Dedekind sums and mean square value of $L(1,\chi)$ over subgroups

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Abstract. An explicit formula for the quadratic mean value at s=1 of the Dirichlet L-functions associated with the odd Dirichlet characters modulo f>2 is known. Here, we present a situation where we could prove an explicit formula for the quadratic mean value at s=1 of the Dirichlet L-functions, associated with the odd Dirichlet characters modulo not necessarily prime moduli f>2 that are trivial on a subgroup H of the multiplicative group $(\mathbb{Z}/f\mathbb{Z})^*$. This explicit formula involves summation S(H,f) of Dedekind sums s(h,f) over the $h\in H$. A result on some cancelation of the denominators of the s(h,f) when computing s(h,f) is known. Here, we prove that for some explicit families of s(h,f) is and s(h,f) is known result on cancelation of denominators is the best result one can expect. Finally, we surprisingly prove that for s(h,f) a prime, s(h,f) and s(h,f) is an s(h,f) do not depend on s(h,f) and s(h,f) of the multiplicative cyclic group s(h,f).

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