Publ. Math. Debrecen
106/1-2 (2025), 103–123
DOI: 10.5486/PMD.2025.9822

On the growth rate of partial quotients in Engel continued fractions

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Abstract. Let $(b_n(x))_{n\geq 1}$ be the sequence of the partial quotients of the Engel continued fraction expansion of an irrational number $x \in (0,1)$. This paper is concerned with the Hausdorff dimension of some exceptional sets related to the growth rate of $(b_n(x))_{n\geq 1}$. As a main result, we obtain the Hausdorff dimension of the set

$$E_{\inf}(\psi) = \left\{ x \in [0,1) : \liminf_{n\to\infty} \frac{\log b_n(x)}{\psi(n)} = 1 \right\},$$

where $\psi : \mathbb{N} \to \mathbb{R}^+$ is a function satisfying $\psi(n) \to \infty$ as $n \to \infty$.

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Mathematics Subject Classification: 11K55, 28A80.
Key words and phrases: Engel continued fractions, growth rate of partial quotients, Hausdorff dimension.