Ottó Varga
In memoriam

1909—1969

On June 14th, 1969, died OTTÓ VARGA. We knew that he had been ailing for some time; nevertheless the news of his untimely death came as a sudden shock, and inflicted a keen sense of loss and bereavment upon the community of Hungarian mathematicians. Ottó Varga was much more than an outstanding and highly sucessful research worker: he was the founder and the leading exponent of differential geometric research in Hungary.

Ottó Varga was born in 1909 at Szepetnek, in what is today Slovakia. He conducted his secondary studies in the nearby town of Késmárk, a picturesque place of old historic traditions at the foot of the Tátra mountains. He started his higher studies at the Architectural Department of the Vienna Polytechnic, but he soon became aware of a certain discrepancy between his individual dispositions and the kind of studies he was conducting there. Accordingly, after a year he left the Vienna Polytechnic for Prague's old Charles University. On concluding his university studies, he obtained his doctorate and his habilitation at a quite young age, in 1933 and in 1937, respectively. In the meantime he spent a year of fruitful research in Hamburg, at the side of W. Blaschke. A few years later, he was entrusted with leading a department at Charles University. After the occupation of Czechoslovakia, he left Prague, and after a short stay in Kolozsvár, he took over the Department of Mathematics at Debrecen University in 1942. During the relatively short time of less than two decades in which he held the post of director, the Department was transformed — in line with the general upsurge of our country — from not much more than a one-man institution, consisting of his prominent personality alone, into a thriving community of mathematicians, a center of research actively participating in international mathematical life. It was during this time that this periodical was founded, and Ottó Varga took a decisive part in organizing and directing the policies of the new journal. In 1958 he left Debrecen for Budapest, working during the last decade of his life as professor of the Budapest Polytechnic and as a member of the Mathematical Research Institute of the Hungarian Academy of Sciences, actively engaged till the very end in mathematical research.

Under the influence of his eminent teacher at Prague University, Professor L. BERWALD, the interests of Ottó Varga soon turned towards differential geometry, in particular towards Fincler geometry, then in its early development. This orientation, adopted in youth, remained decisive for the whole of his scientific career, and on the basis of his life work we can regard him together with his teacher L. Berwald and with E. CARTAN himself the most prominent exponent of Cartan's theory of Finsler spaces.

Although the starting point of this geometry is the 1918 thesis of Finsler, its comprehensive development was largely due to Cartan's ideas. While in Riemannian geometry points having the same infinitesimal distance from a fixed point are situated on an ellipsoid varying with its center, this so called indicatrix surface is replaced in Finsler geometry by a differentiable symmetrical convex surface. Cartan replaced this indicatrix surface in each direction (v) by an ellipsoid osculating the indicatrix surface in the second order in the direction (v), and thus he turned the point-space into a more complicated space of line-elements, but he cleared the way for the application of the apparatus of Riemannian geometry in Finsler geometry.

Ottó Varga started his contributions to Finsler geometry by treating the fundamental problem of the introduction of an affine connection into the Finsler space [1][1]). — One can set up a linear mapping between the vectors defined in the different lineelements by giving an affine transformation. If the functions describing this linear mapping are linear in the differentials of the coordinates of the point and direction, then we are given an affinely connected space of the line-elements, where the vectors ordered to each other by this mapping are called parallel. Cartan enforced the euclidean character of the connection, i.e. the equality of lengths in the Finsler metric of the parallel diplaced vectors by certain formal conditions imposed on the coefficients determining the affine connection.

In one of his first papers on Finsler spaces [8] O. Varga succeeded in finding a most elegant and completely geometrical way for introducing the euclidean connection into the Finsler geometry. In order to define the parallel displacement of the vectors along a one parameter family of line-elements $(x(t), v(t))$ he considers the geodesics of the Finsler space tangent to this one parameter family. Then this set of geodesics will be extended in a neighbourhood $\mathfrak{B}(x)$ of the curve $x(t)$ to a field of geodesics. By the tangents of the geodesic field a direction is determined in every point of $\mathfrak{B}$, and the ellipsoids osculating the indicatrixes of the Finsler space in these directions define an osculating Riemannian space on $\mathfrak{B}$. Varga calls the vectors of the Finsler space parallel displaced along this set of line-elements if they are parallel in the constructed Riemannian space. The coefficients of the connection can easily be calculated from this definition, and they coincide with those, which are derived from Cartan's postulates. Thus his method is also an interesting geometrical interpretation of Cartan's postulates.

The osculating Riemannian space was also instrumental in extending and broadening our knowledge of the geometrical role of more significant quantities of Finsler geometry.

The curvature $R(x, p)$ of a Riemannian space V_n at a point x and plane position p is the curvature of the twodimensional V_2 consisiting of the geodesics tangent to p, and this later curvature equals the Gaussian curvature of the surface representing the V_2 in the euclidean three-space. This $R(x, p)$ was generalized and transfered into the Finsler spaces partly on the basis of its formal expression, and partly on the basis of its role taken in certain variational problems. The result of this gene-

[1]) Numbers in brackets refer to the appended list of the mathematical works of O. Varga.

ralization, the Riemann—Berwald curvature $R(x, v, X)$ of a Finsler space is defined at a line-element (x, v) and a vector X defined at this (x, v). O. Varga has shown, that $R(x, v, X)$ too can be considered as the curvature of a twodimensional subspace, similarly to the case of Riemannian geometry. Namely also the geodesics of the Finsler space F_n tangent to the plane position v, X form a surface X_2, which is turned into an F_2 by the F_n. Let us consider in this F_2 the geodesic C having as its tangent the direction X at x, and let us consider the Riemannian space of Varga osculating the F_2 along C and its tangents. O. Varga has proved [22], that the curvature $R(x)$ of this osculating Riemannian space is equal to $R(x, v, X)$. On the other hand $R(x)$ is equal to the Finsler-curvature in the direction v of the F_2. Thus $R(x, v, X)$ is likewise the curvature of a subspace tagent to the plane position v, X, showing a complete analogy to the $R(x, p)$ of a Riemannian space. This result also provides the $R(x, v, X)$ with a new geometrical content.

Among the Riemannian spaces the spaces of constant curvature, i.e. those for which $R(x, p)$ is independent of the plane position p play an important part. In these spaces, according to a theorem of F. Schur, R is independent also of the point x. These spaces are exactly models for the elliptic, parabolic (euclidean) and hyperbolic spaces. Thus also the Finsler spaces of constant curvature, i.e. the spaces in which $R(x, v, X)$ is constant, and the spaces of scalar curvature, i.e. the spaces in which R depends only on the line-element (x, v) command a well deserved special interest among the Finsler spaces. O. Varga applied with success the parallel displacement of line-elements in their investigation. Let us take a surface in the F_n having v and X as its tangents at x. Let us consider at x an arbitrary line-element (x, v_1) and let us displace it parallel along a closed curve of the surface back to its starting point x. O. Varga has shown in [27], that the F_n is of scalar curvature iff the difference of v_1 and of the v_2 representing the result of the parallel displacement of v_1 is a linear combination of v_1, v and X for an arbitrary curve on the surface. If this differencevector is already expressible by the v and X, then F_n is of constant curvature.

Although this later quality is characteristic of the spaces of constant curvature among the Riemannian spaces as well[2]), this is not the only possible generalization of the Riemannian spaces of constant curvature within the Finsler spaces. O. Varga describes yet another very simple and natural generalization of the Riemannian spaces of constant curvature completely different from the preceding one [14], and gives a criterion for a Finsler space to belong to this class [30], [33]. This criterion is related to the existence of absolute parallel fields of line-elements, and to the Riemannian spaces induced by them.

Already in his youth had O. Varga studied the hypersurfaces of the Finsler spaces [9]. Towards the end of his career he resumed this theme achieving a number of results concerning hypersurfaces. Each of them is a little master piece of Finsler geometry.

The F_n turns any of its hypersurfaces into a Finsler space F_{n-1}, and this Finslerian metric determines on the hypersurface an euclidean connection which Cartan called intrinsic. But the metric of the F_{n-1} allows another remarkable connection too, called induced connection, in which the vectors of the F_{n-1} are displaced paral-

[2]) See E. Bortolotti, *Ann. di Math. Bologna* IV. 8 (1930), 53—101.

lel along the curves of the F_{n-1} according to the connection of the F_n and at last the resulting vectors are projected perpendicularly on the F_{n-1}. O. Varga conducted investigations in order to determine the hypersurfaces, for which these two connections coincide. He found [45], that these hypersurfaces are either totally geodesic (surfaces on which any geodesic of the surface is a geodesic of the F_n too), or they can be characterized by the vanishing of part of the coefficients of a differential form.

In these investigations the method of the repére mobile plays an important part. According to this method the absolute differentials of the tangents of a hypersurface are decomposed according to the tangents and the normal cevtor of the hypersurface, the coefficients being differential forms. If in such a decomposition the coefficients of the normal vector are $A_{\alpha\beta}dx^{\alpha}+B_{\alpha\beta}dv^{\beta}$, then the above criterion is the vanishing of $B_{\alpha\beta}$. — O. Varga has shown [46] that if through any hyperplane position a totally geodesic hypersurface can be laid, then the Finsler space is of constant curvature. This quality characterizes the Finsler spaces of constant curvature. — It through any hyperplane position a hypersurface can be still laid for which $B_{\alpha\beta}=0$, then F_n is a Riemannian space [53].

O. Varga took a special interest in the relation of the Finsler spaces to the simpler Minkowski spaces. Minkowski spaces are those specializations of the Finsler spaces, in which the convex indicatrix is the same at every point. In [11] he presents a very clever and fully geometrical derivation of the euclidean connection in the Minkowskian geometry with the aid of the osculating euclidean metrics attached to the different directions, making no use of the roundabout way of Finsler geometry. — Also within Minkowskian geometry he paid special attention to the hypersurfaces. It is well known, that if a hypersurface in euclidean three-space has constant normalcurvature in every direction, then it is a Riemannian space of constant curvature. O. Varga has shown [50], that if a hypersurface in the Minkowski-space has constant normalcurvature in every line-element, then the geometry of the hypersurface induced by the Minkowski space on it is a Finsler geometry of constant curvature with respect to the induced connection. He studied the case, when a Finsler space induces a Minkowskian geometry on its hypersurface. He has proved [51], [53], that this is the case only if the induced and the intrinsic connections coincide, and the geometry of the hypersurface is a Minkowskian one, exactly when a certain vector explicitly expressed by him is proportional to the normal vector along the hypersurface.

One of his significant achievements is related to the angular metric. By making correspond to each point of the indicatrix the intersection of the tangent plane of the indicatrix and of the eillpsoid osculating the indicatrix surface in second order at the point considered the indicatrix surface becomes a Riemannian space. By the angle of two neighbouring directions starting from one point is meant the distance of the points on the indicatrix corresponding to these directions and measured in the Riemannian geometry of the indicatrix. O. Varga [36] succeeded in proving that the curvature tensor of this Riemannian geometry is in a very close and simple relation to the curvature tensor S of the Finsler space. He has proved, that this Riemannian geometry is then, and only then, of constant curvature, if also a certain scalar made up in a simple way from S is constant, and this curvature has value 1 iff $S=0$. These results are of great importance not only since they revealed the connection between the tensor S and the angular metric, but also because, previously we have had a relatively scarce knowledge about the geometrical role of the tensor S as compared with the other two curvature tensors of the Cartanian theory of the Finsler spaces.

In one of his last papers [56] O. Varga has investigated also the hypersurfaces of a Finsler space from the point of view of the angular metric, and he has determined the effect of the coincidence of the induced and the intrinsic connection on the angular metric of the hypersurface.

O. Varga contributed to the devlopment of nearly all branches of Finsler geometry. He obtained new results concerning the mapping on each other of two Finsler spaces [21], the decomposition of a Finsler space into the product of two others [42], and concerning the metrizability of affinely connected spaces of line-elements [31]. We cannot discuss here each of his achievements. Nevertheless, we must touch upon his results concerning the possibility of introducing normal coordinates and concerning a complete set of invariants, as well as the notion of the quasigeodesic [24].

Normal coordinates are the ones, which may be introduced by analytical transformations and in which the equations of the geodesics starting from a fixed point are linear. Their importance stems from the fact that by them normal affinors and throuhg a replacement theorem a complete set of invariants may be obtained in point spaces, that is, such invariants may be gained, by which any other invariant of the space is expressible. J. Douglas[3]) proved, that normally such a coordinate system does not exist in line-element spaces. Thus it seemed, that insurmountable obstacles stood in the way of employing this succesful method for the determination of a complete set of invariants, a task of fundamental importance from a theoretical point of view. O. Varga was able to surmount these difficulties by showing that the negative result in the case of line-element spaces was not due to this method, but to the inappropriate manner of carrying over the notion of geodesics. For there exists even in line-element spaces a coordinate system in which the equation of the quasigeodesics starting from a fixed point is linear. The quasigeodesic introduced by O. Varga is a curve, whose tangents are parallel displaced with respect to a field of line-elements parallel displaced along the curve so, that the field has a prefixed direction in the center of the coordinatensystem. This notion reduces in point spaces to the notion of a common geodesic. O. Varga managed to show, that this coordinate system can be used in affinelly connected line-element spaces for the determination of a complete system of invariants in the same way as the normal coordinate system constructed with the aid of the geodesics could be in affinely connected pointspaces. According to this result the coefficients determining the affine connection, the main curvature tensor, their partial derivatives and the covariant derivatives of these form a complete set of invariants of an affinely connected line-element space. A. Rapcsák[4]) extended this method to Cartan spaces, and later O. Varga extended it to Kawaguchi spaces and to their affine generalizations [38], [39].

The main field of Varga's scientific activity was Finsler geometry, but he obtained notable results concerning the Riemannian spaces of constant curvature too. It was known for long, that if through any hyperplane position of a Riemannian space

[3]) J. Douglas, *Ann. Math.*, **29** (1928), 143—168. Esp. p. 163.

[4]) A. Rapcsák, *Publ. Math. Debrecen*, **4** (1956), 276—293.

a totally geodesic hypersurface can be laid, i.e. if the axion of plane is fullfiled, then the space is of constant curvature. But this criterion-like quality, does not separate the spaces of constant negative and constant positive curvature. O. Varga managed to characterize these spaces separately by subtler qualities. He proved [34], [43], that the spaces of constant negative curvature may be characterized among the Riemannian spaces by the quality that through any hyperplane position two hypersurfaces can be laid so that the geometry induced on them by the Riemannian space is euclidean (the hypersurfaces are paraspheres), and the spaces of constant positive curvature may be characterized by the condition, that through any hyperplane position a totally geodesic hypersurface can be laid, which is turned by the embedding space into a Riemannian space of constant positive curvature.

Let us finish by mentioning his earliest works. They come last not as if they were of lesser value, but because their subject lies beyond Finsler geometry, strictly speaking even beyond differential geometry. These are works in integral geometry and they are the products of the years 1934—35 spent in Hamburg with W. Blaschke. These were the years of rapid development in integral geometry. O. Varga found parameter transformation and motion invariant measures, so called geometrical densities, on different sets of geometrical configurations, and by disclosing the relations existing between them, i.e. with the aid of the Crofton formulae he established relations between integral invariants [2], [3], [4], [5], [6].

As it can be seen even from this brief survey, Ottó Varga was always intent on seizing and putting into relief the geometric meaning behind the often rather complicated formalism of Finsler geometry. This is perhaps the most characteristic feature of his whole scientific work. His mathematical thinking, for which profundity was rather more characteristic than quickness, was always guided by simple geometric intuition, in his hand formalism remained a means of expressing geometric ideas. The clarity and straightforwardness of his mathematical thinking was reflected by the purity of style and the crystal-clear structure of his papers. The decisive influence Ottó Varga had upon a number of young mathematicians, who as successful geometers of today gladly acknowledge their great debt to him, is certainly not the less imoprtant aspect of his scientific activity. With his objective and unbiased evaluation of mathematical achievements he exercised a far-reaching and benevolent influence on the whole of Hungarian mathematical life. His outstanding qualities earned him many distinctions. At forty he became corresponding member of the Hungarian Academy of Sciences, promoted latter to ordinary membership. In 1952 he was awarded Kossuth prize. These and other marks of esteem, however, did not make him self-satisfied; he remained an active, creative and devoted scientist setting for himself in everything the highest standards throughout his life.

Lajos Tamássy

Mathematical works of Ottó Varga

1. Beiträge zur Theorie der Finslerschen Räume und der affinzusammenhängenden Räume von Linienelementen. *Lotos*, Prag **84** (1936), 1—4.
2. Integralgeometrie 3. Croftons Formeln für den Raum. *Math. Z.* **40** (1935), 384—405.
3. Integralgeometrie 8. Über Masse von Paaren linearer Mannigfaltigkeiten im projektiven Raum P_n. *Rev. Math. Hispano-Americana* (1935), 241—279.
4. Integralgeometrie 9. Über Mittelwerte an Eikörpern, *Mathematica* **12** (1936), 65—80 (with W. Blaschke).
5. Integralgeometrie 19. Mittelwerte an dem Durchschnitt bewegter Flächen. *Math. Z.* **41** (1936), 768—784.
6. Integralgeometrie 24. Über die Schiebungen im Raum. *Math. Z.* **42** (1937), 710—736.
7. Über die Integralinvarianten die zu einer Kurve in der Hermiteschen Geometrie gehören. *Acta Sci. Math. Szeged* **9** (1939), 88—102.
8. Zur Herleitung des invarianten Differentials in Finslerschen Räumen. *Monatshefte f. Math. und Phys.* **50** (1941), 165—175.
9. Zur Differentialgeometrie der Hyperflächen in Finslerschen Räumen. *Deutsche Math.* **6** (1941), 192—212.
10. The establishment of the invariant differential in Finsler spaces. (Hungarian) *Matematikai és Fizikai Lapok* **48** (1941), 165—175.
11. Zur Begründung der Minkowskischen Geometrie. *Acta Sci. Math.* **10** (1943), 149—163.
12. The construction of Finsler geometry with the aid of the osculating Minkowski metric. (Hungarian) *Matematikai és Természettudományi Értesítő* **61** (1942), 14—21.
13. On a way of characterizing the Riemannian spaces of constant curvature. (Hungarian) *Matematikai és Fizikai Lapok* **50** (1943), 34—39.
14. Über eine Klasse von Finslerschen Räumen, die die nichteuklidischen verallgemeinern. *Comm. Math. Helv.* **19** (1946), 367—380.
15. Linienelementräume, deren Zusammenhang durch eine beliebige Transformationsgruppe bestimmt ist. *Acta Sci. Math. Szeged* **11** (1946), 55—62.
16. Über die Lösung differentialgeometrischer Fragen in der nichteuklidischen Geometrie unter gleichzeitiger Verwendung homogener und inhomogener Koordinaten. *Hung. Acta Math.* **1** (1947), 35—52.
17. Vektorfelder, deren kovariante Ableitung längs einer vorgegebenen Kurve verschwindet. *Hung. Acta Math.* **4** (1949) 1—3.
18. Über affinzusammenhängende Mannigfaltigkeiten von Linienelementen insbesondere deren Äquivalenz. *Publ. Math. Debrecen* **1** (1949), 1—17.
19. Bemerkung zur Arbeit des Herrn A. Dinghas „Zur Metrik nichteuklidischer Räume". *Math. Nachrichten* **2** (1949), 386—388.
20. Affinzusammenhängende Mannigfaltigkeiten von Linienlementen, die ein Inhaltsmaß besitzen. *Proc. Acad. Amsterdam* **52** (1949), 316—322.
21. Über den Zusammenhang der Krümmungsaffinoren in zwei eindeutig aufeinander abgebildeten Finslerschen Räumen. *Acta Sci. Math. Szeged* **12** (1950), 132—135.
22. Über das Krümmungsmaß in Finslerschen Räumen. *Publ. Math. Debrecen* **1** (1950), 116—122.
23. The applications of integral geometry in geometrical optics. (Hungarian) *A Magyar Tudományos Akadémia* III. *Osztályának Közleményei* **1** (1951), 192—201.
24. Normalkoordinaten in allgemeinen Räumen und ihre Verwendung zur Bestimmung sämtlicher Differentialinvarianten. *Comptes Rendus de* I. *Congrés des Math. Hongrois, Budapest* (1950), 147—162.
25. Anwendung von *p*-Vektoren auf derivierte Matrizen. *Publ. Math. Debrecen* **2** (1951), 137—145. (with B. Gyires)
26. Soviet results in differential geometry. (Hungarian) *Matematikai Lapok* **2** (1951), 190—218.
27. Eine geometrische Charakterisierung der Finslerschen Räume skalarer und konstanter Krümmung. *Acta Math. Acad. Sci. Hung.* **2** (1951), 281—283.
28. A review of the „Appendix"'s new edition. (Hungarian) *Magyar Tudományos Akadémia III. Osztályának Közleményei* **3** (1953), 281—283.
29. The effect of the geometry of Bolyai—Lobatschevsky on the development of geometry. (Hungarian) A *Magyar Tudományos Akadémia* III. *Osztályának Közleményei* **3** (1953), 151—171.
30. Eine Charakterisierung der Finslerschen Räume mit absolutem Parallelismus der Linienelemente. *Archiv der Math.* **5** (1953), 128—131.

31. Bedingungen für die Metrisierbarkeit von affinzusammenhängenden Linienelementmannigfaltigkeiten. *Acta Math. Acad. Sci. Hung.* **5** (1954), 7—16.
32. Bemerkung zur Cayley-Kleinscher Maßbestimmung. *Publ. Math. Debrecen* **4** (1955), 3—15. (with J. Aczél)
33. A characterization of Finsler spaces having an absolute parallelism of the line elements. (Hungarian) *Acta Univ. Debrecen* **1** (1954), 105—108.
34. Über Riemannsche Räume die freie Beweglichkeit besitzen. *Schriftenreihe des Forschungsinstituts für Math. der Deutschen Akademie der Wissenschaften zu Berlin.* **1** (1957), 124—130.
35. L'influence de la géométrie de Bolyai—Lobatschevsky sur le développement de la géométrie. *Acta Math. Acad. Sci. Hung.* **5** (1954), 71—94.
36. Die Krümmung der Eichfläche des Minkowskischen Raumes und die geometrische Deutung des einen Krümmungstensors des Finslerschen Raumes. *Abh. Math. Sem. Univ. Hamburg* **20** (1955), 41—51.
37. Eine Characterisierung der Kawaguchischen Räume metrischer Klasse mittels eines Satzes über derivierte Matrizen. *Publ. Math. Debrecen* **4** (1956), 418—430.
38. Normalkoordinaten in Kawaguchischen Räumen und seinen affinen Verallgemeinerungen sowie eine Anwendung derselben zur Bestimmung von Differentialinvarianten. *Math. Nachrichten* **18** (1958), 141—151.
39. Ein elementargeometrischer Beweis des Sylvester-Frankeschen Determinantensatzes. *Izvestija Szofia* **3** (1959), 105—107.
40. Verallgemeinerte Riemannsche Normalkoordinaten und einige Anwendungen derselben. *Izvestija Szofia* **4** (1959), 61—69.
41. Hilbertsche verallgemeinerte nicht-euklidische Geometrie und Zusammenhang derselben mit der Minkowskischen Geometrie. *Internat. Congress of Math. Abstracts, Edinburgh* (1958), 111.
42. Über die Zerlegbarkeit von Finslerschen Räume. *Acta Math. Hung.* **11** (1960), 197—203.
43. Über eine Kennzeichnung der Riemannschen Räume konstanter negativer und positiver Krümmung. *Annali di Math.* **53** (1961), 105—117.
44. Bemerkung zur Winkelmetrik in Finslerschen Räumen. *Ann. Univ. Sci. Budapest, Sectio Math.* **3—4**. (1961—61), 379—382.
45. Über den inneren und induzierten Zusammenhang für Hyperflächen in Finslerschen Räumen. *Publ. Math. Debrecen* **8** (1961), 208—217.
46. Über eine Characterisierung der Finslerschen Räume konstanter Krümmung. *Monatshefte für Math.* **65** (1961), 277—286.
47. Zur Begründung der Hilbertschen Verallgemeinerung der nichteuklidischen Geometrie. *Monatshefte für Math.* **66** (1962), 265—275.
48. Herleitung des Cartanschen euklidischen Zusammenhanges in Finslerräumen mit Hilfe der Riemannschen Geometrie. *Acta Physica et Chimica Debrecina* **8** (1962), 121—124.
49. Eine einfache Herleitung der Cartanschen Übertragung der Finslergeometrie. *Math. Notae* **18** (1962), 185—196.
50. Über Hyperflächen konstanter Normalkrümmung in Minkowskischen Räumen. *Tensor N. S.* **13** (1963), 246—250.
51. Hyperflächen mit Minkowskischer Maßbestimmung in Finslerräumen. *Publ. Math. Debrecen* **11** (1964), 301—309.
52. Zur sphärischen Abbildung in Riemannschen Räumen. *Annales de l'Univ. de Jassy* **11 B** (1965), 507—517.
53. Die Methode des beweglichen n-Beines in der Finsler-Geometrie. *Acta Math. Sci. Hung.* **18** (1967), 207—215.
54. F. Kárteszi is 60 years old. (Hungarian) *Matematikai Lapok* **18** (1967), 273—282. (with J. Merza)
55. Hyperflächen konstanter Normalkrümmung in Finslerschen Räumen. *Math. Nachrichten* **38** (1968), 47—52.
56. Zur Invarianz des Krümmungsmaßes der Winkelmetrik in Finsler-Räumen bei Einbettungen. *Math. Nachrichten* **43** (1969), 11—18.
57. Beziehung der ebenen verallgemeinerten nichteuklidischen Geometrie zu gewissen Flächen im pseudominkowskischen Raum. *Aequationes Math.* **3** (1969), 112—117.