## Publicationes Mathematicae Debrecen | Year: 2024

Vol.: 105

Fasc.: 3-4

Title: Rings whose nil-clean and clean elements are uniquely nil-clean

Author(s): Jian Cui, Peter Danchev and Danya Jin

We consider and study those rings in which each nil-clean or clean element is uniquely nil-clean. We establish that those rings are abelian. More precisely, it is shown that the classes of abelian rings and the rings in which nil-clean elements are uniquely nil-clean do coincide. Moreover, we prove that the rings in which clean elements are uniquely nil-clean coincide with the subclass of abelian rings consisting of only unipotent units. In particular, we obtain in the semi-potent case a complete characterization only in terms of the former ring and its divisions. Likewise, some extension properties and group rings for such kinds of rings are also considered.

## Address:

Jian Cui School of Mathematics and Statistics Anhui Normal University Wuhu 241002 China

and Key Laboratory of Financial Mathematics (Putian University) Fujian Province University Fujian Putian 351100 China

## Address:

Peter Danchev Institute of Mathematics and Informatics Bulgarian Academy of Sciences "Acad. G. Bonchev" str., bl. 8 1113 Sofia Bulgaria

## Address:

Danya Jin School of Mathematics and Statistics Anhui Normal University Wuhu 241002 China