(\delta)$ is a compact contraction (see hypothesis $H(A)$). Therefore $\overline{S(\delta)K(t-\delta)}$ is compact. So for every $t \in T$, every $\epsilon > 0$ and every $z \in K(t)$, there exists an element z_ϵ in the compact set $\overline{S(\delta)K(t-\delta)}$ s.t. $\|z - z_\epsilon\| < \epsilon \Longrightarrow \overline{K(t)}$ is compact.

Next, recall that since the semigroup $S(t)$ is compact, for $B \subseteq X$ nonempty, bounded, we have that $t \to \{S(t)x : x \in B\}$ is equicontinuous on $t > 0$. Hence given $\epsilon > 0$, we can find $\delta_1(\epsilon) > 0$ s.t. for $|t' - t| < \delta_1$ and for all $x \in K(t-\delta)$ we have:

$$\begin{aligned}
&\|S(t'-t+\delta)x - S(\delta)x\| < \epsilon \\
\Longrightarrow &\|S(t'-t+\delta)r(h)(t-\delta) - S(\delta)r(h)(t-\delta)\| < \epsilon \\
\Longrightarrow &\|x(\delta)(t') - x(\delta)(t)\| < \epsilon.
\end{aligned}$$

So finally for $\delta_2 = \min(\delta, \delta_1)$ and for $|t' - t| < \delta_2$, we have

$$\begin{aligned}
&\|r(h)(t') - r(h)(t)\| \\
\leq &\|r(h)(t') - x(\delta)(t')\| + \|x(\delta)(t') - x(\delta)(t)\| + \|x(\delta)(t) - r(h)(t)\| < \\
&< \epsilon + \epsilon + \epsilon = 3\epsilon \Longrightarrow K = r(B(\gamma)) \text{ is equicontinuous at each } t > 0.
\end{aligned}$$

Finally for $t = 0$ note that

$$\begin{aligned}
\|r(h)(t) - S(t)x_0\| &\leq \|r(h)(t) - S(t)x_0\| + \|S(t)x_0 - x_0\| \\
&\leq \int_0^t \gamma(s)ds + \|S(t)x_0 - x_0\|
\end{aligned}$$

So we also have equicontinuity at $t = 0$.

Invoking the Arzela–Ascoli theorem, we conclude that $\overline{K}$ is compact in $C(T, X)$. Thus by Mazur's theorem $\overline{K}_c = \overline{\text{conv}}K$ is compact

Next let $R : \overline{K}_c \to 2^{L^1(X)}$ be defined by

$R(x) = S^1_{\hat{F}(\cdot, x(\cdot))}$.

Since $\hat{F}(\cdot,\cdot)$ is graph measurable, it is easy to check as before, that $t \to \hat{F}(t, x(t))$ is graph measurable and integrably bounded by $\gamma(\cdot)$ and so $R(\cdot)$ has nonempty values, in fact $R(\cdot)$ is $P_f(L^1(X))$ –valued. Also since $\hat{F}(t,\cdot)$ is l.s.c. and using theorem 4.1 of [11], we have that if $x_n \to x$ in $\overline{K}_c$, then $R(x) \subseteq s - \underline{\lim} R(x_n)$, where s indicates the strong topology on $L^1(X)$. So $R(\cdot)$ is l.s.c. (see section 2). Hence we can apply Fryszkowski's selection theorem [7], to get $v : \overline{K}_c \to L^1(X)$ continuous s.t. $v(x) \in R(x)$ for all $x \in \overline{K}$. Set $p =$ rov. Clearly $p : \overline{K}_c \to \overline{K}_c$ is continuous. Apply Schauder's fixed point theorem to get $\hat{x} \in \overline{K}_c$ s.t. $\hat{x} = p(\hat{x}) = r(v(\hat{x}))$. Hence we have that $\hat{x}(\cdot)$ is an integral solution of

$$\begin{Bmatrix} \hat{x}(t) \in A\hat{x}(t) + v(\hat{x})(t) \\ \hat{x}(0) = x_0 \end{Bmatrix}$$

with $v(\hat{x})(\cdot) \in S^1_{\hat{F}(\cdot,\hat{x}(\cdot))}$. So $\hat{x}(\cdot) \in C(T, X)$ is an integral solution of $(^{**})'$. From the definition of $\hat{F}(t,x)$ and hypothesis $H(F)$ (3), we see easily that $|\hat{F}(t,x)| \leq a(t) + b(t)\|x\|$ a.e.. So as before, through Gronwall's inequality, we get $\|\hat{x}(t)\| \leq M_2, t \in T \Longrightarrow \hat{F}(t,\hat{x}(t)) = F(t,\hat{x}(t))$, $t \in T \Longrightarrow \hat{x}(\cdot)$ is the desired integral solution of $(^{**})$. Q.E.D.

As we mentioned in section 2, when $X = \mathbf{R}^n$, then every integral solution is a strong solution. So we can state as a corollary to our theorem, an extension of the existence result of CELLINA–MARCHI [6].

So let $T = [0, b], X = \mathbf{R}^n$ and make the following hypothesis about A:

$\mathbf{H}(A)'$: $A : D(A) \subseteq \mathbf{R}^n \to 2^{\mathbf{R}^n}$ is a maximal monotone operator.

Then we get as a corollary to our theorem, the following extension of the work of CELLINA–MARCHI [6].

Corollary. *If hypotheses $H(A)', H(F)$ and H_0 hold, then $(^{**})$ admits a strong solution.*

Remarks. (1) In CELLINA–MARCHI [6], the multivalued perturbation $F(t,x)$ was assumed to be jointly Hausdorff continuous.

(2) Hypotheses $H(F)$ (1) and (2), cover the case where $t \to F(t,x)$ is graph measurable and $x \to F(t,x)$ is Hausdorff continuous (see theorem 3.3 of [10]).

4. An example

Let Ω be a bounded open domain in $\mathbf{R}^n$ with smooth boundary $\partial\Omega = \Gamma, n > 2$.

Let $r > (n-2)/n$ and consider the following multivalued, nonlinear, parabolic partial differential equation on $T \times \Omega$:

$$\left\{ \begin{array}{l} \dfrac{\partial x(t,z)}{\partial t} - \Delta x(t,z)\,|x(t,z)|^{r-1} \in F(t,z,x(t,z)) \\ x(t,z) = 0 \text{ on } T \times \Gamma \\ x(0,z) = x_0(z) \text{ on } \{0\} \times \Omega \end{array} \right\} \quad (^{****})$$

Here $F: T \times \Omega \times \mathbf{R} \to P_f(\mathbf{R})$ is a multifunction which is l.s.c. in the third variable and $(t,y) \to S^1_{F(t,\cdot,y(\cdot))}$ is graph measurable on $T \times L^1(\Omega)$. It is easy to check that this is the case if $(t,z) \to F(t,z,r)$ is measurable and $r \to F(t,z,r)$ is Hausdorff continuous. Also assume that $|F(t,z,r)| = \sup\{\|v\| : v \in F(t,z,r)\} \leq a(t,z) + b(t,z)|r|$ a.e. with $a(\cdot,\cdot) \in L^1_+(T \times \Omega)$ and $b(t,\cdot) \in L^\infty(\Omega)$ while $t \to \|b(t,\cdot)\|_\infty$ belongs in L^1_+. Furthermore let $\hat{x}_0 = x_0(\cdot) \in L^1(\Omega)$.

Take $X = L^1(\Omega)$. This is a separable Banach space. Consider the nonlinear operator $A: D(A) \subseteq X \to X$ defined by $Ax = \Delta x|x|^{r-1}$ with $D(A) = \{x \in X : x|x|^{r-1} \in W^{1,1}_0(\Omega), \Delta x|x|^{r-1} \in L^1(\Omega)\}$. From BREZIS [5] we know that the operator A defined above is m–dissipative and the nonlinear semigroup it generates is compact for $t \in (0,b]$. Also let $\hat{F}: T \times X \to P_f(L^1(X))$ be defined by $\hat{F}(t,x) = S^1_{F(t,\cdot,x(\cdot))}$. Then $\hat{F}(\cdot,\cdot)$ is graph measurable, $\hat{F}(t,\cdot)$ is l.s.c. (see theorem 4.1 of [11]) and

$$|\hat{F}(t,x)| \leq \hat{a}(t) + \hat{b}(t)\|x\|_1 \text{ a.e.}$$

with $\hat{a}(t) = \|a(t,\cdot)\|_1$ and $\hat{b}(t) = \|b(t,\cdot)\|_\infty$.

Rewrite the initial–boundary value problem (****) as the following abstract multivalued evolution equation:

$$\left\{ \begin{matrix} \dot{x}(t) \in Ax(t) + \hat{F}(t,x(t)) \\ x(0) = \hat{x}_0 \end{matrix} \right\} \qquad (****)'$$

We see that all hypotheses of our theorem are satisifed and so we know that (****)′ has an integral solution $\hat{x}(\cdot) \in C(T, L^1(\Omega))$. Set $x(t,z) = \hat{x}(t)(z) z \in \Omega$. This is a generalized solution of (****).

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