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Finsler connection transformations associated to a general Finsler metrical structure of Miron type (II)

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1. Introduction

Let M be an n-dimensional differentiable manifold of class C^∞, (TM, π, M) its tangent bundle, (x^i, y^i) the local coordinates of a point $u \in TM$ in a local map, and let $\{\delta/\delta x^i, \partial/\partial y^i\}$ be an adapted basis, where $\delta/\delta x^i = \partial/\partial x^i - N_i^k \partial/\partial y^k$, N_k^i being the coefficients of a fixed non-linear connection N. Let $\mathcal{T}$ be the general group of Finsler connection transformations [6]: $t : F\Gamma(N) = (N_j^i, F_{jk}^i, C_{jk}^i) \to F\bar{\Gamma}(\bar{N}) = (\bar{N}_j^i, \bar{F}_{jk}^i, \bar{C}_{jk}^i)$ of the form:

$$(1.1)\quad \bar{N}_j^i(x,y) = N_j^i - A_j^i;\ \bar{F}_{jk}^i = F_{jk}^i + C_{jk}^a A_a^i + B_{jk}^i;\ \bar{C}_{jk}^i = C_{jk}^i + D_{jk}^i$$

where $A \in Z_1^1(M)$ and $B, D \in Z_2^1(M)$ are arbitrary Finsler tensors; and $\mathcal{T}_N = \{t \in \mathcal{T}, t = t(0, B, D)\}$ the subgroup of $\mathcal{T}$ formed by transformations $t : (N, F, C) \to (N, \bar{F}, \bar{C})$, which preserve the non-linear connection N. In a previous paper [12] we have introduced a classification method for the Finsler connection transformations $t \in \mathcal{T}_N$, using a metrical Finsler structure of Miron type $g = (g_{ij})$. In the present paper a generalization of this method for the case of the general group $\mathcal{T}$ of the Finsler connection transformations is given.

The notions and notations of M. MATSUMOTO [5] and R. MIRON [6] are used.

2. t_g and $t_{\mathrm{non}\,g}$ transformations

Definition 2.1. A Finsler connection transformation $t \in \mathcal{T}$ is called a t_g transformation if and only if we have:

$$(2.1)\qquad g_{aj\|k} - \bar{N}_k^s g_{aj}\|_s = g_{aj|k} - N_k^s g_{aj}|_s\ ; \quad g_{aj}\|_k = g_{aj}|_k$$

where $|, \Big|$ and $\|, \Big\|$ denote the h- and v-covariant derivatives relative to (N, F, C) and $(\bar{N}, \bar{F}, \bar{C})$ respectively.

Definition 2.2. A Finsler connection transfromation $t \in \mathcal{T}$ is called a $t_{\mathrm{non}\, g}$ transformation if and only if at least one of the conditions (2.1) is not satisfied.

If $\bar{N} = N$, then $t \in \mathcal{T}_N$ and we obtain the Definitions 2.1 and 2.2 from [12].

We denote:

$$G^i_{jk} = \frac{1}{2} g^{ir}(g_{rj|k} + g_{rk|j} - g_{jk|r}) \tag{2.2}$$

$$H^i_{jk} = \frac{1}{2} g^{ir}(g_{rj}\big|_k + g_{rk}\big|_j - g_{jk}\big|_r) \tag{2.3}$$

$$N^i_{jk} = \frac{1}{2} g^{ir}(g_{rj}\big|_a N^a_k + g_{rk}\big|_a N^a_j - g_{jk}\big|_a N^a_r) \tag{2.4}$$

and analogously $\bar{G}^i_{jk}, \bar{H}^i_{jk}$ and $\bar{N}^i_{jk}$ for $F\bar{\Gamma}(\bar{N})$.

From (2.1), (2.2), (2.3) and (2.4) follow the invariants:

$$\bar{G}^i_{jk} + \bar{N}^i_{jk} = G^i_{jk} + N^i_{jk};\ \bar{H}^i_{jk} = H^i_{jk}. \tag{2.5}$$

Thus we have:

Theorem 2.1. *If $t \in \mathcal{T}$ is a t_g-transformation then (2.5) are invariants of this transformation.*

Example 2.1. Any $t(A, 0, 0) \in \mathcal{T}$ transformation i.e. (1.1) transformation with $B = D = 0$, is a t_g-transformation.

If we denote:

$$\overset{*}{U}{}^i_{jk} = \frac{1}{2}(U^i_{jk} + g_{sj} g^{ir} U^s_{kr} + g_{sk} g^{ir} U^s_{jk}); \quad U^i_{jk} \in Z^1_2(M) \tag{2.6}$$

then, using the method from [12], from (1.1), (2.2) and (2.3) it follows the:

Theorem 2.2. *A Finsler connection transformation $t \in \mathcal{T}$ is a t_g-transformation if and only if it is of the form:*

$$\left\{ \begin{array}{l} \bar{N}^i_j = N^i_j - A^i_j; \\ \bar{F}^i_{jk} = F^i_{jk} + C^i_{ja} A^a_k + \overset{*}{\bar{T}}{}^i_{jk} - \overset{*}{T}{}^i_{jk} - \overset{*}{\Theta}{}^i_{jk} \\ \bar{C}^i_{jk} = C^i_{jk} + \overset{*}{\bar{S}}{}^i_{jk} - \overset{*}{S}{}^i_{jk} \end{array} \right. \tag{2.7}$$

where $\overset{*}{\bar{T}}$ is equal to $\overset{*}{U}$ if $U = \bar{T}$; $\overset{*}{T}$ is equal to $\overset{*}{U}$ if $U = T$; $\overset{*}{\Theta}$ is equal to $\overset{*}{U}$ if $U^i_{jk} = C^i_{ja} A^a_k - C^i_{ka} A^a_j$; $\overset{*}{\bar{S}}$ is equal to $\overset{*}{U}$ if $U = \bar{S}$ and $\overset{*}{S}$ is equal to $\overset{*}{U}$ if

$U = S$; $A^a_j \in Z^1_1(M)$ *is an arbitrary Finsler tensor,* T, S *and* $\bar{T}, \bar{S}$ *are the* h- *and* v-*torsions of* $F\Gamma(N)$ *and* $F\bar{\Gamma}(\bar{N})$ *respectively.*

Theorem 2.3. *The set of all Finsler connections* $F\bar{\Gamma}(\bar{N})$ *obtained from a fixed Finsler connection* $F\Gamma(N)$ *by a* t_g-*transformation is given by (2.7), where* $A^a_k \in Z^1_1(M)$ *is an arbitrary Finsler tensor;* $\bar{T}, \bar{S}$ *are arbitrary skewsymmetric tensors:* $\bar{T}^i_{jk} = -\bar{T}^i_{kj}$; $\bar{S}^i_{jk} = -\bar{S}^i_{kj}$ *and* T, S *are the* h- *and* v-*torsions of the fixed Finsler connection* $F\Gamma(N)$.

It follows that the transformations (2.7) are characterized by the invariants (2.1) or (2.5).

Corollary 2.1. *If* $A^i_j = 0$, *then* $\bar{N} = N$ *and we obtain the* g *transformations* $t_g \in \mathcal{T}_N$ *from [12].*

We denote:

$$A^i_{jk} = \frac{1}{2} g^{ia} (g_{aj}|_b A^b_k + g_{ak}|_b A^b_j - g_{jk}|_b A^b_k) \tag{2.8}$$

If $t \in \mathcal{T}$ is a $t_{\text{non}\, g}$-transformation, then using the method from [12] it follows from (1.1) the:

Theorem 2.4. *A Finsler connection transformation* $t \in \mathcal{T}$ *is a* $t_{\text{non}\, g}$-*transformation if and only if it is of the form:*

$$\begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \\ \bar{F}^i_{jk} = F^i_{jk} + C^i_{ja} A^a_k + \overset{*}{\bar{T}}{}^i_{jk} - \overset{*}{T}{}^i_{jk} - \overset{*}{\Theta}{}^i_{jk} + G^i_{jk} - \bar{G}^i_{jk} + A^i_{jk} \\ \bar{C}^i_{jk} = C^i_{jk} + \overset{*}{\bar{S}}{}^i_{jk} - \overset{*}{S}{}^i_{jk} + H^i_{jk} - \bar{H}^i_{jk} \end{cases} \tag{2.9}$$

and:

Theorem 2.5. *Let* $F\Gamma(N)$ *be a fixed Finsler connection with the* h-*torsion* T *and* v-*torsion* S. *Then any other Finsler connection* $F\bar{\Gamma}(\bar{N})$ *which does not possess the property (2.1) is given by (2.9), where* $\bar{T}, \bar{S}$ *are arbitrary skewsymmetric Finsler tensors;* $\bar{G}, \bar{H}$ *are arbitrary symmetric Finsler tensors:* $\bar{G}^i_{jk} = \bar{G}^i_{kj}$; $\bar{H}^i_{jk} = \bar{H}^i_{kj}$; $A^i_j \in Z^1_1(M)$ *is an arbitrary Finsler tensor and* $\bar{H} \neq H$ *or* $G^i_{jk} - \bar{G}^i_{jk} + A^i_{jk} \neq 0$.

If we fix $\bar{T}, \bar{S}, \bar{G}, \bar{H}$ and A^i_j, then we obtain a Finsler connection $F\bar{\Gamma}(\bar{N})$ which has the h-torsion $\bar{T}$, the v-torsion $\bar{S}$ and $\bar{G}, \bar{H}$ satisfies the relations:

$$\begin{aligned} \bar{G}^i_{jk} &= \frac{1}{2} g^{ir} (g_{rj\|k} + g_{rk\|j} - g_{jk\|r}); \\ \bar{H}^i_{jk} &= \frac{1}{2} g^{ir} (g_{rj}\|_k + g_{rk}\|_j - g_{jk}\|_r) \end{aligned} \tag{2.10}$$

From (2.9) follow also special cases of $t_{\text{non}-g}$-transformations.

We obtain the:

Theorem 2.6. (*Separation Theorem*). *Any Finsler connection transformation $t \in \mathcal{T}$ is a t_g-transformation ($t \in \mathcal{T}_g$), or a $t_{\text{non }g}$-transformation ($t \in \mathcal{T}_{\text{non }g}$) and we have: $\mathcal{T} = \mathcal{T}_g \cup \mathcal{T}_{\text{non }g}$; $\mathcal{T}_g \cap \mathcal{T}_{\text{non }g} = \emptyset$.*

If in (2.9) we give up one, or both of the conditions $\bar{H} \neq H$ and $G^i_{jk} - \bar{G}^i_{jk} + A^i_{jk} = 0$, then we obtain the:

Theorem 2.7. (*General Theorem*). *Any Finsler connection transformation $t \in \mathcal{T}$ is of the form (2.9).*

This Theorem is important, since it is expressed solely in the terms of the two Finsler connections $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$ and thus we have the direct interpretation of the terms $\bar{T}, T$; $\bar{S}, S$; $\bar{G}, G$; A, for every definiteness of the connections. Consequently the invariants of these transformations can be studied elegantly in the special cases.

Corollary 2.3. *Any Finsler connection transformation $t \in \mathcal{T}_N$ is of the form:*

$$(2.11)\qquad \begin{aligned} &\bar{N}^i_j = N^i_j; \quad \bar{F}^i_{jk} = F^i_{jk} + \overset{*}{\bar{T}}{}^i_{jk} - \overset{*}{T}{}^i_{jk} + G^i_{jk} - \bar{G}^i_{jk};\\ &\bar{C}^i_{jk} = C^i_{jk} + \overset{*}{\bar{S}}{}^i_{jk} - \overset{*}{S}{}^i_{jk} + H^i_{jk} - \overset{*}{H}{}^i_{jk} \end{aligned}$$

In this way the result of [12] is obtained.

3. Special classes of $t \in \mathcal{T}$ transformations

In the theory of linear connections the Schouten tensor play a prominent part. In the theory of Finsler connections we can define two Finsler-Schouten tensors:

$$(3.1)\quad \underset{1}{I}{}^i_{jk} = T^i_{jk} - \frac{1}{n-1}(\delta^i_j T_k - \delta^i_k T_j); \ \underset{2}{I}{}^i_{jk} = S^i_{jk} - \frac{1}{n-1}(\delta^i_j S_k - \delta^i_k S_j)$$

where $T_k = T^i_{ik}$, $S_k = S^i_{ik}$ and analogously two tensors $\underset{1}{\bar{I}}$ and $\underset{2}{\bar{I}}$ for $F\bar{\Gamma}$. Since we have:

$$(3.2)\qquad \overset{*}{\bar{T}}{}^i_{jk} = \overset{*}{T}{}^i_{jk} = \alpha_j \delta^i_k - g_{jk}\alpha^i; \quad \overset{*}{\bar{S}}{}^i_{jk} = \overset{*}{S}{}^i_{jk} = \beta_j \delta^i_k - g_{jk}\beta^i$$

where $\alpha_j = \frac{1}{n-1}(T_j - \bar{T}_j)$; $\beta_j = \frac{1}{n-1}(S_j - \bar{S}_j)$, in case of $\underset{1}{\bar{I}} = \underset{1}{I}$, $\underset{2}{\bar{I}} = \underset{2}{I}$ we arrive to the following theorems:

Theorem 3.1. *Any t_g-transformation, which admits the invariants* $\underset{1}{\bar{I}} = \underset{1}{I}$ and $\underset{2}{\bar{I}} = \underset{2}{I}$ *is of the form:*

$$(3.3)\qquad \begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \quad \bar{F}^i_{jk} = F^i_{jk} + C^i_{jr}A^r_k - \overset{*}{\Theta}{}^i_{jk} + \alpha_j\delta^i_k - g_{jk}\alpha^i \\ \bar{C}^i_{jk} = C^i_{jk} + \beta_j\delta^i_k - g_{jk}\beta^i \end{cases}$$

Theorem 3.2. *Any $t_{\mathrm{non}\,g}$-transformation, which admits the invariants* $\underset{1}{\bar{I}} = \underset{1}{I}$ and $\underset{2}{\bar{I}} = \underset{2}{I}$ *is of the form:*

$$(3.4)\qquad \begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \\ \bar{F}^i_{jk} = F^i_{jk} + C^i_{jr}A^r_k - \overset{*}{\Theta}{}^i_{jk} + G^i_{jk} - \bar{G}^i_{jk} + A^i_{jk} + \alpha_j\delta^i_k - g_{jk}\alpha^i \\ \bar{C}^i_{jk} = C^i_{jk} + H^i_{jk} - \bar{H}^i_{jk} + \beta_j\delta^i_k - g_{jk}\beta^i \end{cases}$$

If $A^i_j = 0$, then $\bar{N} = N$ and we obtain the corresponding theorems from [12].

Particularly we obtain:

Theorem 3.3. *Any t_g-transformation of semisymmetric* ($\underset{1}{I} = 0$, $\underset{2}{I} = 0$) *Finsler connections is of the form (3.3).*

Theorem 3.4. *Any $t_{\mathrm{non}\,g}$-transformation of semisymmetric Finsler connections is of the form (3.4).*

If $F\Gamma(N)$ is a metrical Finsler connection, then $g_{ij|k} = o$, $g_{ij}\big|_k = o$ and it follows:

Theorem 3.5. *Any transformation $t \in \mathcal{T}$ of metrical Finsler connections is a t_g-transformation.*

Theorem 3.6. *Any transformation $t \in \mathcal{T}$ which transforms a non-metrical Finsler connection $F\Gamma(N)$ in a metrical Finsler connection $F\bar{\Gamma}(\bar{N})$ is a $t_{\mathrm{non}\,g}$-transformation of the form:*

$$(3.5)\qquad \begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \\ \bar{F}^i_{jk} = F^i_{jk} + C^i_{jr}A^r_k + \overset{*}{\bar{T}}{}^i_{jk} - \overset{*}{T}{}^i_{jk} - \overset{*}{\Theta}{}^i_{jk} + G^i_{jk} + A^i_{jk} \\ \bar{C}^i_{jk} = C^i_{jk} + \overset{*}{\bar{S}}{}^i_{jk} - \overset{*}{S}{}^i_{jk} + H^i_{jk} \end{cases}$$

This transformation solves the metrization problem of KAWAGUCHI. From the Theorems 3.1 and 3.5 follow the results of [9], [10].

Obviously any other conformal transformation compatible with a complex structure is obtained from (2.7) or (2.9) by particularization.

In [6], [7], [8] R. MIRON establishes the following metrizations:

Theorem. *If $F\Gamma(N) = (N, F, C)$ is a fixed non-metrical Finsler connection, then any other metrical Finsler connection is given by:*

$$(3.6)\qquad \begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \\ \bar{F}^i_{jk} = F^i_{jk} + C^i_{jr}A^r_k + \frac{1}{2}g^{ir}(g_{rj|k} + g_{kj}\big|_a A^a_k) + \Omega^{ir}_{sj}X^s_{rk} \\ \bar{C}^i_{jk} = C^i_{jk} + \frac{1}{2}g^{ir}g_{rj}\big|_k + \Omega^{ir}_{sj}Y^s_{rk} \end{cases}$$

where $X, Y \in Z^1_2(M)$ are arbitrary Finsler tensor fields and $\Omega^{ir}_{sj} = \frac{1}{2}(\delta^i_s\delta^r_j - g_{sj}g^{ir})$ is the Obata operator $F\Gamma(N)$.

The transformations (3.6) are very important, since for any definiteness of $X, Y \in Z^1_1(M)$ a class of metrical Finsler connections $F\bar{\Gamma}(\bar{N})$ obtained from a non-metrical Finsler connection $F\Gamma(N)$ follows. It is an elegant generalization of the Kawaguchi metrizations method. By definiteness of $A^i_j \in Z^1_1(M)$ we obtain actually the metrical connection $F\bar{\Gamma}(\bar{N})$.

We shall solve now the inverse problem: Let $F\Gamma(N)$ be a fixed non-metrical Finsler connection and $F\bar{\Gamma}(\bar{N})$ an another given metrical Finsler connection. Which relations hold between $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$? What form has the transformation $t : F\Gamma(N) \to F\bar{\Gamma}(\bar{N})$?

Between $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$ obviously exist the relations (3.6), but in what extent are X and Y arbitrary? This is the problem, which has to be solved. From (3.6) it follows:

$$(3.7)\qquad \begin{aligned} \bar{T}^i_{jk} =& T^i_{jk} + C^i_{jr}A^r_k - C^i_{kr}A^r_k + \frac{1}{2}g^{ir}(g_{rj|k} - g_{rk|j}) + \\ &+ \frac{1}{2}g^{ir}(g_{rj}\big|_a A^a_k - g_{rk}\big|_a A^a_j) + \Omega^{ir}_{sj}X^s_{rk} - \Omega^{ir}_{sk}X^s_{rj} \end{aligned}$$

$$(3.8)\qquad \bar{S}^i_{jk} = S^i_{jk} + \frac{1}{2}g^{ir}(g_{rj}\big|_k - g_{rk}\big|_j) + \Omega^{ir}_{sj}Y^s_{rk} - \Omega^{ir}_{sk}Y^s_{rj}$$

Consequently we obtain a system of tensor equations of the form:

$$(3.9)\qquad \Omega^{ir}_{sj}X^s_{rk} - \Omega^{ir}_{sk}X^s_{rj} = \underset{1}{\sigma}{}^i_{jk}$$

$$(3.10)\qquad \Omega^{ir}_{sj}Y^s_{rk} - \Omega^{ir}_{sk}Y^s_{rj} = \underset{2}{\sigma}{}^i_{jk}$$

where:

$$(3.11)\qquad \begin{aligned} \underset{1}{\sigma}{}^i_{jk} =& (\bar{T}^i_{jk} - T^i_{jk}) - (C^i_{jr}A^r_k - C^i_{kr}A^r_j) - \\ &- \frac{1}{2}g^{ir}(g_{rj|k} - g_{rk|j}) - \frac{1}{2}g^{ir}(g_{rj}\big|_a A^a_k - g_{rk}\big|_a A^a_j) \end{aligned}$$

$$(3.12) \qquad \underset{2}{\sigma}{}^i_{jk} = (\bar{S}^i_{jk} - S^i_{jk}) - \frac{1}{2} g^{ir}(g_{rj}\big|_k - g_{rk}\big|_j)$$

The general solution of a systems of the form (3.9)–(3.10) is given in [13] using the theory of fixed points. Here we give a direct solution by comparison of (3.6) and (3.5). It follows that a general solution is of the form:

$$(3.13) \qquad \begin{aligned} \Omega^{ir}_{sj} X^s_{rk} = \overset{*}{\bar{T}}{}^i_{jk} - \overset{*}{T}{}^i_{jk} - \overset{*}{\Theta}{}^i_{jk} + \frac{1}{2} g^{ir}(g_{rk|j} - g_{jk|r}) + \\ + \frac{1}{2} g^{ir}(g_{rk}\big|_a A^a_j - g_{jk}\big|_a A^a_r) \end{aligned}$$

$$(3.14) \qquad \Omega^{ir}_{sj} Y^s_{rk} = \bar{S}^i_{jk} - S^i_{jk} + \frac{1}{2} g^{ir}(g_{rk}\big|_j - g_{jk}\big|_r)$$

Denoting the right-hand member of (3.13) and (3.14) by B^i_{jk} and D^i_{jk} respectively, we have:

$$(3.15) \qquad \overset{*}{\Omega}{}^{ir}_{sj} B^s_{rk} = 0; \quad \overset{*}{\Omega}{}^{ir}_{sj} D^s_{rk} = 0$$

where $\overset{*}{\Omega}{}^{ir}_{sj} = \frac{1}{2}(\delta^i_s \delta^r_j + g^{ir} g_{sj})$ is the Obata operator.

From (3.15) it follows the compatibility of the system (3.13)–(3.14) and it has the solution:

$$(3.16) \qquad X^i_{jk} = B^i_{jk} + \overset{*}{\Omega}{}^{is}_{rj} U^r_{sk}$$

$$(3.17) \qquad Y^i_{jk} = D^i_{jk} + \overset{*}{\Omega}{}^{is}_{rj} V^r_{sk}$$

where $U, V \in Z^1_2(M)$ are arbitrary and B^i_{jk}, D^i_{jk} are given by the right-hand members of (3.13) and (3.14) respectively.

Consequently we have the:

Theorem 3.7. *Between a non-metrical and a metrical Finsler connections $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$, there exists the relation* (3.6), *where X, Y are arbitrary up to a transformation of form* (3.16)–(3.17).

Consequently X, Y are not completly arbitrary. They must be choosen of the form (3.16)–(3.17) in order to satisfy the equations (3.6).

In case when $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$ are both metrical we obtain from [1] and [8] the results:

$$(3.18) \qquad \begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \quad \bar{F}^i_{jk} = F^i_{jk} + C^i_{ja} A^a_k + \Omega^{ir}_{sj} X^s_{rk} \\ \bar{C}^i_{jk} = C^i_{jk} + \Omega^{ir}_{sj} Y^s_{rk} \end{cases}$$

From (3.16) and (3.17) follows the general solution:

$$X^i_{jk} = \overset{*}{\bar{T}}{}^i_{jk} - \overset{*}{T}{}^i_{jk} - \overset{*}{\Theta}{}^i_{jk} + \overset{*}{\Omega}{}^{ir}_{sj} U^s_{rk} \tag{3.19}$$

$$Y^i_{jk} = \overset{*}{\bar{S}}{}^i_{jk} - \overset{*}{S}{}^i_{jk} + \overset{*}{\Omega}{}^{ir}_{sj} V^s_{rk} \tag{3.20}$$

where $U, V \in Z^1_2(M)$. Thus we have:

Theorem 3.8. *Between two given metrical Finsler connections* $F\Gamma(N)$ *and* $F\bar{\Gamma}(\bar{N})$ *the relations* (3.18) *exist, where* X *and* Y *are given by* (3.19) *and* (3.20).

Consequently neither in this case are X, Y completly arbitrary, only up to a transformation (3.19)–(3.20).

Summary

1. The Miron-Hashiguchi transformations are used if we wish to determine all metrical Finsler connections $F\bar{\Gamma}(\bar{N})$ from a fixed Finsler connection $F\Gamma(N)$.

2. The transformations (3.5) (or equivalently (3.6), (3.16), (3.17)) are used if we wish to determine metrical Finsler connections $F\bar{\Gamma}(\bar{N})$ with special properties (for example: $\underset{1}{\bar{I}} = \underset{1}{I}$, $\underset{2}{\bar{I}} = \underset{2}{I}$ or $\underset{1}{\bar{I}} = 0$, $\underset{2}{\bar{I}} = 0$) starting from a fixed non-metrical Finsler connection $F\Gamma(N)$.

3. The transformations (3.18) are used if we wish to determine arbitrary metrical Finsler connections $F\bar{\Gamma}(\bar{N})$, starting from a fixed metrical Finsler connection $F\Gamma(N)$.

4. The transformations (2.7) are used if we wish to determine Finsler connections with the properties (2.1) (particularly also metrical Finsler connections) whose torsions $\bar{T}, \bar{S}$ have given properties (for example: $\underset{1}{\bar{I}} = \underset{1}{I}$, $\underset{2}{\bar{I}} = \underset{2}{I}$ or $\underset{1}{\bar{I}} = 0$, $\underset{2}{\bar{I}} = 0$, and so on).

If $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$ are metrical we can start from (3.18), (3.19), (3.20).

5. The Finsler connection transformations $t : F\Gamma(N) \to F\bar{\Gamma}(\bar{N})$ compatible with a metrical structure or with other structures (complex, symplectic, etc.) studied by G. Atanasiu [1], [2], [3] can be obtained explicitely from (2.7) by imposing of the corresponding conditions.

6. The conformal Finsler connection transformations $t : F\Gamma(N, \omega) \to F\bar{\Gamma}(\bar{N}, \bar{\omega})$ [4] are obtained from (2.9) in an explicit form, without the indetermination of ΩX and ΩY.

7. In the general case, when *we have no* information relative to the relations between these connections and g, then the general transformations from the Theorem 2.7 are used, imposing the desired conditions.

In this way a *synthesis* is obtained in the study or Finsler connection transformations.

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References

[1] GH. ATANASIU, Structures et connexions Finsler presque complexe et presque Hermitiennes, *Proc. Univ. Braşov* (1980), 41–53.

[2] GH. ATANASIU, Variétés differentiables doués de couples Finsler, *Proc. Univ. Braşov* (1982), 35–67; 69–73.

[3] GH. ATANASIU, B. SINHA and S. K. SERIGH, Almost contact metrical structures and connections, *Proc. Univ. Braşov* (1984), 29–36.

[4] I. GHINEA, Conformal Finsler connections, *Proc. Univ. Braşov* (1980), 55–68.

[5] M. MATSUMOTO, Foundations of Finsler geometry and special Finsler spaces, *Kaiseisha Press, Japan* (1986).

[6] R. MIRON, Introduction to the theory of Finsler spaces, *Proc. Nat. Sem. on Finsler Spaces Braşov* **1** (1980), 131–184.

[7] R. MIRON and M. HASHIGUCHI, Metrical Finsler connections, *Rep. Fac. Sci. Kagoshima University No.* **12** (1979), 21–35.

[8] R. MIRON, Metrical Finsler structures and metrical Finsler connections, *J. Math. Kyoto Univ.* **23–2** (1983), 219–214.

[9] P. STAVRE, Metrical Finsler connections $TI(g)$, *An. Univ. "Al. I. Cuza" Iaşi* **4** (1984), 87–90.

[10] P. STAVRE and FR. KLEPP, General Finsler connection transformations with invariant I^i_{jk}-tensors, *Proc. Romanian-Japanese Colloquium, Braşov* (1984), 165–170.

[11] P. STAVRE, On the transformations $t \in \mathcal{T}_{NI}, t \in \mathcal{T}_{NJ}$, *Proc. Nat. Sem. on Finsler Spaces, Braşov* **3** (1984), 153–166.

[12] P. STAVRE and FR. KLEPP, Finsler connection transformations associated to a general Finsler metrical strucrure of R. Miron I, *Publicationes Math., Debrecen* **35** (1988), 103–113.

[13] P. STAVRE and FR. KLEPP, f_Ω-mapping with fixed points, *Proc. of the Symp. of Math. Timişoara* (1986), 145–148.

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