

Generalized group algebras and generalized measure algebras on non-discrete locally compact abelian groups

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Abstract. Let G be a non-discrete LCA group with the dual group Γ . We define a generalized group algebra, $\mathcal{L}^1(G)$, and a generalized measure algebra, $\mathcal{M}(G)$, on G as generalizations of the group algebra $L^1(G)$ and the measure algebra $M(G)$, respectively. Generalized Fourier transforms of elements of $\mathcal{L}^1(G)$ and generalized Fourier–Stieltjes transforms of elements of $\mathcal{M}(G)$ are also defined as generalizations of the Fourier transforms and the Fourier–Stieltjes transforms, respectively. The image $\mathcal{A}(\Gamma)$ of $\mathcal{L}^1(G)$ by the generalized Fourier transform becomes a function algebra on Γ with norm inherited from $\mathcal{L}^1(G)$ through this transform. It is shown that $\mathcal{A}(\Gamma)$ is a natural Banach function algebra on Γ which is BSE and BED. It turns out that $\mathcal{L}^1(G)$ contains all Rajchman measures. Segal algebras in $\mathcal{L}^1(G)$ are defined and investigated. It is shown that there exists the smallest isometrically character invariant Segal algebra in $\mathcal{L}^1(G)$, which (eventually) coincides with the smallest isometrically character invariant Segal algebra in $L^1(G)$, the Feichtinger algebra of G . A notion of locally bounded elements of $\mathcal{M}(G)$ is introduced and investigated. It is shown that for each locally bounded element μ of $\mathcal{M}(G)$ there corresponds a unique Radon measure $\iota\mu$ on G which characterizes μ . We investigate the multiplier algebra $\mathbb{M}(\mathcal{L}^1(G))$ of $\mathcal{L}^1(G)$, and obtain a result that there is a natural continuous isomorphism from $\mathbb{M}(\mathcal{L}^1(G))$ into $A(G)^*$, the algebra of pseudomeasures on G . When G is compact, this map becomes surjective and isometric.

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