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Decomposition of approximately monotone and convex sequences

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Abstract. In this paper, we primarily deal with approximately monotone and convex sequences. We start by showing that any sequence can be expressed as the difference of two nondecreasing sequences. One of these two monotone sequences acts as the majorant of the original sequence, while the other possesses non-negativity. Another result establishes that an approximately monotone (increasing) sequence can be closely approximated by a nondecreasing sequence. A similar assertion can be made for an approximately convex sequence. A sequence $\langle u_n \rangle_{n=0}^{\infty}$ is called approximately convex (or ε-convex) if the following inequality holds under the mentioned assumptions:

$$u_i - u_{i-1} \leq u_j - u_{j-1} + \varepsilon, \qquad \text{where } i, j \in \mathbb{N}, \quad \text{with } i < j.$$

We prove that an approximately convex sequence can be written as the algebraic difference of two specific types of sequences. The initial sequence contains sequential convexity property, while the other sequence possesses the Lipschitz property. Moreover, we introduce an operator $\mathcal{T}$, which is termed as a twisting operator. In a compact interval $I(\subseteq \mathbb{R})$, we characterize the convex function with this newly introduced operator. In addition to various results on sequence decomposition and the study of the $\mathcal{T}$-operator, a characterization regarding non-negative sequential convexity, a fractional inequality, the implication of the $\mathcal{T}$ operator on different types of functions, the relationship between a convex function and a convex sequence are also included.

Motivation, previous research in this direction, various applications and linkage with some other branches of mathematics are discussed in the Introduction.

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