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On Diophantine equations involving Thabit and Williams numbers base b

By KOUÈSSI NORBERT ADÉDJI (Abomey-Calavi), MARIAMA NDAO FAYE (Saint-Louis) and ALAIN TOGBÉ (Hammond)

Abstract. Let $b \geq 2$ be a positive integer. Let r and s be two integers with $r \geq 1$, $s \in \{-1,1\}$ and $\Delta = r^2 + 4s > 0$, let $\{U_n\}_{n \geq 0}$ be the Lucas sequence given by $U_{n+2} = rU_{n+1} + sU_n$, with $U_0 = 0$ and $U_1 = 1$. In this paper, we investigate the solutions of the Diophantine equations

$$U_n \pm U_m = (b \pm 1) \cdot b^{\ell} \pm 1,$$

by giving effective bounds for the variables n, m and ℓ in terms of b, r and s. Moreover, we solve the above equation in the cases where $2 \le b \le 10$, by considering the Fibonacci, Pell and balancing sequences.

KOUÈSSI NORBERT ADÉDJI INSTITUT DE MATHÉMATIQUES ET DE SCIENCES PHYSIQUES UNIVERSITÉ D'ABOMEY-CALAVI BÉNIN

MARIAMA NDAO FAYE UFR OF APPLIED SCIENCES AND TECHNOLOGY UNIVERSITY GASTON BERGER SAINT-LOUIS SÉNÉGAL

ALAIN TOGBÉ
DEPARTMENT OF MATHEMATICS AND STATISTICS
PURDUE UNIVERSITY NORTHWEST
2200 169TH STREET
46323 HAMMOND, IN
USA

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