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Left invariant conic Finsler metrics and related Landsberg Problem on surfaces

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Abstract. In this paper, we study left invariant conic Finsler metrics on the 2-dimensional non-Abelian Lie group G with nowhere vanishing spray vector fields, and classify those satisfying the constant curvature condition, the Landsberg condition or the Berwald condition, respectively. We prove that any left invariant conic Landsberg metric on G must be Berwaldian. This discovery suggests a homogeneous conic Landsberg Problem, i.e., searching for homogeneous conic Landsberg metrics and exploring if they are Berwaldian. We solve the 2-dimensional case of this problem, and prove that all homogeneous conic Landsberg surfaces are Berwaldian.

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