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Jacobson's lemma for commuting outer inverses in rings

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Abstract. Let a,b be elements in a ring R with unity 1, and let X_{1-ab} denote the set of commuting outer inverses of 1-ab. In this paper, under the condition that R is an abelian ring, we establish a bijection $\Omega: X_{1-ab} \to X_{1-ba}$, which is an isomorphism of posets with respect to Mitsch's partial order $\leq_{\mathcal{M}}$, and is a semigroup isomorphism with respect to the von Neumann product. It is also shown that there exists an isomorphism between certain subsets of $(X_{1-ab}, \leq_{\mathcal{M}})$ and $(X_{1-ba}, \leq_{\mathcal{M}})$ without any restriction on R.

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