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**Title:** Relative cohomology dimensions of complexes based on degreewise cotorsion pairs

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Let  $R$  be an associative ring with identity, and  $(\mathcal{A}, \mathcal{B})$  a hereditary cotorsion pair generated by a set in  $R\text{-Mod}$ . Then  $(\text{dw } \tilde{\mathcal{A}}, (\text{dw } \tilde{\mathcal{A}})^\perp)$  is a complete and hereditary cotorsion pair (we call it the degreewise cotorsion pair) in the category of  $R$ -complexes, where  $\text{dw } \tilde{\mathcal{A}}$  denotes the class of all complexes  $X$  with components  $X_n \in \mathcal{A}$  for all  $n \in \mathbb{Z}$ . For any complexes  $X$  and  $Y$  and any  $n \in \mathbb{Z}$ , we define the relative cohomology groups  $\text{Ext}_{\text{dw } \tilde{\mathcal{A}}}^n(X, Y)$  based on the degreewise cotorsion pair and investigate the vanishing of the relative cohomology groups. Specifically, we introduce the relative cohomology dimension of  $X$  related to  $\text{dw } \tilde{\mathcal{A}}$ -precovers, and then show that such a dimension of  $X$  is equal to the least integer  $n$  for which  $\text{Ext}_{\text{dw } \tilde{\mathcal{A}}}^i(X, Y) = 0$  for all  $i > n$  and all  $R$ -modules  $Y \in \mathcal{B}$ , which recovers the result on relative cohomology dimensions (defined by Liu) of complexes related to Gorenstein projective precovers.

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