Tibor Szele and his mathematical life-work.

On the 5th of April, 1955, a short but grave illness caused the death of Dr. TIBOR SZELE, the eminent young professor of Debrecen University, the noted algebraist. Death overcame him in the middle of an ascending career full of restless work and creative energy. His departure is an incalculable loss to this country's mathematical learning in which he, despite his youthful age, played so prominent a part, and there is no relief for the sorrow with which his parents, students and friends lament his passing.

I feel honoured and deeply moved when I comply with the request of the Editors of the *Publicationes Mathematicae* to review the life-work of TIBOR SZELE in an article that heads this memorial issue the rich contents of which are dedicated to his memory by his colleagues, friends and pupils. It is rather difficult to survey, within the limited space at my disposal, the imposing range of his many-sided activity, and my task is not rendered easirer by the obligatory attitude of impersonal detachment which is so hard to assume for a grateful disciple and admiring friend.

Let us begin with the main facts of TIBOR SZELE's short life. He was born on the 2lst of June, 1918, in Debrecen. In this town he had his elementary and secondary schooling and completed his university studies, with the highest scholastic records all the way through his schools. Already as a secondary school student he became enthusiastically interested in mathematics and won several mathematical prizes in nationwide competitions. He received his diploma as secondary school teacher of mathematics and physics in 1941, and, in the same year, he was appointed assistant to the Szeged University. Although attached to the Institute of Theoretical Physics his actual work was devoted chiefly to mathematical, particularly algebraic studies. His scholarly development received here strong impulses from the vigorous scientific spirit of the Mathematical Institute and from the outstanding mathematicians of the Szeged University, especially L. RÉDEI and L. KALMÁR. He completed here, already in 1941, his doctor's thesis [1][1]) in which he successfully solved a difficult graph-theoretical problem proposed to him by RÉDEI.

[1]) Both here and in the sequel, the numbers in brackets refer to the appended list of the scientific publications of T. SZELE.

Unfortunately, his oral examination, owing to a long-term military service, had to be postponed. So it was only after the end of World War II that the doctors degree, *sub laurea almae matris,* was conferred on him. At this time he had already been engaged in the Mathematical Institute of the Szeged University. From this time on an unhindered ascent marked his scientific career. In 1948 he became lecturer of the Debrecen University. In 1950 he was put in charge of the Second Mathematical Chair of the University to which he was appointed ordinary professor in 1952. In the same year, in recognition of his scientific activity, the Government of the Hungarian People's Republic awarded him a Kossuth Prize.

The short life of TIBOR SZELE was marked by a remarkable concentration of his energies. His scientific work comprised but ten years, yet the publication of 62 papers and of a book witnesses to the energy of his efforts. One of the most characteristic traits of his research work was his unfailing ability to gauge the main trends of scientific development. As a result of his close contacts, by correspondence, with the leading algebraists of our time he often learned of their latest findings even before they appeared in print. He was particularly keen on general theories. He possessed the ability to perceive analogies and establish relations between apparently distant phenomena and he knew how to make a fruitful use of these observations. He was never weary of research. Even in making his best accomplishments, he displayed the facility of genuine talent, as though these were but gratuituos additions to the fullness of his life. Another feature of his work was that, even in dealing with the most complicated problems, he always strove for the greatest possible simplicity. The problems he raised were always interesting, and his papers, which were composed in an exemplary manner, captured the interest of many young mathematicians for abstract algebra. In a number of his papers he offered new and essentially simplified proofs of important algebraic theorems, as, for instance, [3], [10], [20], [51].

With the death of TIBOR SZELE we lost not only an outstanding man of mathematical research but also an excellent teacher. His enthusiasm for mathematics and his love for his students made him a great teacher. Lucidity and delightful liveliness marked his lectures. In helping his students, he never spared himself. He readily gave them his time and energy. He posed problems, gave hints as to their solution, and unselfishly assisted his students in order to awaken their interest for research. It took but a short time and an algebraical school centering around his person came into being. His book „*Introduction to Algebra*“, which was published in Hungarian, in 1953, as a textbook for university students, also bears the stamp of his educational skill. The subject matter of the book is classical algebra, yet, by virtue of the careful thoroughness of exposition, the well-selected examples and instructive comments, it also affords a good preparation for studying higher algebra.

Beside his scientific and educational activities great importance must be attached to the indefatigable labours which TIBOR SZELE expended for the development of the Mathematical Institute of the Debrecen University. Then he was one of the founders and an always enthusiastic and active member of the *János Bolyai Mathematical Society* under whose auspices he often lectured in many towns of the country. His widespread activity also included his work as the managing editor of the *Publicationes Mathematicae Debrecen,* the editor of the periodical *Matematikai Lapok* and a reviewer of the *Mathematical Reviews* and of the *Zentralblatt für Mathematik.*

The rich legacy of his work accomplished will never be separated from his memory as a man, as a friend. He was a man of a rare nobility of mind and great kindness whose life was consumed in incessant labours and in exertions for others. We shall greatly miss his genial presence and gracious spirit, and his modest, sympathetic and self-effacing personality will remain an unforgettable and shining example for all who knew him.

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The main field of TIBOR SZELE's scientific activity was the theory of infinite abelian groups, but we are also indebted to him for some important investigations into other branches of abstract algebra. We shall review each of these in turn. We cannot aim at completeness, and must content ourselves with a discussion of the most important results. We hope that our comments will be found helpful by those perusing the appended bibliographical list.[2])

1. The theory of abelian (or commutative) groups is one of the most important chapters in the theory of algebraic structures. Important new investigations have placed this discipline in the foreground, especially during the last 10—15 years. In investigating the structure of infinite abelian groups, it is of importance to know the conditions under which such a group can be split up into the direct sum of cyclic groups. Two papers of SZELE deal with this question. In [28] he gives a sufficient condition for a generating system of an arbitrary abelian group to be a basis. By an interesting corollary to his theorem, a torsion-free abelian group G is a direct sum of cyclic groups if and only if G has a generating system S such that each finite subsystem $a_1, \ldots, a_n$ of S is a generating system with the minimal number of elements of $\{a_1, \ldots, a_n\}$. The note [30] contains a short and simple proof of a theorem of PONTRJAGIN, formulated by SZELE as follows: a countable torsion-free abelian group is the direct sum of cyclic groups if and only if in it every subgroup of finite rank is finitely generated. By a well known theo-

[2]) An excelent description of the scientific work of TIBOR SZELE was given earlier by L. FUCHS, of which the present writer made use at some places. (See L. FUCHS, Life and works of T. Szele, *Mat. Lapok* **6** (1955), 97—129. (Hungarian).)

rem of PRÜFER, a countable primary abelain group is a direct sum of cyclic groups if and only if it does not contain elements of infinite height. PRÜFER himself was aware of the fact that this theorem does not remain valid for the non-countable case. The note [40] contains a counter example relevant to this. The counter example itself coincides essentially with that of KUROŠ, the proof however is simpler, moreover it is shown that there exists also a counter example of power $\aleph_1$.

In investigations on abelian groups the question often arises under what conditions does a group possess a direct summand of a given property. In his paper [11] SZELE proves that if an abelain group has a non-zero element of finite order, then it has a direct summand of the form $C(p^k)$ ($k=1, 2, \dots,$ or ∞).[3]) By developing further this result, he later obtains the following theorem [43]: Let H be a subgroup of the abelain group G, and let this subgroup be a direct sum of cyclic groups, all of order $r<\infty$; then *H is a direct summand of G if and only if $H \cap rG = 0$ holds.* The proof is elementary and very simple. From this result many important decomposition theorems easily follow. Among them we find the result of KULIKOV, according to which of all non-torsion-free abelian groups only the groups $C(p^k)$ ($k=0, 1, 2 \dots$ and ∞) are directly indecomposable; the theorem of KUROŠ on the structure of abelian groups with descending chain condition for subgroups; the theorem of BAER and FOMIN on mixed abelain groups with torsion subgroup of elements of bounded order, etc. — As a further application of the above decomposition theorem SZELE gives a new and very natural method of introducing the concept of *basic subgroup* [52]. As it is known, the basic subgroups, introduced by KULIKOV, are structural invariants playing a fundamental role in the theory of primary abelian groups of arbitrary cardinality. In SZELE's treatment a basic subgroup B of an abelian primary group G appears as a direct sum $B=B_1+B_2+\dots+B_n+\dots$, where, for every natural n, B_n is a direct sum of cyclic groups of order p^n, and $B_1+B_2+\dots+B_n$ is a maximal p^n-bounded direct summand of G. The main result of paper [52] says that if B is a basic subgroup of the primary abelian group G, then B is a homomorphic image of G. This theorem has already made possible the solution of several problems, and so it seems to be one of the fundamental results of the theory of primary abelain groups of arbitrary cardinality.

Paper [41] also deals with direct decompositions of abelian groups, namely, with the structure of the direct sum of infinite cyclic groups with one amalgamated subgroup. He achieves an essential simplification of the problem by reducing it to the case of countable groups.

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[3]) We denote by $C(p^k)$ (where p is a prime) the cyclic group of order p^k if k is a natural number and PRÜFER's group of type p^∞ (i. e. the additive group mod 1 of all rational numbers with powers of p as denominators) if k is ∞.

His investigations led to the following generalization of the concept of direct sum (see [31] and [32])[4]): the abelian group G is *an interdirect sum*[5]) of its subgroups H_λ if there exist endomorphisms ε_λ of G sucht that

1) $$\varepsilon_\lambda G = H_\lambda;$$

2) $$\varepsilon_\lambda \varepsilon_\mu = \begin{cases} \varepsilon_\lambda \text{ if } \lambda = \mu; \\ 0 \text{ if } \lambda \neq \mu; \end{cases}$$

3) $g \in G$ and $\varepsilon_\lambda g = 0$ for every λ, imply $g = 0$.

This generalization proves serviceable in investigations into the structure of mixed groups, and also in describing the structure of primary abelian groups without elements of infinite height.[6]) The concept of interdirect sum also enables us to construct a very simple example of a mixed abelian group, the torsion subgroup of which is not a direct summand of the group.

Characteristic of the present phase of development of algebra are the unifying tendencies, striving to establish connections between more or less independetly evolved branches of this discipline. This unification is the aim of analogous theories built up for different kinds of algebraic structures. Investigations of this type have also the advantage of demonstrating more clearly, besides the analogies, also the important differences between different kinds of structures. In his paper of great importance [23] SZELE works out for abelian groups a theory analogous to the classical theory of fields created by STEINITZ. He starts with the remark that a general algebraic equation in one unknown over an abelian group G has the form $nx = a$ with $a, x \in G$ (x is the unknown) and n a rational integer. This suggests the definition of algebraic and transcendental extensions of groups, and it is shown in the paper that, on the basis of these definitions, a number of fundamental results of the theory of fields immediately carry over to abelian groups. It is shown, e. g., that an arbitrary group extension can always be realized by a pure transcendental and a succesive algebraic extension; that an arbitrary group has one (and up to equivalent extensions only one) algebraically closed algebraic extension. It is clear that the algebraically closed abeiian groups in the sense of SZELE coincide with the complete abelian groups, i. e., the groups G satisfying $nG = G$ (for every integer n). Despite the far-reaching analogy discovered by SZELE, group theory and the theory of fields exhibit numerous differences. The cause of this is, of course, the

[4]) Although we discovered later that this generalization was alreadly made in earlier investigations into the theory of rings, it is to SZELE's credit that he furnished an "inner" characterization of this construction and that applied it in the theory of the abelian groups.

[5]) This suggestive terminology is due to J. SURÁNYI.

[6]) See [31], [32] and L. FUCHS, On the structure of abelian p-groups, *Acta Math. Acad. Sci. Hungar.* **4** (1953), 267—288.

fundamental disparity between groups and fields as structures with one, resp., two operations. Thus, e. g. the concepts of prime field and characteristic have no analogies for groups; the theorem, according to which an algebraic equation in a field has at most as many solutions as its degree, also ceases to be valid for groups, since in a group an equation can have arbitrarily many solutions. — This theory of SZELE is rendered important, beside the numerous and interesting new results it yields, by the fact that it enables us to treat some well-known theorems and methods, which have been developed and applied independently of each other, as organic parts of an integrated theory of comprehensive scope.

Besides a general inquiry into the structure of abelain groups, investigations of a more specialized nature, aiming at the description of all groups with a given property, also deserve attention. SZELE always found his pleasure in investigations of this kind, and numerous are his papers [9], [14], [21], [32], [35], [45], [46], [48], [55] (written sometimes jointly with fellow algebraists) publishing his results in this direction. This is also the place to mention the investigations concerning rings of endomorphisms of abelian groups. In his papers [9], [14], [15], [31] he investigates questions like these: which are the abelian groups whose endomorphism ring is without divisors of zero, or is commutative ect.

Of his results in the theory of non-commutative groups, we mention but two. In [19] he shows that there exists one and only one group, containing all generalized quaternion groups[7]) as subgroups and being minimal among the groups with this property. SZELE calls this group the *infinite quaternion* group, and states a conjecture according to which this is the only infinite non-commutative p-group possessing exactly one minimal subgroup. — In the note [24] it is shown that if an arbitrary group has a perfect subgroup, then it has also a perfect normal subgroup.

2. TIBOR SZELE obtained results of abiding value also in the theory of rings. His first ring-theoretical paper, written jointly with L. RÉDEI, appeared in two parts: [4] and [13]. In these the authors investigate the possibility of generalizing number theoretical functions to arbitrary rings. They obtain a number of interesting results, but we cannot go into details. We mention only one of their results, stating that in the class of all rings the finite fields are characterized by the following feature: any function whose range and values are ring elements is a polynomial. In another of his early papers [5] the structure of finite rings is discussed. He constructs matrix ring the subrings of which exhaust all finite rings.

In 1948 he raised the interesting question, to what extent is a ring determined by its additive group, or, in other words, the question of deter-

[7]) See. e g. H. ZASSENHAUS, The theory of groups, *New York*, 1949.

mining all rings which can be built on a given abelian group. It is a trivial fact that on any abelian group there can be built a ring, namely the zero-ring, and in the case of finite groups also at least one non-zeroring. In the paper [12] he describes all non-torsion-free abelian groups which determine the rings over them in the greatest degree, i. e. which admit only the zero-ring. Pursuing the same line of investigation, he determines in [27] all abelian groups, on which there can be built exactly two rings. The only defect of the characterization given is due to the fact that the problem becomes reduced partly to the case of a torsion-free group, the structure of which is not known more explicitly. To this set of problems belongs also the paper [17] written jointly with RÉDEI. In this they give an explicit description of all rings the additive group of which is of rank 1, i. e. is isomorphic to a subgroup of the additive group of rationals or to a subgroup of a group $C(p^\infty)$.

Paper [54] also has some bearing on the investigation of the additive group of a ring. In this paper he investigates the nilpotent rings with descending chain condition for left ideals. Improving on the results of C. HOPKINS he shows that the additive group of such a ring also satisfies the descending chain condition for subgroups, and hence by a theorem of KUROŠ, it is a direct sum of finite number of groups $C(p_i^{k_i})$ $(0 \leqq k_i \leqq \infty, p_i$ prime). By virtue of this important remark the sructure problem of the rings under consideration can be reduced to that of finite nilpotent p-rings.

Of great importance in ring theory are the semi-simple rings, i. e. the rings containing no non-zero nilpotent left ideals and satisfying the descending chain condition for left ideals. Three papers of SZELE are devoted to this important category of rings. In the paper [38], written jointly with L. FUCHS, among others a theorem is proved, according to which *a ring is semi-simple if and only if any of its left ideals has a right unit element*. This characterization of semi-simple rings has proved useful also in other investigations. Papers [50] and [51] contain a very simple proof of the important structure theorem of WEDDERBURN-ARTIN on semi-simple rings. The proof makes use of the CHEVALLEY-JACOBSON density theorem, this idea being originally due to ARTIN.

ARTIN and SCHREIER succeeded in 1926 in giving a purely algebraic characterization of all commutative fields capable of ordering. Developing further this result [34], SZELE proved that a skew field can be ordered if and only if -1 cannot be represented as a sum of products of squares. Starting with this result, R. E. JOHNSON and V. D. PODDERYUGIN solved the problem of the possibility of ordering of a ring without divisors of zero and of arbitrary rings, respectively.

3. In the last two years of his life, TIBOR SZELE became interested in topological algebra. In a paper written jointly with A. KERTÉSZ, the question of introducing a topology into an abelian group is treated. It is proved that every infinite abelian group admits a non-discrete regular topology satisfying the first axion of countability. Moreover, an abelian group admits a non-discrete subgroup-topology if and only if it does not satisfy the minimum condition (for subgroups).

In his last address, held at Szeged on March 26, 1955 at the festivity commemorating the 50th birthday of Professor L. KALMÁR, T. SZELE treated topological rings. He introuced into the endomorphism ring of an abelian group a topology which shows the well known phenomenon occurring for p-adic valuation, namely that those and only those infinite series are summable whose terms form a zero-sequence.

4. Of the papers treating different other topics, we mention yet the following.

In his doctor's thesis [1] he gives an account of his investigations concerning the directed complete graph. He gives several generalizations of a theorem produced by RÉDEI, by which the number of directed open paths passing through every vertex in a directed complete (finite) graph is odd. Investigating the maximal number of such paths he succeeds in giving very good estimates for this maximum.

In his paper [20] he gives a simple proof of ZORN's lemma, deducing it directly from the axiom of choice.

In [39] he generalizes the classical theory of linear equation systems over skew fields to the case of systems with an arbitrary cardinality of equations and also of unknowns, and subject only to the natural restriction that in each equation all but a finite number of the unknowns must have zero coefficients.

Death overtook TIBOR SZELE in the midst of his creative activity. Numerous were his investigations being in progress, unfortunately however the notes he made of these are very scarce. Based on these notes and the reminiscences of mathematician friends, the posthumous papers [59], [60], [61] contain some of these results.

ANDOR KERTÉSZ *(Debrecen)*

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