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Classification of abelian finite-dimensional C^*-algebras by orthogonality

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Abstract. The main goal of the article is to prove that if $\mathcal{A}_1$ and $\mathcal{A}_2$ are Birkhoff–James isomorphic C^*-algebras over the fields $\mathbb{F}_1$ and $\mathbb{F}_2$, respectively, and if $\mathcal{A}_1$ is finite-dimensional, abelian of dimension greater than one, then $\mathbb{F}_1 = \mathbb{F}_2$, and $\mathcal{A}_1$ and $\mathcal{A}_2$ are (isometrically) $*$-isomorphic C^*-algebras. Furthermore, it is also proved that for a finite-dimensional C^*-algebra $\mathcal{A}$, we have $\mathcal{L}_{\mathcal{A}}^{\perp}$ is the sum of minimal ideals which are not skew-fields, and $\mathcal{L}_{\mathcal{A}}^{\perp\perp}$ is the sum of minimal ideals which are skew-fields, where $\mathcal{L}_{\mathcal{A}}$ denotes the set of all left-symmetric elements in $\mathcal{A}$, and for any subset $\mathcal{S} \subseteq \mathcal{A}$, the set $\mathcal{S}^{\perp}$ represents the set of all elements of $\mathcal{A}$ which are Birkhoff–James orthogonal to $\mathcal{S}$. A procedure to extract the minimal ideals which are (commutative) fields is also given.

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