

**Title:** Congruences for sums of powers of an integer

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For coprime integers  $q$  and  $e$ , let  $m(q, e)$  denote the least positive integer  $t$  such that there exists a sum of  $t$  powers of  $q$  which is divisible by  $e$ . We prove that  $m(q, e) \leq \lceil e / \text{ord}_e(q) \rceil$  where  $\text{ord}_e(q)$  denotes the (multiplicative) order of  $q$  modulo  $e$ . We apply this in order to classify, for any positive integer  $r$ , the cases where  $m(q, e) \geq \frac{e}{r}$  and  $e > r^4 - 2r^2$ . In particular, we determine all pairs  $(q, e)$  such that  $m(q, e) \geq \frac{e}{6}$ . We also investigate in more detail the case where  $e$  is a prime power.

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