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Connected relator spaces

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Introduction

In this paper, we study an appropriate notion of connectedness of a relator space which is a generalized uniform space lacking all the axioms of uniform space except the reflexivity of the corresponding relations [11].

The results obtained extend some classical results of Riesz and Hausdorf on topological connectedness and some recent results of Mrówka and Pervin [8] on uniform and proximal connectednesses.

Thus, our purpose here is very similar to that of Sieber and Pervin [10] who studied a related concept of connectedness of Császár's syntopogenous spaces.

0. Notations and terminology

A relator space is an ordered pair $X(\mathcal{R}) = (X, \mathcal{R})$ consisting of a set X and a nonvoid family $\mathcal{R}$ of reflexive relations on X which is called a relator on X.

If (x_α) and (y_α) are nets, A and B are sets and x is a point in $X(\mathcal{R})$, then we write:

(i) $(y_\alpha) \in \operatorname{Lim}_{\mathcal{R}}(x_\alpha)$ $\quad((y_\alpha) \in \operatorname{Adh}_{\mathcal{R}}(x_\alpha))$ if $((x_\alpha, y_\alpha))$ is eventually (frequently) in each $R \in \mathcal{R}$;

(ii) $x \in \lim_{\mathcal{R}}(x_\alpha)$ $\quad(x \in \operatorname{adh}_{\mathcal{R}}(x_\alpha))$ if $(x) \in \operatorname{Lim}_{\mathcal{R}}(x_\alpha)$ $((x) \in \operatorname{Adh}_{\mathcal{R}}(x_\alpha))$;

(iii) $B \in \operatorname{Cl}_{\mathcal{R}}(A)$ $\quad(B \in \operatorname{Int}_{\mathcal{R}}(A))$ if $R(B) \cap A \neq \emptyset$ $(R(B) \subset A)$ for all (some) $R \in \mathcal{R}$;

(iv) $x \in \operatorname{cl}_{\mathcal{R}}(A)$ $\quad(x \in \operatorname{int}_{\mathcal{R}}(A))$ if $\{x\} \in \operatorname{Cl}_{\mathcal{R}}(A)$ $(\{x\} \in \operatorname{Int}_{\mathcal{R}}(A))$.

If $\mathcal{R}$ is a relator on X, then the relators

$$\mathcal{R}^* = \left\{ S \subset X^2 : \exists R \in \mathcal{R} : R \subset S \right\},$$
$$\mathcal{R}^\# = \left\{ S \subset X^2 : \forall A \subset X : \exists R \in \mathcal{R} : R(A) \subset S(A) \right\}$$

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and

$$\hat{\mathcal{R}} = \{S \subset X : \forall x \in X : \exists R \in \mathcal{R} : R(x) \subset S(x)\}$$

are called the uniform, proximal and topological refinements of $\mathcal{R}$, respectively.

Namely, $\mathcal{R}^*$, $\mathcal{R}^\#$ and $\hat{\mathcal{R}}$ are the largest relators on X such that $\mathrm{Lim}_{\mathcal{R}^*} = \mathrm{Lim}_{\mathcal{R}}$ $(\mathrm{Adh}_{\mathcal{R}^*} = \mathrm{Adh}_{\mathcal{R}})$, $\mathrm{Cl}_{\mathcal{R}^\#} = \mathrm{Cl}_{\mathcal{R}}$ $(\mathrm{Int}_{\mathcal{R}^\#} = \mathrm{Int}_{\mathcal{R}})$ and $\mathrm{lim}_{\hat{\mathcal{R}}} = \mathrm{lim}_{\mathcal{R}}$ $(\mathrm{adh}_{\hat{\mathcal{R}}} = \mathrm{adh}_{\mathcal{R}})$ or $\mathrm{cl}_{\hat{\mathcal{R}}} = \mathrm{cl}_{\mathcal{R}}$ $(\mathrm{int}_{\hat{\mathcal{R}}} = \mathrm{int}_{\mathcal{R}})$, respectively.

Therefore, a relator $\mathcal{R}$ on X, or a relator space $X(\mathcal{R})$, is called uniformly, proximally and topologically fine if $\mathcal{R}^* = \mathcal{R}$, $\mathcal{R}^\# = \mathcal{R}$ and $\hat{\mathcal{R}} = \mathcal{R}$, respectively.

A function f from a relator space $X(\mathcal{R})$ into another $Y(\mathcal{S})$ is called continuous if $f^{-1} \circ S \circ f \in \mathcal{R}$ for all $S \in \mathcal{S}$. Moreover, f is called uniformly, proximally and topologically continuous if f is continuous as a function of $X(\mathcal{R}^*)$, $X(\mathcal{R}^\#)$ and $X(\hat{\mathcal{R}})$ into $Y(\mathcal{S})$, respectively.

It is not very hard to see that these latter continuity properties agree with the usual ones in the sense that:

(i) f is uniformly continuous iff $(y_\alpha) \in \mathrm{Lim}_{\mathcal{R}}(x_\alpha)$ implies $(f(y_\alpha)) \in \mathrm{Lim}_{\mathcal{S}}(f(x_\alpha))$;

(ii) f is proximally continuous iff $B \in \mathrm{Cl}_{\mathcal{R}}(A)$ implies $f(B) \in \mathrm{Cl}_{\mathcal{S}}(f(A))$;

(iii) f is topologically continuous iff $x \in \mathrm{lim}_{\mathcal{R}}(x_\alpha)$ $(x \in \mathrm{cl}_{\mathcal{R}}(A))$ implies $f(x) \in \mathrm{lim}_{\mathcal{S}}(f(x_\alpha))$ $(f(x) \in \mathrm{cl}_{\mathcal{S}}(f(A)))$.

Finally, a relator $\mathcal{R}$ on X, or a relator space $X(\mathcal{R})$, is called

(i) uniformly directed if for each $R, S \in \mathcal{R}$ there exists a $T \in \mathcal{R}$ such that $T \subset R \cap S$;

(ii) proximally symmetric if for each $A \subset X$ and $R \in \mathcal{R}$ there exists an $S \in \mathcal{R}$ such that $S(A) \subset R^{-1}(A)$.

The importance of these properties lies mainly in the fact that in a uniformly directed (proximally symmetric) relator space $X(\mathcal{R})$ the relation $\mathrm{Lim}_{\mathcal{R}}$ may be restricted to directed nets ($\mathrm{Cl}_{\mathcal{R}}$ is symmetric).

1. Uniform, proximal and topological connectednesses

Definition 1.1. A relator $\mathcal{R}$ on X, or a relator space $X(\mathcal{R})$, will be called connected if

$$A^2 \cup (X \setminus A)^2 \notin \mathcal{R}$$

for any proper nonvoid subset A of X.

Moreover, a relator $\mathcal{R}$ on X, or a relator space $X(\mathcal{R})$, will be calleduniformly, proximally and topologically connected if the relators $\mathcal{R}^*$, $\mathcal{R}^\#$ and $\hat{\mathcal{R}}$ are connected, repectively.

Remark 1.2. Because of the inclusions $\mathcal{R} \subset \mathcal{R}^* \subset \mathcal{R}^\# \subset \hat{\mathcal{R}}$, it is clear that 'topologically connected' $\Longrightarrow$ 'proximally connected' $\Longrightarrow$ 'uniformly connected' $\Longrightarrow$ 'connected'.

The fact that the converse implications do not, in general, hold will be cleared up by our forthcoming Examples 1.8, 4.9, and 4.10.

Remark 1.3. Because of the equalities $(\mathcal{R}^\#)^* = \mathcal{R}^\#$ and $(\hat{\mathcal{R}})^\# = \hat{\mathcal{R}}$, it is clear that $\mathcal{R}$ is proximally (topologically) connected if and only if $\mathcal{R}^\#$ $(\hat{\mathcal{R}})$ is uniformly (proximally) connected.

The appropriateness of Definition 1.1 is apparent from the next theorem and its subsequent corollary.

Theorem 1.4. *If $X(\mathcal{R})$ is a relator space, and moreover $Y = \{0, 1\}$ and $\mathcal{S} = \{\triangle_Y\}$, then the following assertions are equivalent:*
(i) $X(\mathcal{R})$ is connected;
(ii) each continuous function f from $X(\mathcal{R})$ into $Y(\mathcal{S})$ is constant.

Proof. If f is a function from X into Y, and $A = f^{-1}(1)$, then one can easily check that

$$f^{-1} \circ \triangle_Y \circ f = f^{-1} \circ f = A^2 \cup (X \setminus A)^2.$$

And hence, by the corresponding definitions, the equivalence of the assertions (i) and (ii) is obvious.

As an immediate consequence of this theorem, we can at once state

Corollary 1.5. *A relator space $X(\mathcal{R})$ is uniformly, proximally, resp. topologically connected if and only if each uniformly, proximally, resp. topologically continuous function f from $X(\mathcal{R})$ into $Y(\mathcal{S})$, with $Y = \{0, 1\}$ and $\mathcal{S} = \{\triangle_Y\}$, is constant.*

At this point, it seems also convenient to state the next obvious

Theorem 1.6. *If X is an arbitrary set and $\mathcal{R} = \{\triangle_X\}$, then*
(i) $\mathcal{R}$ is connected if and only if card$(X) \neq 2$;
(ii) $\mathcal{R}$ is uniformly connected if and only if card$(X) \leq 1$.

Remark 1.7. Note that if $\operatorname{card}(X) \leq 1$, then any relator $\mathcal{R}$ on X is topologically connected.

In this respect, it is also worth mentioning that the relator $\mathcal{R} = \{X^2\}$ is always topologically connected.

Now, as a trivial consequence of Theorem 1.6, it is also convenient to state

Example 1.8. If X is a set such that $\operatorname{card}(X) \geq 3$, and $\mathcal{R} = \{\triangle_X\}$, then $\mathcal{R}$ is a connected relator on X such that $\mathcal{R}$ is not uniformly connected.

2. Characterizations of proximal and topological connectednesses

To state briefly our subsequent characterization theorems, it seems convenient to introduce two different kinds of clopen sets.

Definition 2.1. A subset A of a relator space $X(\mathcal{R})$ will be called proximally (topologically) open if

$$A \in \operatorname{Int}_{\mathcal{R}}(A) \quad (A \subset \operatorname{int}_{\mathcal{R}}(A)).$$

Moreover, a subset A of a relator space $X(\mathcal{R})$ will be called proximally (topologically) clopen if both A and $X \setminus A$ are proximally (topologically) open.

Remark 2.2. Because of the corresponding definitions, it is clear that each proximally open set is also topologically open.

Clearly, the converse is not true. However, as an immediate consequence of [11, Theorem 6.7], we still have the next useful

Proposition 2.3. *If $X(\mathcal{R})$ is a relator space and $A \subset X$, then the following assertions are equivalent:*

(i) A is a topologically open subset of $X(\mathcal{R})$;

(ii) A is a proximally open subset of $X(\hat{\mathcal{R}})$.

Remark 2.4. Hence, it is clear that a subset of a topologically fine relator space is topologically open if and only if it is proximally open.

Now, it is easy to state and prove our basic characterization theorems of proximal and topological connectednesses.

Theorem 2.5. *If $X(\mathcal{R})$ is a relator space, then the following assertions are equivalent:*

(i) $X(\mathcal{R})$ is proximally connected;

(ii) no proper nonvoid subset of $X(\mathcal{R})$ is proximally clopen.

Proof. If (i) does not hold, then there exists a proper nonvoid subset A of X such that

$$R = A^2 \cup (X \setminus A)^2 \in \mathcal{R}^{\#}.$$

Hence, in particular, it follows that there exist R_1, $R_2 \in \mathcal{R}$ such that

$$R_1(A) \subset R(A) = A \quad \text{and} \quad R_2(X \setminus A) \subset R(X \setminus A) = X \setminus A.$$

This shows that both A and $X \setminus A$ are proximally open subsets of $X(\mathcal{R})$. And thus, (ii) cannot hold.

Conversely, if (ii) does not hold, then there exists a proper nonvoid subset A of X for which there exists $R_1,\, R_2 \in \mathcal{R}$ such that

$$R_1(A) \subset A \quad \text{and} \quad R_2(X \setminus A) \subset X \setminus A.$$

Hence, by defining $R_B \in \mathcal{R}$ for each $B \subset X$ such that

$$R_B = R_1 \quad \text{if} \quad \emptyset \neq B \subset A, \quad \text{and} \quad R_B = R_2 \quad \text{if} \quad \emptyset \neq B \subset X \setminus A,$$

we can at once state that the relation

$$R = A^2 \cup (X \setminus A)^2$$

has the property

$$R_B(B) \subset R(B)$$

for all $B \subset X$. Consequently, $R \in \mathcal{R}^{\#}$, and thus (i) cannot hold.

From this theorem, using Remark 1.3 and Proposition 2.3, we can at once derive

Theorem 2.6. *If $X(\mathcal{R})$ is a relator space, then the following assertions are equivalent:*
(i) $X(\mathcal{R})$ is topologically connected;
(ii) no proper nonvoid subset of $X(\mathcal{R})$ is topologically clopen.

Remark 2.7. Because of [11, Theorem 2.6], it is clear that a subset A of a relator space $X(\mathcal{R})$ is proximally clopen if and only if

$$X \setminus A \notin \mathrm{Cl}_{\mathcal{R}}(A) \quad \text{and} \quad A \notin \mathrm{Cl}_{\mathcal{R}}(X \setminus A)$$

Hence, we can state that a subset A of a proximally symmetric relator space $X(\mathcal{R})$ is proximally clopen if and only if A is proximally open.

Therefore, from Theorem 2.5, we can also at once derive the next

Theorem 2.8. *A proximally symmetric relator space $X(\mathcal{R})$ is proximally connected if and only if no proper nonvoid subset of $X(\mathcal{R})$ is proximally open.*

Hence, by Remarks 1.3 and 2.4, it is clear that we can also state

Theorem 2.9. *A proximally symmetric, topologically fine relator space $X(\mathcal{R})$ is topologically connected if and only if no proper nonvoid subset of $X(\mathcal{R})$ is topologically open.*

Remark 2.10. Note that a relator space may be called proximally (topologically) indiscrete if no proper nonvoid subset of it is proximally (topologically) open.

3. Characterizations of uniform connectedness

Theorem 3.1. *If $X(\mathcal{R})$ is a relator space, then the following assertions are equivalent:*

(i) $X(\mathcal{R})$ is uniformly connected;

(ii) for each proper nonvoid subset A of X there exists a net $((x_\alpha, y_\alpha))$ in $A \times (X \setminus A) \cup (X \setminus A) \times A$ such that $(x_\alpha) \in \mathrm{Lim}_{\mathcal{R}}(y_\alpha)$;

(iii) for each proper nonvoid subset A of X there exists a net $((x_\alpha, y_\alpha))$ in $A \times (X \setminus A) \cup (X \setminus A) \times A$ such that $(x_\alpha) \in \mathrm{Adh}_{\mathcal{R}}(y_\alpha)$.

Proof. If (i) holds and A is a proper nonvoid subset of X, then

$$A^2 \cup (X \setminus A)^2 \notin \mathcal{R}^*.$$

Thus, for each $R \in \mathcal{R}$, there exists an $(x_R, y_R) \in R$ such that $(x_R, y_R) \notin A^2 \cup (X \setminus A)^2$, i.e.,

$$(x_R, y_R) \in A \times (X \setminus A) \cup (X \setminus A) \times A.$$

Hence, by preordering $\mathcal{R}$ with the reverse set inclusion (the largest possible preorder), we can at once state that $((x_R, y_R))_{R \in \mathcal{R}}$ is a net in $A \times (X \setminus A) \cup (X \setminus A) \times A$ such that

$$(x_R) \in \mathrm{Lim}_{\mathcal{R}}(y_R) \quad ((x_R) \in \mathrm{Adh}_{\mathcal{R}}(y_R)).$$

Thus, (i) implies (ii) and (iii).

The converse implications (ii) $\Longrightarrow$ (i) and (iii) $\Longrightarrow$ (i) are even more obvious.

Remark 3.2. In the assertion (iii), we may always assume that the net $((x_\alpha, y_\alpha))$ is directed.

Moreover, assuming the uniform directedness of $X(\mathcal{R})$, we can supplement Theorem 3.1 with the next useful

Theorem 3.3. *If $X(\mathcal{R})$ is a uniformly directed relator space, then the following assertions are equivalent:*

(i) $X(\mathcal{R})$ is uniformly connected;

(ii) for each proper nonvoid subset A of X there exists a directed net $((x_\alpha, y_\alpha))$ in $A \times (X \setminus A)$ or $(X \setminus A) \times A$ such that $(x_\alpha) \in \mathrm{Lim}_{\mathcal{R}}(y_\alpha)$;

(iii) for each proper nonvoid subset A of X there exists a directed net $((x_\alpha, y_\alpha))$ in $A \times (X \setminus A)$ or $(X \setminus A) \times A$ such that $(x_\alpha) \in \mathrm{Adh}_{\mathcal{R}}(y_\alpha)$.

Proof. From the proof of Theorem 3.1, it is clear that in this case the net $((x_\alpha, y_\alpha))$ may be assumed to be directed not only in the assertion (iii), but also in the assertion (ii) of Theorem 3.1.

Therefore, to obtain Theorem 3.3, we need only to note that each directed net being in a finite union has a directed subnet being in some member of that union, and to apply [11, Theorem 1.8] and [12, Theorem 1.5].

From this theorem, by using [11, Theorem 3.1], Remark 2.7 and Theorem 2.5, we can at once derive the next striking

Theorem 3.4. *A uniformly directed relator space $X(\mathcal{R})$ is uniformly connected if and only if it is proximally connected.*

Remark 3.5. Our forthcoming Example 4.9 will show that even a proximally directed and uniformly connected relator space $X(\mathcal{R})$ need not be proximally connected.

4. Connectedness properties of the Davis-Pervin relators

Theorem 4.1. *If $\mathcal{A}$ is a nonvoid family of subsets of a set X and*

$$R_A = A^2 \cup (X \setminus A) \times X$$

for all $A \in \mathcal{A}$, then

$$\mathcal{R}_{\mathcal{A}} = \{R_A : A \in \mathcal{A}\}$$

is a uniformly connected relator on X.

Proof. If $B \subset X$ such that

$$B^2 \cup (X \setminus B)^2 \in \mathcal{R}^*_{\mathcal{A}},$$

then there exists an $A \in \mathcal{A}$ such that

$$A^2 \cup (X \setminus A) \times X \subset B^2 \cup (X \setminus B)^2.$$

And this implies that $B = X$ or $\emptyset$.

Remark 4.2. The relator $\mathcal{R}_{\mathcal{A}}$ has been called the Davis-Pervin relator on X generated by $\mathcal{A}$ in [12, p. 195].

The proximal and topological connectedness properties of $\mathcal{R}_{\mathcal{A}}$ can be easily derived from Theorems 2.5 and 2.6 by using the next simple

Proposition 4.3. *If $\mathcal{A}$ is a nonvoid family of subsets of a set X and B is a proper nonvoid subset of X, then*

(i) B is a proximally open subset of $X(\mathcal{R}_{\mathcal{A}})$ if and only if $B \in \mathcal{A}$;

(ii) B is a topologically open subset of $X(\mathcal{R}_{\mathcal{A}})$ if and only if B is a union of certain members of $\mathcal{A}$.

Proof. If B is a proximally open subset of $X(\mathcal{R}_{\mathcal{A}})$, then there exists an $A \in \mathcal{A}$ such that

$$R_A(B) \subset B.$$

And hence, since $B \neq X$ and $B \neq \emptyset$, it follows that $B = A$.

Conversely, if $B \in \mathcal{A}$, then since

$$R_B(B) = B,$$

it is clear that B is a proximally open subset of $X(\mathcal{R}_{\mathcal{A}})$.

This proves (i). The proof of (ii) is even more obvious.

Remark 4.4. Hence, it is clear that if $\mathcal{A}$ is closed under arbitrary unions, then a subset B of $X(\mathcal{R}_{\mathcal{A}})$ is proximally open if and only if it is topologically open.

Now, as an immediate consequence of Theorem 2.5 and 2.6 and Proposition 4.3, we can at once state the next useful

Theorem 4.5. *If $\mathcal{A}$ is a nonvoid family of subsets of a set X, then*

(i) $\mathcal{R}_{\mathcal{A}}$ is proximally connected if and only if there is no proper nonvoid subset B of X such that both B and $X \setminus B$ are in $\mathcal{A}$;

(ii) $\mathcal{R}_{\mathcal{A}}$ is topologically connected if and only if there is no proper nonvoid subset B of X such that both B and $X \setminus B$ are unions of certain members of $\mathcal{A}$.

Remark 4.6. Hence, it is clear that if $\mathcal{A}$ is closed under arbitrary unions, then $\mathcal{A}$ is proximally connected if and only if it is topologically connected.

Remark 4.7. From Theorem 4.5, it is also clear that if $X \neq A_1 \cup A_2$ for all $A_1, A_2 \in \mathcal{A}$ $\quad (X \neq \cup\mathcal{A})$, then $\mathcal{R}_{\mathcal{A}}$ is proximally (topologically) connected.

However, in view of Theorem 1.6, it is more interesting to state now another easy consequence of Theorem 4.5.

Corollary 4.8. *If $\mathcal{A}$ is a partition of a nonvoid set X, then*
(i) $\mathcal{R}_{\mathcal{A}}$ is proximally connected if and only if card $(\mathcal{A}) \neq 2$;
(ii) $\mathcal{R}_{\mathcal{A}}$ is topologically connected if and only if card $(\mathcal{A}) = 1$.

Hence, by Theorem 4.1, we can at once state

Example 4.9. If $\mathcal{A}$ is a partition of a set X such that $\text{card}(\mathcal{A}) = 2$, then $\mathcal{R}_{\mathcal{A}}$ is a uniformly connected relator on X such that $\mathcal{R}_{\mathcal{A}}$ is not proximally connected.

Moreover, from Corollary 4.8, we can also at once state

Example 4.10. If $\mathcal{A}$ is a partition of a set X such that $\text{card}(\mathcal{A}) \geq 3$, then $\mathcal{R}_{\mathcal{A}}$ is a proximally connected relator on X such that $\mathcal{R}_{\mathcal{A}}$ is not topologically connected.

Remark 4.11. Note that if $\mathcal{A}$ is as in Corollary 4.8, then $\mathcal{R}_{\mathcal{A}}$ is not only strongly transitive, but also proximally directed [12, Example 1.3].

In this respect, it is also worth mentioning that if $\mathcal{A}$ is as in Example 4.9, then $\mathcal{R}_{\mathcal{A}}$ is in addition properly symmetric in the sense that $\mathcal{R}_{\mathcal{A}}^{-1} = \mathcal{R}_{\mathcal{A}}$.

5. Notes and comments

It is commonly accepted but quite unreasonable practice to call a uniform space connected if the underlying topological space is connected.

Proper definitions for connectedness of a uniform space have only been considered by Lubkin [7, p. 207], Mrówka and Pervin [8], Sieber and Pervin [10] and James [3, p. 125].

The first part of our Definition 1.1 is partly due to Levine [5] who showed that a topological space is connected iff its Pervin quasi-uniformity is connected.

The second part of Definition 1.1 has mainly been suggested by some former results on uniform, proximal and topological continuities [11].

Because of Theorem 1.4 and Corollary 1.5, it is clear that our present definitions of the various connectednesses agree with those of the above authors.

The proximally open sets, Proportion 2.3 and the derivation of Theorem 2.6 appear to be completely new. They well illustrate the appropriateness of our treatment.

Theorems 2.6 and 2.5, together with Corollary 1.5, greatly extend a basic fact from topology [3, p. 114] and a part of [8, Theorem 1], respectively.

Theorems 3.1 and 3.3* seem to have no analogues in the existing literature, while a particular case of Theorem 3.4 was already proved in [8].

If $\mathcal{A}$ is a topology, then the relator $\mathcal{R}'_{\mathcal{A}}$ consisting of all finite intersections of members of $\mathcal{R}_{\mathcal{A}}$ was first utilized by Davis [2] and Pervin [9].

However, the relationship between $\mathcal{A}$ and $\mathcal{R}'_{\mathcal{A}}$ was more fully explored only by Levine [5] who actually showed that $\mathcal{R}'_{\mathcal{A}}$ is uniformly connected iff $\mathcal{A}$ is connected.

Finally, we remark that Levine [6] also proved several relevant theorems concerning well-chained uniformities which will as well be extended to relator spaces.

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*Meantime, we observed that our Theorem 3.3 is closely related to Theorem 82 of L.E. Ward, Topology, Marcel Dekker, New York, 1972.