Publ. Math. Debrecen
39 / 3–4 (1991), 315–323

Normal locally conformal almost cosymplectic manifolds

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Summary. By an f-Kenmotsu manifold we mean an almost contact metric manifold which is normal and locally conformal almost cosymplectic. The local structure of such manifolds is described explicitly, and a geometric interpretation is given. Next, after deriving auxiliary curvature properties, we study f-Kenmotsu manifolds being $C(\lambda)$-manifolds (in particular, of constant curvature) or locally symmetric or Ricci-symmetric.

§1. Preliminary definitions

Let M be an almost contact metric manifold, i.e. M is a connected $(2n+1)$-dimensional differentiable manifold endowed with an almost contact metric structure (φ, ξ, η, g) (cf.[4]). As usually, denote by Φ the second fundamental form of M, $\Phi(X,Y) = g(\varphi X, Y)$, $X, Y \in \mathcal{X}(M)$. $\mathcal{X}(M)$ is the Lie algebra of differentiable vector fields on M.

For futher use, we recall the following definitions (cf. [14], [6], or [4]). The manifold M (and its structure (φ, ξ, η, g)) is said to be:

1) normal if the almost complex structure defined on the product manifold $M \times \mathbf{R}$ is integrable (equivalently, $[\varphi, \varphi] + 2d\eta \otimes \xi = 0$),

2) almost cosymplectic if $d\eta = 0$ and $d\Phi = 0$,

3) cosymplectic if it is normal and almost cosymplectic (equivalently, $\nabla\varphi = 0$, ∇ being the covariant differentiation with respect to the Levi–Civita connection).

We also need the following definition (cf. [10], [15]): The manifold M is called locally conformal, l.c. in short, cosymplectic (resp., almost cosymplectic) if M has an open covering $\{U_t\}$ endowed with differentiable functions $\sigma_t : U_t \to \mathbf{R}$ such that over each U_t the almost contact metric structure $(\varphi_t, \xi_t, \eta_t, g_t)$ defined by

$$(1.1) \qquad \varphi_t = \varphi, \quad \xi_t = e^{\sigma_t}\xi, \quad \eta_t = e^{-\sigma_t}\eta, \quad g_t = e^{-2\sigma_t}g,$$

is cosymplectic (resp., almost cosymplectic).

An almost contact metric manifold M is l.c. almost cosymplectic if and only if there exists a 1-form ω on M such that $d\omega = 0$, $d\eta = \omega \wedge \eta$, $d\Phi = 2\omega \wedge \Phi$. If the form ω verifying the above conditions exists, then it is unique. So, this is a characteristic form of a l.c. almost cosymplectic manifold. On such a manifold the form ω is given locally by $\omega | U_t = d\sigma_t$ (cf. (1.1)).

§2. f-Kenmotsu manifolds

In [10] one of the present authors has proved the following theorem:

Theorem 2.1. *For an almost contact metric manifold M, the following conditions are mutually equivalent:*

a) the manifold is normal l.c. almost cosymplectic,

b) the manifold is l.c. cosymplectic with the characteristic form $\omega = f\eta$, f being a function on M,

c) the covariant derivative of the tensor field φ is of the form

$$(\nabla_X \varphi)Y = f\{g(\varphi X, Y)\xi - \eta(Y)\varphi X\} \tag{2.1}$$

for $X, Y \in \mathcal{X}(M)$, where f is a function on M such that $df \wedge \eta = 0$.

The class of normal l.c. almost cosymplectic manifolds contains the all α-Kenmotsu manifolds, for which the characterizing analytic condition is just (2.1) with $f = \alpha = const \neq 0$ (cf. [7]). A 1-Kenmotsu manifold is Kenmotsu ([7],[8]). Considering this and simplifying the terminology a normal l.c. almost cosymplectic manifold, i.e. an almost contact metric manifold fulfilling the condition (2.1) with a function f such that $df \wedge \eta = 0$, will be called an f-Kenmotsu manifold.

Note that for an f-Kenmotsu manifold, from (2.1) it follows that

$$\nabla_X \xi = f\{X - \eta(X)\xi\}. \tag{2.2}$$

Here and in the sequel $X, Y, Z, \ldots$ denote arbitrary differentiable vector fields on the manifold unless otherwise stated.

The condition $df \wedge \eta = 0$, occurring in (2.1) and (2.2), follows in fact from (2.1) if $\dim M \geq 5$. This does not hold in general if $\dim M = 3$. Indeed, by (2.1) and (2.2) we have $d\eta = 0$ and $d\Phi = 2f\eta \wedge \Phi$, and consequently $0 = d^2\Phi = 2df \wedge \eta \wedge \Phi$, which gives the assertion. As a consequence of $df \wedge \eta = 0$, we get $df = f'\eta$ and $X(f) = f'\eta(X)$, where $f' = \xi(f)$. We also have $df' = f''\eta$ and $X(f') = f''\eta(X)$, where $f'' = \xi(f')$.

Now consider the following

Example. Let $\mathbf{R}$ be the real line with coordinate s. Fix a function σ on $\mathbf{R}$, and consider the Riemannian metric $e^{-2\sigma}ds \otimes ds$ on $\mathbf{R}$. Let N be a Kähler manifold, J its almost complex structure and G its Kähler metric. Define a cosymplectic structure $(\tilde{\varphi}, \tilde{\xi}, \tilde{\eta}, \tilde{g})$ on $\mathbf{R} \times N$ by $\tilde{\varphi}\frac{\partial}{\partial s} = 0$, $\tilde{\varphi}X = JX$ if X is a vector tangent to N, $\tilde{\xi} = e^{\sigma}\frac{\partial}{\partial s}$, $\tilde{\eta} = e^{-\sigma}ds$ and let $\tilde{g}$ be the product of the Riemannian metrics $e^{-2\sigma}ds \otimes ds$ and G. Now, consider the conformal deformation of the structure (φ, ξ, η, g) given by

$$\varphi = \tilde{\varphi}, \quad \xi = e^{-\sigma}\tilde{\xi}, \quad \eta = e^{\sigma}\tilde{\eta}, \quad g = e^{2\sigma}\tilde{g}.$$

The structure (φ, ξ, η, g) is (globally) conformal cosymplectic, its characteristic form $\omega = d\sigma$ has the property $\omega = f\eta$, where $f = \sigma'$, and it can be written in the following matrix form:

$$(2.3) \qquad \varphi = \begin{pmatrix} 0 & 0 \\ 0 & J \end{pmatrix}, \quad \xi = \begin{pmatrix} \frac{\partial}{\partial s} \\ 0 \end{pmatrix}, \quad \eta = (ds \ 0), \quad g = \begin{pmatrix} ds\otimes ds & 0 \\ 0 & e^{2\sigma}G \end{pmatrix}.$$

So, (φ, ξ, η, g) is an f-Kenmotsu structure on $\mathbf{R} \times N$. Clearly, if the function σ occurring in the above is a periodic function, then the structure (φ, ξ, η, g) can be projected on $S^1 \times N$.

Our next results characterizes locally an f-Kenmotsu manifold.

Proposition 2.2. *Let M be an f-Kenmotsu manifold. Then an arbitrary point of M has a neighborhood $U = (a, b) \times V$, where (a, b) is an open interval, V is a Kähler manifold and the structure (φ, ξ, η, g) is given on U as in (2.3), s being the coordinate on (a, b), σ a function on (a, b) and (J, G) the Kähler structure on V.*

PROOF. This follows indeed from the following two facts: a) M is l.c. cosymplectic with the characteristic form $\omega(= d\sigma_t) = f\eta$, and b) a cosymplectic manifold is locally a product of an open interval and a Kähler manifold. Q.E.D.

The following theorem provides a geometric interpretation of an f-Kenmotsu structure.

Theorem 2.3. *Let M be an almost contact metric manifold. Then M is f-Kenmotsu if and only if it satisfies the conditions:*

a) *any integral curve of the vector field ξ is a geodesic, and the tensor field φ is invariant by any local 1-parameter group of local transformations generated by ξ (analytically, $\nabla_\xi \xi = 0$ and $\mathcal{L}_\xi \varphi = 0$, where $\mathcal{L}$ is the Lie derivative),*

b) *the distribution $\mathcal{D} = \ker \eta$ is integrable, and any leaf of the foliation $\mathcal{F}$ corresponding to the distribution $\mathcal{D}$ is a totally umbilical hypersurface with constant mean curvature,*

c) *the almost Hermitian structure (J, G), induced on an arbitrary leaf $\tilde{M} \in \mathcal{F}$ by $J\tilde{X} = \varphi\tilde{X}$, $G(\tilde{X}, \tilde{Y}) = g(\tilde{X}, \tilde{Y})$, $\tilde{X}, \tilde{Y} \in \mathcal{X}(\tilde{M})$, is Kähler.*

PROOF. Assume that M is an almost contact metric manifold, for which the conditions a)–c) hold. We shall show that the identity (2.1) is fulfilled on M. Indeed note that ξ is unit and orthogonal to any leaf $\tilde{M} \in \mathcal{F}$. Thus, the shape operator A of $\tilde{M}$ is given by $A\tilde{X} = -\nabla_{\tilde{X}}\xi$, $\tilde{X} \in \mathcal{X}(\tilde{M})$. Since a leaf is totally umbilical and its mean curvature is constant, we get $\nabla_{\tilde{X}}\xi = \lambda_{\tilde{M}}\tilde{X}$ for $\tilde{X} \in \mathcal{X}(\tilde{M})$, where $\lambda_{\tilde{M}}$ is a constant depending, maybe, on the choice of the leaf. Hence and from $\nabla_\xi\xi = 0$ we see that the relation (2.2) holds on M, if $f : M \to \mathbf{R}$ is defined by $f(p) := \lambda_{\tilde{M}}$, for $p \in \tilde{M}$. From this definition it is clear that $df \wedge \eta = 0$. Now, using $\tilde{\nabla}J = 0$ and the Gauss equation

$$\nabla_{\tilde{X}}\tilde{Y} = \tilde{\nabla}_{\tilde{X}}\tilde{Y} - \lambda_{\tilde{M}}G(\tilde{X}, \tilde{Y})\xi,$$

$\tilde{\nabla}$ being the covariant differentiation with respect to the Levi-Civita connection on $\tilde{M}$, we obtain

$$(\nabla_{\tilde{X}}\varphi)\tilde{Y} = \lambda_{\tilde{M}}g(\varphi\tilde{X}, \tilde{Y})\xi$$

for $\tilde{X}, \tilde{Y} \in \mathcal{X}(\tilde{M})$. Consequently, we see that (2.1) is satisfied for $X, Y \in \mathcal{X}(M)$ if $X \perp \xi$, $Y \perp \xi$. For $Y = \xi$, (2.1) follows from (2.2). Finally, applying (2.2) and $\mathcal{L}_\xi\varphi = 0$, we derive

$$(\nabla_\xi\varphi)Y = \nabla_\xi\varphi Y - \varphi\nabla_\xi Y = [\xi, \varphi Y] - \varphi[\xi, Y] = (\mathcal{L}_\xi\varphi)Y = 0,$$

i.e. (2.1) for $X = \xi$. Thus, (2.1) holds for any $X, Y \in \mathcal{X}(M)$.

The converse statement follows by applying Proposition 2.2. Q.E.D.

Remarks. 1. In [11] one of the present authors has proved that a 3-dimensional almost contact metric manifold satisfies the identity

$$(\nabla_X\varphi)Y = g(\varphi\nabla_X\xi, Y)\xi - \eta(Y)\varphi\nabla_X\xi.$$

Therefore, for such a manifold, the relations (2.1) and (2.2) are equivalent.

2. The Riemannian manifold appearing in the Example is locally a warped product space in the sense of BISHOP and O'NEILL [3], or a semi-reducible space in the sense of KRUČKOVIČ [9].

3. The terminology used in the present paper is the same as in the paper [10]. It should be added that structures defined by certain stronger conditions than (2.1) (but under another name) were studied in [13].

4. Recently, almost contact metric manifolds whose structure tensors satisfy the condition (2.1) have been treated in relation with contact conformal transformations by ALEXIEV and GANCHEV [1], [2]. These manifolds are trans-Sasakian in the sense of OUBIÑA [5], [12].

§3. Three auxiliary propositions

In this section we collect the main curvature identities fulfilled by an arbitrary normal l.c. almost cosymplectic manifold. For such a manifold, let R denote the usual curvature operator by $R(X,Y) := [\nabla_X, \nabla_Y] - \nabla_{[X,Y]}$, and let $X \wedge Y$ be the linear operator defined by $(X \wedge Y)Z = g(Y,Z)X - g(X,Z)Y$.

Proposition 3.1. *The curvature operator of an f-Kenmotsu manifold satisfies the relations*

$$R(X,Y)\xi = -(f^2 + f')(X \wedge Y)\xi, \tag{3.1}$$

$$\begin{aligned} R(\varphi X, \varphi Y) - R(X,Y) = -f^2\{(\varphi X) \wedge (\varphi Y) - X \wedge Y\}+ \\ +f'\{\eta(X)(\xi \wedge Y) + \eta(Y)(X \wedge \xi)\}. \end{aligned} \tag{3.2}$$

PROOF. (3.1) follows, by direct calculations, from (2.2). To prove (3.2) note that we have in general

$$\begin{aligned} \varphi R(Z,W)\varphi X + R(Z,W)X = \varphi(\nabla^2_{ZW}\varphi X - \varphi\nabla^2_{ZW}X)- \\ - \varphi(\nabla^2_{WZ}\varphi X - \varphi\nabla^2_{WZ}X) - g(R(Z,W)\xi, X)\xi, \end{aligned} \tag{3.3}$$

where $\nabla^2_{ZW} = \nabla_Z\nabla_W - \nabla_{\nabla_Z W}$ is the second covariant derivative. On the other hand, rewriting (2.1) in the form

$$\nabla_W\varphi X - \varphi\nabla_W X = fg(\varphi W, X)\xi - f\eta(X)\varphi W,$$

differentiating this covariantly and using again (2.1) and (2.2), we find

$$\begin{aligned} \nabla^2_{ZW}\varphi X - \varphi\nabla^2_{ZW}X = -f\eta(\nabla_W X)\varphi Z - f\eta(\nabla_Z X)\varphi W+ \\ + f^2\eta(X)\{\eta(Z)\varphi W + \eta(W)\varphi Z\} - f^2\{g(W,\varphi X)Z+ \\ + g(Z,X)\varphi W\} - f'\eta(X)\eta(Z)\varphi W + (\,\cdot\,)\xi, \end{aligned} \tag{3.4}$$

where $(\,\cdot\,)$ denotes an expression depending on Z, W, X but playing no role whatever in what follows. In virtue of (3.4) and (3.1), the equality (3.3) takes the form

$$\begin{aligned} \varphi R(Z,W)\varphi X + R(Z,W)X = -f^2\{g(\varphi X, W)\varphi Z - g(\varphi X, Z)\varphi W+ \\ +g(X,W)Z - g(X,Z)W\} + f'\eta(X)\{\eta(Z)W - \eta(W)Z\}+ \\ +f'\{\eta(W)g(X,Z) - \eta(Z)g(X,W)\}\xi. \end{aligned} \tag{3.5}$$

Finally, from

$$g((R(\varphi X, \varphi Y) - R(X,Y))Z, W) = -g(\varphi R(Z,W)\varphi X + R(Z,W)X, Y),$$

after using (3.5), we can deduce (3.2). Q.E.D.

Consider the Ricci curvature tensor ϱ given by $\varrho(X,Y) = \text{trace}\,\{Z \to R(Z,X)Y\}$, and the Ricci operator $\tilde{\varrho}$ defined by $g(\tilde{\varrho}X, Y) = \varrho(X,Y)$.

Proposition 3.2. *The Ricci operator $\tilde{\varrho}$ of an f-Kenmotsu manifold satisfies the identities*

$$\tilde{\varrho}\xi = -2n(f^2 + f')\xi, \tag{3.6}$$

$$\varphi \circ \tilde{\varrho} = \tilde{\rho} \circ \varphi. \tag{3.7}$$

PROOF. (3.6) easily follows from (3.1). To prove (3.7) we introduce the auxiliary (0,2)-tensor ρ^* by

$$\rho^*(X,Y) = \text{ trace}\,\{Z \to -\varphi R(Z,X)\varphi Y\}.$$

One verifies that we also have

$$\rho^*(X,Y) = \text{ trace}\,\{Z \to \varphi R(Z,\varphi Y)X\}.$$

With the help of the above formulas and (3.1), we find

$$\rho^*(\varphi X, \varphi Y) = \rho^*(Y,X). \tag{3.8}$$

On the other hand, the following expression of the tensor ρ^* is a consequence of (3.5)

$$\rho^* = \varrho + \{(2n-1)f^2 + f'\}g + \{(2n-1)f' + f^2\}\eta \otimes \eta.$$

Using this in (3.8) we get

$$\varrho(\varphi X, \varphi Y) = \varrho(X,Y) + 2n(f^2 + f')\eta(X)\eta(Y),$$

whence the relation (3.7) follows. Q.E.D.

Proposition 3.3. *The curvature operator R and the Ricci operator $\tilde{\rho}$ of a 3-dimensional f-Kenmotsu manifold are given by*

$$\begin{aligned} R(X,Y) = (\frac{\tau}{2} + 2f^2 + 2f')(X \wedge Y) - \\ - (\frac{\tau}{2} + 3f^2 + 3f')\{\eta(X)(\xi \wedge Y) + \eta(Y)(X \wedge \xi)\}, \end{aligned} \tag{3.9}$$

$$\tilde{\varrho} = (\frac{\tau}{2} + f^2 + f')Id - (\frac{\tau}{2} + 3f^2 + 3f')\eta \otimes \xi, \tag{3.10}$$

where $\tau = $ trace $\tilde{\varrho}$ is the scalar curvature.

PROOF. As is known, in any 3-dimensional Riemannian manifold the curvature operator $R(X,Y)$ can be given by

$$R(X,Y) = (\tilde{\varrho}X) \wedge Y + X \wedge (\tilde{\varrho}Y) - \frac{\tau}{2}(X \wedge Y). \tag{3.11}$$

For $X \perp \xi$, using (3.1) we find $R(\xi,X)\xi = (f^2 + f')X$, and using (3.11) and (3.6) we get $R(\xi,X)\xi = (\frac{\tau}{2} + 2f^2 + 2f')X - \tilde{\varrho}X$. Comparing the equalities obtained we see that $\tilde{\varrho}X = (\frac{\tau}{2} + f^2 + f')X$ for $X \perp \xi$. This and (3.6) imply (3.10). (3.9) follows from (3.11) in view of (3.10). Q.E.D.

§4. Curvature properties

In this section various curvature conditions on f-Kenmotsu manifolds are studied.

In [7] JANSSENS and VANHECKE introduced the notion of almost $C(\lambda)$-manifolds, λ being a real number. An almost contact metric manifold M is said to be an almost $C(\lambda)$-manifold if its Riemann curvature tensor has the following property

$$R(\varphi X, \varphi Y) = R(X,Y) + \lambda\{(\varphi X) \wedge (\varphi Y) - X \wedge Y\}. \tag{4.1}$$

A normal almost $C(\lambda)$-manifold is called a $C(\lambda)$-manifold. It is known that an α-Kenmotsu manifold is a $C(-\alpha^2)$-manifold and a cosymplectic manifold is a $C(0)$-manifold. JANSSENS and VANHECKE [7] proved a theorem concerning an orthogonal decomposition of the space of the curvature tensors satisfying the condition (4.1), into irreducible components with respect to the action of the group $U(n) \times 1$. One of the components contains so-called Bochner curvature tensors.

Theorem 4.1. *Let M be an f-Kenmotsu manifold and λ a real number. Then we have:*

a) If $\dim M = 3$, then M is a $C(\lambda)$-manifold if and only if the function f satisfies the equation $f^2 + f' = -\lambda$.

b) If $\dim M \geq 5$ and M is additionally a $C(\lambda)$-manifold, then M is α-Kenmotsu and $\lambda = -\alpha^2$ ($\alpha = const \neq 0$), or M is cosymplectic and $\lambda = 0$.

PROOF. a) Let $(E_0 = \xi, E_1, E_2 = \varphi E_1)$ be an orthonormal φ-basis. Then (4.1) is satisfied trivially and independently of λ for $X = E_1$ and $Y = E_2$. In view of (3.1), the condition (4.1) holds for $X = \xi$ and an arbitrary Y if and only if $f^2 + f' = -\lambda$. b) Comparing (3.2) vith (4.1) and taking $\dim M \geq 5$ into account, we get the assertion. Q.E.D.

The following corollary is an immediate consequence of Theorem 4.1 and Proposition 3.3.

Corollary 4.2. *Let M be an f-Kenmotsu manifold and K a real number. Then we have:*

a) If $\dim M = 3$, then M is of constant curvature K if and only if the function f and the scalar curvature τ of M fulfil the equations $K = \frac{\tau}{6} = -(f^2 + f')$.

b) If $\dim M \geq 5$ and M is moreover of constant curvature K, then M is α-Kenmotsu and $K = -\alpha^2$, or M is cosymplectic and flat.

Theorem 4.3. *Let M be an f-Kenmotsu manifold. If M is locally symmetric and non-cosymplectic, then it is of constant curvature.*

PROOF. Differentiating (3.1) covariantly and using the relations $\nabla R = 0$, (2.2) and (3.1), we have

$$fR(X,Y)Z = -\xi(f^2 + f')\eta(Z)(X \wedge Y)\xi - f(f^2 + f')(X \wedge Y)Z. \tag{4.2}$$

Putting $Z = \xi$ into (4.2) and applying (3.1) we find $\xi(f^2 + f') = 0$. Consequently, using also Schur's theorem, we deduce from (4.2) that the Riemannian metric g is of constant curvature on the open and non-empty subset of M, on which $f \neq 0$. By the parallelity of R, the metric is of constant curvature on the whole of M. Q.E.D.

Theorem 4.4. *Let M be an f-Kenmotsu manifold. If M is Ricci-symmetric (i.e. $\nabla \varrho = 0$) and non-cosymplectic, then it is Einstein.*

PROOF. The scheme of this proof is the same as that of Theorem 4.3. But instead of the formula (3.1) one needs (3.6). Q.E.D.

References

[1] V.A. ALEXIEV, Almost contact metric manifolds with some conditions concerning the structure tensor fields, *Serdica, Bulg. Math. Publ.* **13** (1987), 287–292.

[2] V.A. ALEXIEV and G.T. GANCHEV, Contact conformal transformations of the almost contact metric structures, *C. R. Acad. Bulg. Sci.* **39** no. 9 (1986), 27–30.

[3] R.L. BISHOP and B. O'NEILL, Manifolds of negative curvature, *Trans. Amer. Math. Soc.* **145** (1969), 1-50.

[4] D.E. BLAIR, Contact manifolds in Riemannian geometry, *Lect. Notes in Math. 509, Springer–Verlag, Berlin–Heidelberg–New York*, 1976.

[5] D.E. BLAIR and J.A. OUBIÑA, Conformal and related changes of metric on the product of two almost contact metric manifolds, *(in press)*.

[6] S.I. GOLDBERG and K. YANO, Integrability of almost cosymplectic structures, *Pacific J. Math.* **31** (1969), 373–382.

[7] D. JANSSENS and L. VANHECKE, Almost contact structures and curvature tensors, *Kodai Math. J.* **4** (1981), 1–27.

[8] K. KENMOTSU, A class of almost contact Riemannian manifolds, *Tôhoku Math. J.* **24** (1972), 93–102.

[9] G.I. KRUČKOVIČ, On semi-reducible Riemannian spaces, *Dokl. Akad. Nauk SSSR* **115** (1957), 862–865. (in Russian)

[10] Z. OLSZAK, Locally conformal almost cosymplectic manifolds, *Colloq. Math.* **57** (1989), 73–87.

[11] Z. OLSZAK, Normal almost contact metric manifolds of dimension three, *Ann. Polon. Math.* **47** (1986), 41–50.

[12] J.A. OUBIÑA, New classes af almost contact metric manifolds, *Publ. Math. Debrecen* **32** (1985), 187–193.

[13] R. ROSCA, Exterior concurrent vector fields on a conformal cosymplectic manifold endowed with a Sasakian structure, *Libertas Math.* **6** (1986), 167–174.

[14] S. SASAKI and Y. HATAKEYAMA, On differentiable manifolds with certain structures which are closely related to almost contact structures II., *Tohoku Math. J.* **13** (1961), 281–294.

[15] I. VAISMAN, Conformal changes of almost contact metric manifolds, Geometry and Differential Geometry Haifa, 1979, pp. 435–443; Lect. Notes in Math. 792, *Springer–Verlag, Berlin–Heidelberg–New York*, 1980.

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(Received June 15, 1989)