Title: Supercongruences concerning lacunary sums of Catalan numbers and binomial coefficients

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We consider two conjectures of Sun in [22] concerning lacunary sums of Catalan numbers and binomial coefficients. As a conclusion, we completely confirm one of Sun's conjecture and partially confirm the other one. For example, suppose that $p \geq 3$ is prime and $0 \leq r \leq p - 1$. Then, for any $a \geq 0$, we have

$$S_r(p^{a+2}) \equiv S_r(p^a) \pmod{p^{1+a}},$$

where

$$S_r(p^a) = \sum_{\substack{0 < k < p^a \\ k \equiv r \pmod{p-1}}} C_k,$$

and $C_k = \binom{2k}{k}/(k+1)$ is the k-th Catalan number. Furthermore, when $p = 2$, we have

$$S_r(2^{a+2}) \equiv S_r(2^a) \pmod{2^{2(1+a)}}.$$

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