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**Title:** On  $s$ -maximal asymptotic nonbases of density zero

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Let  $A$  and  $G$  be sets of nonnegative integers. The set  $A$  is called an asymptotic basis of order 2 if every sufficiently large integer can be written as the sum of two elements of  $A$ . However,  $A$  is called an asymptotic nonbasis of order 2 if there are infinitely many positive integers that cannot be written as the sum of two elements of  $A$ . Let  $s$  be a positive integer. An asymptotic nonbasis  $A$  of order 2 is  $s$ -maximal if  $A \cup G$  is an asymptotic nonbasis of order 2 whenever  $|G \setminus A| < s$ , but  $A \cup G$  is an asymptotic basis of order 2 if  $|G \setminus A| \geq s$ . We denote by  $A(x)$  the number of positive elements of  $A$  not exceeding  $x$ . In 1977, Nathanson constructed an  $s$ -maximal asymptotic nonbasis  $A$  of order 2 such that  $A(x) = O(\sqrt{x})$ . In this paper, we construct an  $s$ -maximal asymptotic nonbasis  $A$  of order 2 such that  $A(x) < 7.887\sqrt{x}$ , for all  $x \geq 1$ .

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