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Uniform distribution of $\alpha\phi(n)$ and $\alpha\sigma(n)$ modulo 1, for non-Liouville numbers α

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Abstract. There have been results on uniform distribution modulo 1 of sequences of the form $\{\alpha f(n)\}_{n=1}^{\infty}$, where $f(n)$ is an arithmetic function, and α is an irrational number. For example, $\{\alpha n\}_{n=1}^{\infty}$ (Bohl, Sierpiński and Weyl) and $\{\alpha\Omega(n)\}_{n=1}^{\infty}$ (Erdős and Delange) have been shown to be uniformly distributed modulo 1 for all irrational numbers α. De Koninck and Kátai have shown that $\{\alpha\phi(n)\}_{n=1}^{\infty}$ and $\{\alpha\sigma(n)\}_{n=1}^{\infty}$ are uniformly distributed modulo 1 for a subset of irrational numbers α. In this article, we will extend their result by showing that the sequences $\{\alpha\phi(n)\}_{n=1}^{\infty}$ and $\{\alpha\sigma(n)\}_{n=1}^{\infty}$ are uniformly distributed modulo 1 when α is a non-Liouville number. The proof will use Weyl's criterion, upper bounds of exponential functions established by Vinogradov and Vaughan, and the notion of a thin set established by Pollack and Vandehey. There are two corollaries that arise from the result of this article: $\{10^{\alpha\phi(n)}\}_{n=1}^{\infty}$ and $\{10^{\alpha\sigma(n)}\}_{n=1}^{\infty}$ are strong Benford sequences for all non-Liouville numbers α, and the sequences $\{F(n)+\alpha\phi(n)\}_{n=1}^{\infty}$ and $\{F(n)+\alpha\sigma(n)\}_{n=1}^{\infty}$ are uniformly distributed modulo 1 for all non-Liouville numbers α and additive function F.

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