Béla Barna
(1909 – 1990)

Professor Béla Barna was born on March 30, 1909 in Máramarossziget (now in Roumania), and he died in Debrecen on June 9, 1990. Both his father and his grandfather have been schoolteachers. Coming from the intellectual athmosphere of the paternal house, he entered the Piarist's Gymnasium in the city of his birth, where he spent two years. The remaining years of his secondary studies he made in the Kossuth Gymnasium of Nyíregyháza, where he obtained his baccalaureate in 1926.

He started his university studies at the Philosophical Faculty of the Pázmány Péter University in Budapest. Following his special interests, he devoted himself to the study of mathematics, physics and chemistry. The mathematical lectures of L. Fejér, G. Rados and J. Suták had a decisive influence on his intellectual development. In 1928 he made his first examination for the teaching profession in Budapest, and continued his studies at the University of Debrecen. It was here that he obtained his high school teachers diploma in mathematics and physics in the spring of 1931, and in the fall of the following year he made his Ph. D., with a dissertation entitled "On the theory of the medium aritmetiko–geometricum". (Debrecen, 1932). He spent the academic year 1942/43 with a state fellowship at the Collegium Hungaricum of Vienna.

Proceeding further in his scientific career, he defended in 1957 another dissertation "On the approximation of the roots of algebraic equations by Newton's procedure" gaining by it a higher, second scientific degree, and in 1967 he obtained his D. Sc.

After having obtained his Ph. D. he became an unsalaried assistent in the Mathematical Seminary of Debrecen University where he lectured on introductory number theory. At that time there existed no textbook or monograph on this subject in Hungarian. In order to help his students, he got his lecture notes printed at his own expense.

It was only in 1935 that he obtained a high school teaching post in Nagykálló. At the end of 1941 he became an ordinary professor of the Fazekas Gymnasium in Debrecen. In 1951 he joined the staff of the Mathematical Institute of Debrecen University, where he became a professor in 1970. Back in 1966 he detained also a second job, acting for a year as head of the chair for mathematics at the Teachers Training College in Eger.

For long years, he has been the Editor of our periodical "Publicationes Mathematicae, Debrecen". He continued in this position also after his retirement, almost until the age of eighty.

His one time teacher, professor Lajos Dávid, was interested, among other things, in the theory of modul functions. Influenced by him, professor Barna became involved in this theory, devoting his attention to the following set of problems.

We consider two complex numbers a and b satisfying $ab(a^2 - b^2) \neq 0$, and with the help of the Lagrange–Gauss algorithm we form the sequences $a_{n+1} = \frac{1}{2}(a_n + b_n)$, $b_{n+1} = \sqrt{a_n b_n}$ $(n = 0, 1, 2, \dots ;\ a_0 = a,\ b_0 = b)$. Then the sequences $\{a_n\}_0^\infty$ and $\{b_n\}_0^\infty$ will converge to the same value $M(a, b)$, called the arithmetic–geometric mean of the numbers a and b. The function $M(a, b)$ has infinitely many values. Let $\bar{M}(a, b)$ denote its branch satisfying $|\bar{M}(a, b)| \geq |M(a, b)|$. By investigating the relation existing between these two quantities Professor Béla Barna has obtained beautiful results. These he has published both in Hungarian and in German (see [1], [3] and [2], [4] respectively). The two German papers have appeared in the old and prestigious Journal of Crelle.

He returned to these questions in 1955, and in his paper [6] he established several interesting properties of the iteration

$$a_{n+1} = \frac{1}{2}\left[a_n + \sqrt{a_n b_n}\right], \quad b_{n+1} = \frac{1}{2}\left[b_n + \sqrt{a_n b_n}\right]$$
$$(n = 0, 1, 2, \dots ;\ \ a_0 = a,\ b_0 = b).$$

His extensive knowledge of iteration theory enabled him to undertake penetrating research into Newton's method for the determination of the reals roots of equations.

Let $f(x)$ be a polynomial having only simple real roots. Put x_0 and form the sequence $x_{n+1} = N(x_n)$, $N(x) = \frac{f(x)}{f'(x)}$ $(n = 1, 2, \dots)$. A point

x_0 is called a convergence point if the sequence $\{x_n\}_0^\infty$ converges to some root of $f(x)$, otherwise it is a divergence point. Alfred Rényi has raised the following two questions: 1) Let $f(x)$ be a polynomial having only real simple roots. Is it true that the set of divergence points is countable ? 2) Does there exist a polynomial having only real simple roots for which the set of divergence points contains some interval ?

Béla Barna has investigated and answered these questions in his papers [5], [7], [8], [9] and [10].

The papers [11] and [12] are devoted to the questions raised by Rényi, and to many other problems from the general theory of iteration, problems which have attracted the attention of many authors.

Let $f(x)$ be a continuous function defined on the interval $a \le x \le b$ and taking its values in this same interval. Suppose that $f(x)$ is not constant on any subinterval of the interval $a \le x \le b$. Then $f(x)$ is called the basic function of the iteration. Now, for each point x_0 of the interval $a \le x \le b$ we can form the iteration sequence

$$f_{n+1}(x) = f(f(x_n)), \quad f_0(x) = x_0 \quad (n = 0, 1, 2, \dots).$$

What can be said about the sequences $\{x_n\}_0^\infty$, when x_0 runs through the interval $a \le x \le b$? This is the question Béla Barna investigates in his two papers just mentioned.

One of his often cited results says: For a polynomial $f(x)$ with all roots real, Newton's method itself converges to a zero starting with almost every real number. The exceptional set of starting points is homeomorphic to the Cantor set.

The problems he raised and the answers he found in connection with the iteration of polynomials and real functions have influenced the international scene, attracting the attention of specialists throughout the world. They have been taken into account both in monographs and at international conferences. Their coverage in the international journals of reference was also highly positive and appreciating.

Summing up we can say that the whole scientific career of Béla Barna was closely connected with the Mathematical Institute of Debrecen University. It is here that he laid the foundations for his scientific activity, which in the sequel led to new and interesting results, internationally appreciated, within the field of iteration theory. Mention is also due to his indefatigable activity as editor of our periodical Publicationes Mathematicae, Debrecen, by which he substantially contributed to the development of our Institute, and assured us valuable publicational possibilities. We shall preserve his memory with gratitude and with the respect due to his likeable personality.

B. Gyires

List of the papers of Béla Barna

[1] Zur Theorie des Medium Arithmetico–Geometricum. Dissertationes Davidianae Nr. 7. (1932), 23 Seiten. (ungarisch)

[2] Ein Limessatz aus der Theorie des arithmetisch–geometrischen Mittels, *J. Reine Angew. Math.* **172** (1934), 86–88.

[3] Über die elementare Theorie des Medium Arithmetico–Geometricum und der Modulfunktionen, *MTA Matematikai és Természettudományi Értesítő* **56** (1937), 893–909. (ungarisch)

[4] Zur elementaren Theorie des arithmetisch–geometrischen Mittels, *J. Reine Angew. Math.* **178** (1938), 129–134.

[5] Über das Newtonsche Verfahren zur Annäherung von Wurzeln algebraischer Gleichungen, *Publicationes Mathematicae, Debrecen* **2** (1951), 50–63.

[6] Über die Funktionaliteration in zwei Veränderlichen, *Acta Univ. Debreceniensis* **3** (1956), 13–21. (ungarisch)

[7] Über die Divergenzenpunkte des Newtonchen Verfahrens zur Bestimmung von Wurzeln algebraischer Gleichungen. I., *Publicationes Mathematicae, Debrecen* **3** (1953), 109–118.

[8] Über die Divergenzenpunkte des Newtonchen Verfahrens zur Bestimmung von Wurzeln algebraischer Gleichungen. II., *Publicationes Mathematicae, Debrecen* **4** (1956), 384–397.

[9] Über die Divergenzenpunkte des Newtonchen Verfahrens zur Bestimmung von Wurzeln algebraischer Gleichungen. III., *Publicationes Mathematicae, Debrecen* **8** (1961), 193–207.

[10] Über die Divergenzenpunkte des Newtonchen Verfahrens zur Bestimmung von Wurzeln algebraischer Gleichungen. IV., *Publicationes Mathematicae, Debrecen* **14** (1967), 91–97.

[11] Über die Iteration reeller Funktionen. I., *Publicationes Mathematicae, Debrecen* **7** (1960), 16–40.

[12] Über die Iteration reeller Funktionen. II., *Publicationes Mathematicae, Debrecen* **13** (1966), 169–172.