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**Title:**  $D(-1)$ -tuples in the ring  $\mathbb{Z}[\sqrt{-k}]$  with  $k > 0$

**Author(s):** Yasutsugu Fujita and Ivan Soldo

Let  $n$  be a non-zero integer and  $R$  a commutative ring. A  $D(n)$ - $m$ -tuple in  $R$  is a set of  $m$  non-zero elements in  $R$  such that the product of any two distinct elements plus  $n$  is a perfect square in  $R$ . In this paper, we prove that there does not exist a  $D(-1)$ -quadruple  $\{a, b, c, d\}$  in the ring  $\mathbb{Z}[\sqrt{-k}]$ ,  $k \geq 2$  with positive integers  $a < b \leq 8a - 3$  and negative integers  $c$  and  $d$ . By using that result, we were able to prove that such a  $D(-1)$ -pair  $\{a, b\}$  cannot be extended to a  $D(-1)$ -quintuple  $\{a, b, c, d, e\}$  in  $\mathbb{Z}[\sqrt{-k}]$  with integers  $c, d$  and  $e$ . Moreover, we apply the obtained result to the  $D(-1)$ -pair  $\{p^i, q^j\}$  with arbitrary different primes  $p, q$  and positive integers  $i, j$ .

**Address:**

Yasutsugu Fujita  
Department of Mathematics  
College of Industrial Technology  
Nihon University  
2-11-1 Shin-ei, Narashino, Chiba  
Japan

**Address:**

Ivan Soldo  
Department of Mathematics  
University of Osijek  
Trg Ljudevita Gaja 6  
HR-31 000 Osijek  
Croatia