Publicationes Mathematicae Debrecen | Year: 2023 | Vol.: 103 | Fasc.: 3-4

Title: A note on the asymptotic behavior of nonoscillatory solutions of half-linear ordinary differential equations

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The asymptotic behavior of nonoscillatory solutions of the half-linear differential equation

$$(p(t)|x'|^{\alpha}\operatorname{sgn} x')' + q(t)|x|^{\alpha}\operatorname{sgn} x = 0, \qquad t \ge t_0,$$

is discussed. It is assumed that $P(t) \equiv \int_{t_0}^{t} p(s)^{-1/\alpha}ds$ $(t \ge t_0)$ diverges to ∞ as $t \to \infty$, and that $Q(t) \equiv \int_{t}^{\infty} q(s)ds$ $(t \ge t_0)$ exists and is finite. It is shown that, under certain conditions on $P(t)$ and $Q(t)$, if a nonoscillatory solution $x(t)$ of the above equation satisfies the asymptotic property of the type $p(t)^{1/\alpha}P(t)[x'(t)/x(t)] \to \lambda \ne 0$ $(t \to \infty)$, then $x(t) \sim cP(t)^{\lambda}$ and $x'(t) \sim c\lambda p(t)^{-1/\alpha}P(t)^{\lambda-1}$ $(t \to \infty)$, where c is a nonzero constant.

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