Publ. Math. Debrecen
108/1-2 (2026), 99–119
DOI: 10.5486/PMD.2026.10211

Unconditional convergence of general Fourier series

By VAKHTANG TSAGAREISHVILI (Tbilisi), GIORGI TUTBERIDZE (Tbilisi) and GIORGI CAGAREISHVILI (Tbilisi)

Abstract. S. Banach, in particular, proved that for any function – even for the constant function $f(x) = 1$, where $x \in [0, 1]$ – the convergence of its Fourier series with respect to a general orthonormal system (ONS) is not guaranteed.

In this paper, we establish conditions on the functions φ_n of an orthonormal system (φ_n) under which the Fourier series of functions $f \in \mathrm{Lip}_1$ converges unconditionally almost everywhere.

Our investigation primarily relies on the properties of sequences of linear functionals in Banach spaces, which we use to prove the main results presented in this article. We show that the aforementioned conditions not only exist but are also optimal in a certain sense. Furthermore, we demonstrate that any orthonormal system contains a subsystem for which the Fourier series of every function $f \in \mathrm{Lip}_1$ converges unconditionally.

VAKHTANG TSAGAREISHVILI &
GIORGI CAGAREISHVILI
DEPARTMENT OF MATHEMATICS
FACULTY OF EXACT AND NATURAL SCIENCES
IVANE JAVAKHISHVILI TBILISI
STATE UNIVERSITY
CHAVCHAVADZE STR. 1
TBILISI 0128
GEORGIA

GIORGI TUTBERIDZE
VIKTOR KUPRADZE INSTITUTE
OF MATHEMATICS
SCHOOL OF SCIENCE AND TECHNOLOGY
THE UNIVERSITY OF GEORGIA
77A MERAB KOSTAVA ST
TBILISI 0128
GEORGIA
AND
DEPARTMENT OF COMPUTER SCIENCE
FACULTY OF EXACT AND NATURAL SCIENCES
IVANE JAVAKHISHVILI TBILISI
STATE UNIVERSITY
CHAVCHAVADZE STR. 1
TBILISI 0128
GEORGIA

Mathematics Subject Classification: 42C10, 46B07.
Key words and phrases: unconditional convergence, orthonormal systems, Fourier coefficients, sequence of linear functionals, Banach space.