Sum of elements in finite Sidon sets. II

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Abstract. A set $S \subset \{1, 2, \ldots, n\}$ is called a Sidon set if all the sums $a + b$ are different for different unordered pairs $(a, b) \in S^2$. Let S_n be the largest cardinality of a Sidon set in $\{1, 2, \ldots, n\}$. In a former article, the author proved the following asymptotic formula

$$\sum_{a \in S, \, |S| = S_n} a = \frac{1}{2} n^{3/2} + O(n^{111/80+\varepsilon}),$$

where $\varepsilon > 0$ is an arbitrarily small constant. In this note, we improve the error term by showing that $O\left(n^{11/8} \log n\right)$ is true for almost all integers n in the above formula. Besides, we give some extensions of the former results. For any positive integers ℓ and s, we obtain the asymptotic formulae of the following summations

$$\sum_{\substack{S = \{a_1 < a_2 < \cdots < a_t\} \\ S \subset [1,n] \text{ Sidon}}} a_i^\ell, \qquad \text{and} \qquad \sum_{\substack{S = \{a_1 < a_2 < \cdots < a_t\} \\ S \subset [1,n] \text{ Sidon}}} i^s a_i^\ell,$$

when t is near the magnitude $n^{1/2}$.

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