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Orientation-reversing actions on a solid torus which extend to a lens space

By JOHN KALLIONGIS (St. Louis) and RYO OHASHI (Wilkes-Barre)

Abstract. We consider all the finite orientation-reversing actions on a solid torus V_1, write explicit representations for each equivalence class of action, and identify the quotient space. For fixed positive integers n and s, these groups divide into seven families ($1 \leq i \leq 7$) with two groups in each family $\widetilde{G_{(i,j)}}(n,s)$ and $G_{(i,j)}(n,s')$, where $s' = s$ or $2s$ depending on certain conditions on s or n. The integer j relates to the quotient type (Bj, n). We consider the question of extending these actions to a lens space $L(p,q) = V_1 \cup_\alpha V_2$. We show $\widetilde{G_{(i,j)}}(n,s)$ extends to the 3-sphere $\mathbb{S}^3$ but not to $L(2,1)$, and $G_{(i,j)}(n,s')$ extends to $L(2,1)$ but not to $\mathbb{S}^3$. All the non-orientable orbifolds having an Euler number zero Heegaard decomposition with finite fundamental group appear as quotient spaces of these actions. In addition, no orientation-reversing action on V_1 extends to $L(p,q)$ for $p > 2$.

JOHN E. KALLIONGIS
DEPARTMENT OF MATHEMATICS
AND STATISTICS
SAINT LOUIS UNIVERSITY
220 NORTH GRAND BOULEVARD
ST. LOUIS, MO 63103
U.S.A.

RYO OHASHI
DEPARTMENT OF MATHEMATICS
AND COMPUTER SCIENCE
KING'S COLLEGE
133 NORTH RIVER STREET
WILKES-BARRE, PA 18711
U.S.A.

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