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On the explicit Galois group of $\mathbb{Q}(a_1^{\frac{1}{m}}, a_2^{\frac{1}{m}}, \dots, a_n^{\frac{1}{m}}, \zeta_m)$ over $\mathbb{Q}$

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Abstract. Let $m > 1$ be an integer, and ζ_m denote a primitive m-th root of unity. For any finite set $S = \{a_1, a_2, \dots, a_n\}$ and arbitrary choice of m-th roots of unity $\zeta_m^{r_i}$, $0 \le r_i < m$ for $i = 1, \dots, n$, we study the density of primes $\mathbf{p}$ of $\mathbb{Q}(\zeta_m)$ such that the m-th power residue symbol $\left(\frac{a_i}{\mathbf{p}}\right)_m = \zeta_m^{r_i}$. We calculate the explicit structure of the Galois group $\operatorname{Gal}\mathbb{Q}(a_1^{\frac{1}{m}}, \dots, a_n^{\frac{1}{m}}, \zeta_m)/\mathbb{Q}(\zeta_m)$ in terms of its action on $a_i^{\frac{1}{m}}$ for $1 \le i \le n$.

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