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# Analytic Besov space $B^p$, $0 < p < 1$

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**Abstract.** For a measurable function $f$ on the unit ball $B$ in $C^n$, $n \geq 1$, we define $M_1 f(z)$, $z \in B$, to be the mean modulus of $f$ over a hyperbolic ball with center at $z$ and of a fixed radius. The space $L^{1,p}(\tau)$, $0 < p < 1$, where $\tau$ is $M$-invariant measure on $B$, is defined by the requirement that $M_1 f \in L^p(\tau)$. The analytic Besov space $B^p$, $0 < p < 1$, can be naturally embedded as a complemented subspace of $L^{1,p}(\tau)$ by a topological embedding $V_{m,s} : B^p \mapsto L^{1,p}(\tau)$. We show that $V_{m,s} \circ P_s$, where $P_s$ is an integral operator whose reproducing kernel is $\gamma_s(1-|w|^2)^s(1-\langle z,w\rangle)^{-(n+1+s)}$, is projection on this embedded copy. The embedding is applied to show that for each $0 < p < 1$ the dual space of the Besov space $B^p$ is isomorphic to the Bloch space $B^\infty$ (with equivalent norms) under certain integral pairing.

## 1. Introduction

Let $B$ be the open unit ball in $C^n$, $n \geq 1$, and $\nu$ the $2n$-dimensional Lebesgue measure on $B$ normalized so that $\nu(B) = 1$.

For $f$ analytic on $B$, $f \in H(B)$, and any positive integer $m$ we write $\partial^m f(z) = (\partial^\alpha f(z))_{|\alpha|=m}$ and $|\partial^m f(z)| = \sum_{|\alpha|=m} |\partial^\alpha f(z)|$, where $\partial^\alpha f(z) = \dfrac{\partial^{|\alpha|} f(z)}{\partial z^\alpha}$, $\alpha$ a multiindex.

Let $0 < p \leq \infty$ and let $m$ be a positive integer, $m > n/p$. We define the analytic Besov space $B^p$ by

$$B^p = \{f \in H(B) : (1-|z|^2)^m |\partial^m f(z)| \in L^p(\tau)\},$$

where $d\tau(z) = (1-|z|^2)^{-n-1}\, d\nu(z)$ is $M$-invariant measure on $B$ (see [8]). We note that the definition is independent of $m$.

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For $z \in B$, $0 < r < 1$, $E_r(z) = \{w \in B : |\varphi_z(w)| < r\}$. As usual, $\varphi_z$ is the standard automorphism of $B$ taking 0 to $z$ (see [8]). An integration in polar coordinates shows that $\tau(E_r(z)) = r^{2n}(1-r^2)^{-n} =: \tau(r)$, $z \in B$.

For a complex measurable function $f$ on $B$ we define

$$M_\infty f(z) = M_{\infty,r} f(z) = \operatorname{ess\,sup}\{|f(w)| : w \in E_r(z)\}$$

and

$$M_p f(z) = M_{p,r} f(z) = \left(\frac{1}{\tau(r)} \int\limits_{E_r(z)} |f(w)|^p \, d\tau(w)\right)^{1/p}, \quad 0 < p < \infty.$$

For $0 < p, q \le \infty$, we define $L_r^{p,q}(\tau)$ to be the space of all measurable functions $f$ on $B$ for which

$$\|f\|_{L_r^{p,q}(\tau)} = \|M_{p,r} f\|_{L^q(\tau)} < \infty.$$

Since the definition is independent of $r$, $0 < r < 1$, we will write $L^{p,q}(\tau)$ instead of $L_r^{p,q}(\tau)$ (see [3] and [11]).

Throughout this paper we assume $s$ is a real number satisfying $s > -1$. Let $\gamma_s$ be a positive normalizing constant such that the measure $d\nu_s(z) = \gamma_s(1-|z|^2)^s \, d\nu(z)$ has total mass 1 on $B$.

Following Forelli and Rudin ([4]) we let

$$P_s f(z) = \gamma_s \int\limits_B \frac{(1-|w|^2)^s f(w) \, d\nu(w)}{(1-\langle z, w\rangle)^{n+1+s}}, \quad z \in B, \ f \in L^1(\nu_s).$$

We now introduce some differential operators that are of great importance in the rest of the paper.

Let $s > -1$ and $m \ge 0$ . We define a linear operator $R_s^m$ on $L^1(\nu_s)$ by

$$R_s^m f(z) = \gamma_s \int_B \frac{(1-|w|^2)^s f(w) \, d\nu(w)}{(1-\langle z, w\rangle)^{n+1+s+m}}.$$

If $1 \le p \le \infty$ and $s > -1$ then $B^p = P_s L^p(\tau)$ (see [7], [10]). In this note we show that the analytic Besov space $B^p$, $0 < p < 1$, can be naturally embedded as a complemented subspace of $L^{1,p}(\tau)$ by a topological embedding $V_{m,s} : B^p \to L^{1,p}(\tau)$ defined by $V_{m,s} f(z) = (1-|z|^2)^m R_s^m f(z)$, where $mp > n$ and $s > -1$ . We show that $V_{m,s} \circ P_s$ is projection on this embedded copy and $B^p = P_s L^{1,p}(\tau)$.

Our work was motivated by the paper [6] where the coresponding problem for the Bergman space was considered.

Further, we show that for each $0 < p < 1$ the dual space of the Besov space $B^p$ is isomorphic to the Bloch space $B^\infty$ (with equivalent norms) under certain integral pairing.

## 2. Analytic Besov space $B^p$, $0 < p < 1$

First, we give a characterization of the Besov space $B^p$ in terms of the differential operators $R_s^m$ .

**Lemma 2.1.** *Let $0 < p < \infty$, $s > -1$ and let $m$ be a positive integer, $m > n/p$. If $f \in H(B)$ then the following are equivalent:*

*(i)* $f \in B^p$,

*(ii)* $\int\limits_B (1-|z|^2)^{mp} |R^m f(z)|^p \, d\tau(z) < \infty$, *where $R$ denotes the radial derivative* $R = \sum\limits_{j=1}^n z_j \dfrac{\partial}{\partial z_j}$,

*(iii)* $\int\limits_B (1-|z|^2)^{mp} |R_s^m f(z)|^p \, d\tau(z) < \infty$.

Proof. In [2] it is shown that $(i) \Longleftrightarrow (ii)$

The implication $(ii) \Longrightarrow (iii)$ follows from the following identities:

$$
\begin{aligned}
&R_s^0 f = P_s f = f \text{ and} \\
&R_s^m f = \big[(n+1+s+m)^{-1} R + I\big] R_s^{m-1} f, \quad f \in B^p,
\end{aligned}
\tag{2.1}
$$

(a straightforward calculation, see [7]).

Assume now that $(iii)$ holds. Then using the reproducing property of the operator $P_s$ and Fubini's theorem we obtain

$$
\begin{aligned}
&\int\limits_B \frac{(1-|w|^2)^{s+m} R_s^m f(w)}{(1-\langle z,w\rangle)^{n+1+s}} \, d\nu(w) \\
&= \int\limits_B \frac{(1-|w|^2)^{s+m}}{(1-\langle z,w\rangle)^{n+1+s}} \left[\gamma_s \int\limits_B \frac{(1-|\xi|^2)^s f(\xi)}{(1-\langle w,\xi\rangle)^{n+1+s+m}} d\nu(\xi)\right] d\nu(w)
\end{aligned}
\tag{2.2}
$$

$$= \gamma_s \int\limits_B (1-|\xi|^2)^s f(\xi)$$
$$\times \left[ \int\limits_B \frac{(1-|w|^2)^{s+m}\, d\nu(w)}{(1-\langle w,\xi\rangle)^{n+1+s+m}(1-\langle z,w\rangle)^{n+1+s}} \right] d\nu(\xi)$$
$$= \frac{\gamma_s}{\gamma_{s+m}} \int\limits_B \frac{(1-|\xi|^2)^s f(\xi)}{(1-\langle z,\xi\rangle)^{n+1+s}}\, d\nu(\xi) = \frac{1}{\gamma_{s+m}} f(z).$$

Taking derivatives inside integral gives

$$|\partial^m f(z)| \le C \int\limits_B \frac{(1-|w|^2)^{s+m} |R_s^m f(w)|}{|1-\langle z,w\rangle|^{n+1+s+m}}\, d\nu(w).$$

By Theorem 1.1 ([5]), $f \in B^p$.

Here and elsewhere constants are denoted by $C$ which may indicate a different constant from one occurrence to the next.

*Remark.* Carefully examining the proof of Lemma 2.1 above we actually see that the following are equivalent "norms" on $B^p$ for the appropriate $p$'s:

$$\left( \int\limits_B (1-|z|^2)^{mp} |\partial^m f(z)|^p \, d\tau(z) \right)^{1/p} + \sum_{|\alpha|<m} |\partial^\alpha f(0)|, \tag{1}$$

$$\left( \int\limits_B (1-|z|^2)^{mp} |R^m f(z)|^p \, d\tau(z) \right)^{1/p} + |f(0)|, \tag{2}$$

$$\left( \int\limits_B (1-|z|^2)^{mp} |R_s^m f(z)|^p \, d\tau(z) \right)^{1/p} + |f(0)|. \tag{3}$$

In the sequel by $\|f\|_{B^p}$ we will mean any of the expressions (1), (2) and (3).

A similar argument shows that

$$\|f\|_{B^\infty} \cong |f(0)| + \sup_{z\in B} (1-|z|^2)^m |R_s^m f(z)|.$$

**Theorem 2.2.** *Let $0 < p < 1$. Then for any $s > -1$, $P_s : L^{1,p}(\tau) \mapsto B^p$ is a continuous linear map. Moreover if $m$ is an integer, $m > n/p$, $V_{m,s} : B^p \mapsto L^{1,p}(\tau)$ is a topological embedding.*

To prove Theorem 2.2 we need the following two lemmas.

**Lemma 2.3.** *Let $s > -1$ and let $k$ and $m$ be non-negative integers. If $f \in L^1(\nu_s)$, then $R^m_{s+k} R^k_s f = R^{m+k}_s f$.*

Proof. Same as the equality (2.2).

The following lemma is proven in [6]. The proof is simple and it will be included for a reader convenience.

**Lemma 2.4** [6]. *If $0 < p < 1$, then $L^{1,p}(\tau) \subset L^1(\tau)$ and the inclusion map is continuous.*

Proof. Let $0 < \delta < \epsilon < 1$, $2\delta = \epsilon(1+\delta^2)$, and let $f \in L^{1,p}_\epsilon(\tau)$. The invariance of the measure $\tau$ and Fubini's theorem shows that

$$L^q(\tau) = L^{q,q}(\tau), \quad \text{for any } q,\ 0 < q < \infty. \tag{2.3}$$

Thus, we have

$$\|f\|_{L^1(\tau)} \cong \|f\|_{L^{1,1}_\delta(\tau)} \le C \int\limits_B \Bigg[ \int\limits_{E_\delta(z)} |f(\xi)|\, d\tau(\xi) \Bigg] d\tau(z)$$

$$\le C \operatorname*{ess\,sup}_{z\in B} \Bigg[ \int\limits_{E_\delta(z)} |f(\xi)|\, d\tau(\xi) \Bigg]^{1-p} \int\limits_B \Bigg[ \int\limits_{E_\delta(z)} |f(\xi)|\, d\tau(\xi) \Bigg]^p d\tau(z).$$

It is easy to see that if $w \in E_\delta(z)$ then $E_\delta(w) \subset E_\epsilon(z)$. Thus,

$$\int\limits_B \big[ M_{\infty,\delta}(M_{1,\delta} f)(z) \big]^p\, d\tau(z) \le C \int\limits_B \big[ M_{1,\epsilon} f(z) \big]^p\, d\tau(z).$$

On the other hand,

$$\begin{aligned}
\operatorname*{ess\,sup}_{w\in B} \big(M_{1,\delta} f(w)\big)^p \tau(\delta) &= \operatorname*{ess\,sup}_{w\in B} \int\limits_B \mathcal{X}_{E_\delta(w)}(z) \big(M_{1,\delta} f(w)\big)^p d\tau(z) \\
&\le \int\limits_B \operatorname*{ess\,sup}_{w\in B} \mathcal{X}_{E_\delta(z)}(w) \big(M_{1,\delta} f(w)\big)^p d\tau(z) \\
&= \int\limits_B \big[ M_{\infty,\delta}(M_{1,\delta} f)(z) \big]^p d\tau(z).
\end{aligned}$$

Combining the above inequalities we find that $\|f\|_{L^1(\tau)} \le C\|f\|_{L^{1,p}(\tau)}$.

Proof of Theorem 2.2. Let $f \in L^{1,p}(\tau)$ and let $m$ be any integer larger than $n/p$. Using Lemma 2.4 we see that

$$|R_s^m f(z)| \le C \int_B \frac{(1-|w|^2)^{n+1+s}|f(w)|}{|1-\langle z,w\rangle|^{n+1+s+m}}\, d\tau(w)$$

$$\le C\Bigg[\int_B \Bigg[\int_{E_\epsilon(w)} \frac{(1-|\xi|^2)^{n+1+s}|f(\xi)|}{|1-\langle z,\xi\rangle|^{n+1+s+m}}\, d\tau(\xi)\Bigg]^p d\tau(w)\Bigg]^{1/p},$$

for some fixed $\epsilon$, $0<\epsilon<1$.

Since $1-|\xi|^2 \cong 1-|w|^2$ and $|1-\langle z,\xi\rangle| \cong |1-\langle z,w\rangle|$, if $\xi \in E_\epsilon(w)$, we have

$$|R_s^m f(z)|^p \le C \int_B \frac{(1-|w|^2)^{p(n+1+s)}}{|1-\langle z,w\rangle|^{p(n+1+s+m)}}\Bigg[\int_{E_\epsilon(w)} |f(\xi)|\, d\tau(\xi)\Bigg]^p d\tau(w).$$

Using Lemma 2.3 and (2.1) we obtain

$$\|(1-|z|^2)^m R_s^m P_s f\|_{L^p(\tau)} = \|(1-|z|^2)^m R_s^m R_s^0 f\|_{L^p(\tau)}$$

$$= \|(1-|z|^2)^m R_s^m f\|_{L^p(\tau)} \le C\Bigg[\int_B (1-|z|^2)^{mp}$$

$$\times\Bigg[\int_B \frac{(1-|w|^2)^{p(n+1+s)}}{|1-\langle z,w\rangle|^{p(n+1+s+m)}}\Bigg[\int_{E_\epsilon(w)} |f(\xi)| d\tau(\xi)\Bigg]^p d\tau(w)\Bigg] d\tau(z)\Bigg]^{1/p}$$

$$= C\Bigg[\int_B (1-|w|^2)^{p(n+1+s)}\Bigg[\int_{E_\epsilon(w)} |f(\xi)| d\tau(\xi)\Bigg]^p$$

$$\times\Bigg[\int_B \frac{(1-|z|^2)^{mp-n-1} d\nu(z)}{|1-\langle z,w\rangle|^{p(n+1+s+m)}}\Bigg] d\tau(w)\Bigg]^{1/p},$$

by Fubini's theorem.

By standard estimates ([8], p. 17)

$$\int_B \frac{(1-|z|^2)^{mp-n-1}\, d\nu(z)}{|1-\langle z,w\rangle|^{p(n+1+m+s)}} \le C(1-|w|^2)^{-p(n+1+s)}.$$

Therefore

$$\|(1-|z|^2)^m R_s^m P_s f\|_{L^p(\tau)} \le C\,\|f\|_{L^{1,p}(\tau)}.$$

Using Lemma 2.4 we find that

$$|P_s f(0)| \le C\,\|f\|_{L^1(\nu_s)} \le C\,\|f\|_{L^1(\tau)} \le C\,\|f\|_{L^{1,p}(\tau)}.$$

Thus, $\|P_s f\|_{B^p} \le C\|f\|_{L^{1,p}(\tau)}$, by the Remark following Lemma 2.1, and we have proved that $P_s : L^{1,p}(\tau) \mapsto B^p$ is continuous.

Let now $f \in B^p$. By (2.3) $\int_B (1-|z|^2)^{mp}\big[M_{p,\epsilon} R_s^m f(z)\big]^p d\tau(z) \le C\|f\|_{B^p}^p$, for some $\epsilon$, $0<\epsilon<1$. Let $0<\delta<\epsilon<1$, $2\delta = \epsilon(1+\delta^2)$. The function $R_s^m f \in H(B)$ and therefore the function $|R_s^m f(\varphi_w)|^p$, $w \in B$, is subharmonic, whence

$$|R_s^m f(w)|^p = |R_s^m f(\varphi_z(0))|^p \le \delta^{-2n} \int\limits_{\delta B} |R_s^m f(\varphi_w(z))|^p d\nu(z)$$

$$\le \delta^{-2n} \int\limits_{\delta B} |R_s^m f(\varphi_w(z))|^p d\tau(z) = \delta^{-2n} \int\limits_{E_\delta(w)} |R_s^m f(z)|^p d\tau(z).$$

From this we find that $M_{1,\delta} R_s^m f(z) \le M_{\infty,\delta} R_s^m f(z) \le C M_{p,\epsilon} R_s^m f(z)$, $z \in B$.

Thus, $\|(1-|z|^2)^m M_{1,\delta} R_s^m f\|_{L^p(\tau)} \le C\|f\|_{B^p}$.

Since

$$\|(1-|z|^2)^m R_s^m f\|_{L_\delta^{1,p}(\tau)} \cong \|(1-|z|^2)^m M_{1,\delta} R_s^m f\|_{L^p(\tau)},$$

we see that $\|V_{m,s} f\|_{L^{1,p}(\tau)} \le C\|f\|_{B^p}$.

Using again the fact that $1-|z|^2 \cong 1-|w|^2$, if $z \in E_\epsilon(w)$, we get

$$\begin{aligned}
\|V_{m,s} f\|_{L^{1,p}(\tau)} &= \Bigg[\int\limits_B \Bigg[\int\limits_{E_\epsilon(w)} (1-|z|^2)^m |R_s^m f(z)| \tau(z)\Bigg]^p d\tau(w)\Bigg]^{1/p} \\
&\ge C\Bigg[\int\limits_B (1-|w|^2)^{mp} \Bigg[\int\limits_{E_\epsilon(w)} |R_s^m f(z)| d\tau(z)\Bigg]^p d\tau(w)\Bigg]^{1/p} \\
&\ge C\Bigg[\int\limits_B (1-|w|^2)^{mp} \Bigg[\int\limits_{E_\epsilon(w)} |R_s^m f(z)|^p d\tau(z)\Bigg] d\tau(w)\Bigg]^{1/p} \\
&\ge C\,\|(1-|z|^2)^m R_s^m f\|_{L^p(\tau)}.
\end{aligned}$$

Here we have used the estimate $M_{p,\epsilon}R_s^m f(w) \le M_{1,\epsilon}R_s^m f(w)$, $w \in B$.

Using (2.2) and Lemma 2.4 we see that

$$|f(0)| \le C\|V_{m,s}f\|_{L^{1,p}(\tau)}.$$

Thus, $\|f\|_{B^p} \cong \|V_{m,s}f\|_{L^{1,p}(\tau)}$, i.e. $V_{m,s}$ is a topological embedding.

We note that $V_{m,s} \circ P_s$ is projection from $L^{1,p}(\tau)$ onto $V_{m,s}(B^p)$. See the next section. As a corollary of (2.2) and Lemma 2.4 we have that $B^p = P_s L^{1,p}(\tau)$.

## 3. Duality

A linear functional $\lambda$ on $B^p$, $0 < p < 1$, is said to be bounded if

$$\|\lambda\| = \sup\{\,|\lambda(f)| : \|f\|_{B^p} \le 1\} < \infty.$$

The dual space of $B^p$, denoted $(B^p)^\star$, is then the space of all bounded linear functionals on $B^p$.

In [12] it is shown that each dual $(L_a^{p,s})^\star$, $0 < p < 1$, $s > -1$, of the weighted Bergman space $L_a^{p,s} = L^p(\nu_s) \cap H(B)$, can be indentified with $B^\infty$ via volume integral pairing

$$\langle f, g\rangle_\beta = \lim_{r\to 1} \int_B f(rz)\overline{g(z)}\, d\nu_\beta(z), \quad f \in L_a^{p,s},\ g \in B^\infty,$$

where $\beta = \frac{n+1+s}{p} - (n+1)$. In this note we show that $(B^p)^\star$, $0 < p < 1$, can also be indentified with $B^\infty$, but via a different pairing.

**Theorem 3.1.** *Let $0 < p < 1$ ,let $m$ be a positive integer, $mp > n$, and $s = m - n - 1$. The integral pairing*

$$\langle f, g\rangle_\tau = \int_B V_{m,s}f(z)\,\overline{V_{m,s}g(z)}\, d\tau(z)$$

*induces the following duality: $(B^p)^\star = B^\infty$.*

Proof. First, assume that $g$ is a function in $B^\infty$. Then

$$\sup_{z\in B}|V_{m,s}g(z)| = \sup_{z\in B}(1-|z|^2)^m|R_s^m g(z)| \le C\,\|g\|_{B^\infty}.$$

We show that $g$ gives rise to a bounded linear functional on $B^p$ under the pairing $\langle \cdot, \cdot \rangle_\tau$. By Theorem 2.2 and Lemma 2.4 if $f \in B^p$ then

$$\|f\|_{B^p} \geq C \, \|V_{m,s} f\|_{L^{1,p}(\tau)} \geq C \, \|V_{m,s} f\|_{L^1(\tau)}.$$

Thus if $f \in B^p$, then we have

$$|\langle f, g\rangle|_\tau \leq \sup_{z\in B} |V_{m,s} g(z)| \, \|V_{m,s} f\|_{L^1(\tau)} \leq C \, \|f\|_{B^p} \, \|g\|_{B^\infty}$$

Conversely, assume that $\lambda$ is a bounded linear functional on $B^p$; we show that $\lambda$ arises from a function in $B^\infty$. Since $V_{m,s}$ is a topological embedding of $B^p$ into $L^{1,p}(\tau)$, $\lambda \circ V_{m,s}^{-1}$ is a bounded linear functional on the image space of $V_{m,s}$ in $L^{1,p}(\tau)$. Since $L^{1,p}(\tau) \subset L^1(\tau)$, by the Hahn–Banach theorem $\lambda \circ V_{m,s}^{-1}$ extends to a bounded linear functional on $L^1(\tau)$. Thus, there exists a function $\varphi \in L^\infty(\tau)$ such that

$$\lambda \circ V_{m,s}^{-1}(\psi) = \int_B \psi(z) \, \overline{\varphi(z)} \, d\tau(z), \quad \psi \in L^1(\tau).$$

When $f \in B^p$, then $V_{m,s} f \in L^{1,p}(\tau) \subset L^1(\tau)$. Therefore

$$\lambda(f) = \int_B V_{m,s} f(z) \, \overline{\varphi(z)} \, d\tau(z), \quad f \in B^p.$$

Let $h = P_s(\varphi)$. Then $h \in B^\infty$ and by Lemma 2.3

$$V_{m,s} h(z) = (1 - |z|^2)^m R_s^m (P_s \varphi)(z) = (1 - |z|^2)^m R_s^m \varphi(z) = V_{m,s} \varphi(z).$$

To finishe the proof of Theorem 3.1 it remains to show that

$$\langle V_{m,s} f, V_{m,s} \varphi\rangle_\tau = \langle V_{m,s}^2 f, \varphi\rangle_\tau \quad \text{and} \quad \gamma_{m+s} \gamma_s^{-1} V_{m,s}^2 f = V_{m,s} f.$$

Note that $s = m - n - 1$. This follows easily from Fubini's theorem and the reproducing property of $P_s$. We leave the details to the interested reader.

Thus,

$$\lambda(f) = \int_B V_{m,s} f(z) \, \overline{V_{m,s} g(z)} \, d\tau(z), \quad f \in B^p,$$

where $g = \gamma_{s+m} \gamma_s^{-1} h \in B^\infty$.

The proof of Theorem 3.1 shows that the dual space of $B^1$ can also be identified with $B^\infty$ under the integral pairing $\langle \cdot \, , \cdot \rangle_\tau$. See also [13].

Using the term "Mackey topology" (see [9]), we can refrase Theorem 3.1 as follows.

**Corollary 3.2.** *Suppose $0 < p < 1$. Then the norm of $B^1$ induces the Mackey topology on $B^p$; and $B^1$ is the Mackey completion of $B^p$.*

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