*Publ. Math. Debrecen*
**41 / 1–2** (1992), 173–180

# Ricci coefficients of rotation in a generalized Riemannian space

By SVETISLAV M. MINČIĆ (Niš)

**Abstract.** Because of nonsymmetry of the fundamental tensor in a generalized Riemannian space ($GR_N$), the connexion, defined on the base of such a fundamental tensor, is nonsymmetric too. Therefore, it is possible to define in this space two kinds of Ricci coefficients of rotation.

In this work we define the mentioned coefficients in a $GR_N$ and examine their properties as well as their connection with the Ricci coefficients of rotation in the associated Riemannian space, that is in the Riemannian space whose fundamental tensor is the symmetric part of the fundamental tensor of the $GR_N$.

## Introduction

An $N$-dimensional manifold equipped with a nonsymmetric fundamental tensor $(g_{ij})$ is said to be a generalized Riemannian space $GR_N$ (see [1], [2]). We denote the symmetric and the antisymmetric part of $(g_{ij})$ by $(g_{\underline{ij}})$ and $(g_{\underset{\smile}{ij}})$, resp. Lowering and raising the indices of these tensors will be used automaticly; then e.g.

$$g_{\underline{ij}}g^{\underline{jk}} = \delta_i^k. \tag{1}$$

Denoting the ordinary partial derivative by a comma ",", the Christoffel symbols of $(g_{ij})$ are defined by the well-known formula

$$\Gamma_{i.jk} = \frac{1}{2}(g_{ji,k} - g_{jk,i} + g_{ik,j}), \quad \Gamma^i_{jk} = g^{\underline{ip}}\Gamma_{p.jk}. \tag{2}$$

These are also nonsymmetric in the indices $j, k$.

On the base manifold of $GR_N$ one can also introduce a usual Riemannian metric, namely the Riemannian metric defined by $(g_{\underline{ij}})$. The resulting

1980 *Mathematics Subject Classification* (1985 *Revision*): 54E15.
*Keywords*: Common fixed points, metric space.

Riemannian space will be denoted by $R_N$ and will be mentioned as the *associated space* of $\mathrm{GR}_N$. The canonical connection of $R_N$ is just the symmetric part of the connection $\Gamma^i_{jk}$, i.e. $\Gamma^i_{\underline{jk}}$, constructed from $g_{\underline{ij}}$. Denoting by a semicolon "$;$" the covariant derivative w.r.t. $\Gamma^k_{\underline{ij}}$, we have two kinds of covariant derivatives for a vector $u^i$ in $\mathrm{GR}_N$, namely

$$u^{\;i}_{\underset{\substack{1\\2}}{|}n} = u^i_{,n} + \Gamma^i_{\substack{pn\\np}} u^p = u^i_{,n} + (\Gamma^i_{\underline{pn}} \pm \Gamma^i_{\underset{\smile}{pn}}) u^p,$$

that is

$$u^i_{\underset{\theta}{|}n} = u^i_{;n} + (-1)^\theta \Gamma^i_{\underset{\smile}{np}} u^p, \quad \theta = 1, 2. \tag{3}$$

Analogously, for a form $v_i$

$$v_{i\underset{\theta}{|}n} = v_{i;n} + (-1)^\theta \Gamma^p_{\underset{\smile}{in}} v_p, \quad \theta = 1, 2. \tag{4}$$

## 1. Congruence of curves and orthogonal ennuple

*Definition 1.* A congruence of curves in a $\mathrm{GR}_N$ is such a family of curves that trough each point of $\mathrm{GR}_N$ passes one curve of the family. $N$ mutually orthogonal congruences of curves constitute an *orthogonal ennuple*. Instead of congruences of curves, we shall sometimes speak about congruences of the corresponding tangent vectors.

If $\lambda_{(h)}$ $(h = 1, \dots, N)$ are unit tangent vectors of congruences of curves of an orthogonal ennuple, then, in virtue of the previous definition

$$g_{\underline{ij}} \lambda^i_{(h)} \lambda^j_{(k)} = e_{(k)} \delta_{hk}, \quad e_{(k)} = \pm 1, \tag{5}$$

or

$$e_{(k)} \lambda^i_{(h)} \lambda_{(k)i} = \delta_{hk}, \tag{5'}$$

where $\delta_{hk}$ are the Kronecker symbols. (Of course, we do not mean summation w.r.t. $(k)$ in (5), (5') and in similar formulas later on.) The next theorem expresses the basic properties of orthogonal ennuples.

**Theorem 1.** *For the unit tangent vectors $\lambda_{(h)}$ $(h = 1, \dots, N)$ of congruences of curves of an orthogonal ennuple the relations*

$$\sum_{k=1}^{N} e_{(k)} \lambda_{(k)i} \lambda^j_{(k)} = \delta^j_i, \quad \sum_k e_{(k)} \lambda_{(k)i} \lambda_{(k)j} = g_{\underline{ij}}, \tag{6 a,b}$$

$$\sum_k e_{(k)} \lambda^i_{(k)} \lambda^j_{(k)} = g^{\underline{ij}} \tag{6c}$$

*are valid.*

Proof. In the determinant det $(\lambda^i_{(h)})$, whose value is 1, we can regard $e_{(k)}\lambda_{(k)i}$ as the cofactor of the element $\lambda^i_{(k)}$. Developing the determinant either by rows or by columns it (6a) follows.

Further, we have

$$\sum_k e_{(k)}\lambda_{(k)i}\lambda_{(k)j} = g_{\underline{jl}}\sum_k e_{(k)}\lambda_{(k)i}\lambda^l_{(k)} \underset{(6a)}{=} g_{\underline{jl}}\ \delta^l_i \Rightarrow (6b).$$

(6c) can be obtained in the same manner.

## 2. Definition and basic properties of the coefficients of a rotation

Using the two kinds of covariant derivative of a vector in a $\mathrm{GR}_N$, we can define two kinds of coefficients of rotation ([3], §32, [4], ch.VI), as two systems of invariants (for $\theta = 1, 2$).

*Definition 2.* The invariants

$$\underset{\theta}{\gamma}_{(hkm)} = \lambda_{(h)i\underset{\theta}{|}j}\lambda^i_{(k)}\lambda^j_{(m)} = \lambda^i_{(h)\underset{\theta}{|}j}\lambda_{(k)i}\lambda^j_{(m)}, \quad \theta = 1, 2 \tag{7}$$

are said to be the *coefficients of rotation* of the given orthogonal ennuple.

**Theorem 2.** *Both kinds of coefficients of rotation are antisymmetric in their first two indices, i.e.*

$$\underset{\theta}{\gamma}_{(hkm)} = -\underset{\theta}{\gamma}_{(khm)} \implies \underset{\theta}{\gamma}_{(hhm)} = 0. \tag{8}$$

Proof. By covariant differentiation we get from (5') the relation

$$e_{(k)}\left[\lambda^i_{(h)\underset{\theta}{|}j}\lambda_{(k)i} + \lambda^i_{(h)}\lambda_{(k)i\underset{\theta}{|}j}\right] = 0,$$

from where, transvecting by $\lambda^j_{(m)}$,

$$e_{(k)}\left[\lambda^i_{(h)\underset{\theta}{|}j}\lambda_{(k)i}\lambda^j_{(m)} + \lambda^i_{(h)}\lambda_{(k)i\underset{\theta}{|}j}\lambda^j_{(m)}\right] \underset{(7)}{=} e_{(k)}\left[\underset{\theta}{\gamma}_{(hkm)} + \underset{\theta}{\gamma}_{(khm)}\right] = 0,$$

that is

$$\underset{\theta}{\gamma}_{(hkm)} + \underset{\theta}{\gamma}_{(khm)} = 0 \quad \implies \quad (8).$$

**Theorem 3.** *If*

$$\gamma_{(hkm)} = \lambda_{(h)i;j}\lambda^i_{(k)}\lambda^j_{(m)} \tag{9}$$

*are the coefficients of rotation in the associated Riemannian space* $R_N$, *we have the relations*

$$\underset{\theta}{\gamma}_{(hkm)} = \gamma_{(hkm)} + (-1)^\theta \Gamma^p_{\underset{\smile}{ij}}\,\lambda_{(h)p}\lambda^i_{(k)}\lambda^j_{(m)}, \quad \theta = 1,2, \tag{10}$$

$$\gamma_{(hkm)} = (\underset{1}{\gamma}_{(hkm)} + \underset{2}{\gamma}_{(hkm)})/2, \tag{11}$$

$$\underset{\theta}{\gamma}_{(hkh)} = \gamma_{(hkh)},\ \underset{\theta}{\gamma}_{(hhk)} = \gamma_{(hhk)},\ \underset{\theta}{\gamma}_{(hkk)} = \gamma_{(hkk)}, \quad \theta = 1,2. \tag{12}$$

Proof. In virtue of (7) and (4)

$$\underset{\theta}{\gamma}_{(hkm)} = [\lambda_{(h)i;j} + (-1)^\theta \Gamma^p_{\underset{\smile}{ij}}\,\lambda_{(h)p}]\lambda^i_{(k)}\lambda^j_{(m)} \quad \underset{(9)}{\Longrightarrow} \quad (10).$$

In virtue of (10), for two coinciding indices there follows (12), because it is for example (for $h = m$) :

$$\begin{aligned}\Gamma^p_{\underset{\smile}{ij}}\,\lambda_{(h)p}\lambda^i_{(k)}\lambda^j_{(h)} = \Gamma_{\underset{\smile}{p.ij}}\,\lambda^p_{(h)}\lambda^i_{(k)}\lambda^j_{(h)} = \Gamma_{\underset{\smile}{j.ip}}\,\lambda^j_{(h)}\lambda^i_{k)}\lambda^p_{(h)} \\ = -\Gamma_{\underset{\smile}{p.ij}}\,\lambda^p_{(h)}\lambda^i_{(k)}\lambda^j_{(h)} = 0.\end{aligned}$$

Here we applied the fact that $\Gamma_{\underset{\smile}{p.ij}}$ is antisymmetric in all pairs of indices (which can be easily seen from (2)).

### 3. Expression of the derivative of the vectors of a congruence by the coefficients of rotation

**Theorem 4.** *In a* $GR_N$ *the relation*

$$\lambda_{(h)i\underset{\theta}{|}j} = \sum_{k,m=1}^{N} e_{(k)}e_{(m)}\underset{\theta}{\gamma}_{(hkm)}\lambda_{(k)i}\lambda_{(m)j} \tag{13}$$

*is valid.*

Proof. Multiplying the relation (7) by $e_{(k)}e_{(m)}\lambda_{(k)p}\lambda_{(m)_q}$ and summing with respect to $k, m$, we get

$$\begin{aligned}\sum_{k,m}\underset{\theta}{\gamma}_{(hkm)}e_{(k)}e_{(m)}\lambda_{(k)p}\lambda_{(m)_q} = \sum_{k,m}\lambda_{(h)i\underset{\theta}{|}j}\lambda^i_{(k)}\lambda^j_{(m)}e_{(k)}e_{(m)}\lambda_{(k)p}\lambda_{(m)_q} = \\ = \lambda_{(h)i\underset{\theta}{|}j}\left\{\sum_k e_{(k)}\lambda_{(k)p}\lambda^i_{(k)}\right\}\cdot\left\{\sum_m e_{(m)}\lambda_{(m)_q}\lambda^j_{(m)}\right\}\underset{(6a)}{=}\end{aligned}$$

$$= \lambda_{(h)i\underset{\theta}{|}j}\delta^i_p\delta^j_q = \lambda_{(h)p\underset{\theta}{|}q} \quad\Longrightarrow\quad (13).$$

**Theorem 5.** *The covariant derivatives of the vectors $\lambda_{(h)i}$ and $\lambda^i_{(h)}$ in the direction of the vector $\lambda^j_{(p)}$ can be expressed by the coefficients of rotation as a linear combination of the vectors of the ennuple as follows:*

$$\lambda_{(h)i\underset{\theta}{|}j}\lambda^j_{(p)} = \sum_k e_{(k)}\underset{\theta}{\gamma}_{(hkp)}\lambda_{(k)i}, \tag{14a}$$

$$\lambda^i_{(h)\underset{\theta}{|}j}\lambda^j_{(p)} = \sum_k e_{(k)}\underset{\theta}{\gamma}_{(hkp)}\lambda^i_{(k)}. \tag{14b}$$

Proof. Transvecting the equation (13) by $\lambda^j_{(p)}$, we obtain

$$\begin{aligned}\lambda_{(h)i\underset{\theta}{|}j}\lambda^j_{(p)} &= \sum_{k,m} e_{(k)}e_{(m)}\underset{\theta}{\gamma}_{(hkm)}\lambda_{(k)i}\lambda_{(m)j}\lambda^j_{(p)} \underset{(5')}{=}\\ &= \sum_{k,m} e_{(k)}\underset{\theta}{\gamma}_{(hkm)}\lambda_{(k)i}\delta_{mp} = \sum_k e_{(k)}\underset{\theta}{\gamma}_{(hkp)}\lambda_{(k)i} \quad\Longrightarrow\quad (14)\end{aligned}$$

## 4. Integrability conditions of the equation (13)

The relation (13) is a partial differential equation with respect to the unknown functions $\lambda_{(h)^i}$. Now we are going to examine its integrability conditions.

In [5] we have obtained 10 Ricci-type identities in a $\mathrm{GR}_N$. In three of these identities appear the curvature tensors $\underset{1}{R}, \underset{2}{R}, \underset{3}{R}$, and in the others appear the quantities $\underset{1}{A}, \dots, \underset{15}{A}$, which have the form and the role of the curvature tensors, but they are not tensors. In [6] we have obtained combined Ricci-type identities, in which appear "derived" curvature tensors $\underset{1}{\tilde{R}}, \dots, \underset{8}{\tilde{R}}$. In [7] it is proved that only five are independent among the mentioned curvature tensors, for example $\underset{1}{R}, \underset{2}{R}, \underset{3}{R}, \underset{3}{\tilde{R}}, \underset{4}{\tilde{R}}$, while the others are linear combinations of these five tensors. We shall use further those of the Ricci-type identities in which appear the above tensors (the tensor $\underset{1}{\tilde{R}}$ is a linear combination of $\underset{1}{R}, \underset{2}{R}$, while the tensor $\underset{2}{\tilde{R}}$ does not appear in the identities which we need).

**Theorem 6.** *In a $GR_N$ the first two integrability conditions ($\theta = 1, 2$) of equation (13) are*

$$
\begin{aligned}
&\Big[ -\underset{\theta}{R}{}^{s}{}_{ijr}\lambda_{(h)s} + 2(-1)^{\theta}\Gamma^{s}_{\underset{\smile}{jr}}\lambda_{(h)i\underset{\theta}{|}s} \Big]\, \lambda^{i}_{(p)}\lambda^{j}_{(q)}\lambda^{r}_{(t)} \\
&= \underset{\theta}{\gamma}_{(hpq),j}\lambda^{j}_{(t)} - \underset{\theta}{\gamma}_{(hpt),j}\lambda^{j}_{(q)} + \\
&+ \sum_{k=1}^{N} e_{(k)} \Big\{ \underset{\theta}{\gamma}_{(hkq)}\underset{\theta}{\gamma}_{(kpt)} - \underset{\theta}{\gamma}_{(hkt)}\underset{\theta}{\gamma}_{(kpq)} + \underset{\theta}{\gamma}_{(hpk)}\big[\underset{\theta}{\gamma}_{(kqt)} - \underset{\theta}{\gamma}_{(ktq)}\big] \Big\}, \\
&\theta = 1, 2,
\end{aligned}
\tag{15}
$$

*where*

$$
\underset{1}{R}{}^{s}{}_{ijr} = \Gamma^{s}_{ij,r} - \Gamma^{s}_{ir,j} + \Gamma^{p}_{ij}\Gamma^{s}_{pr} - \Gamma^{p}_{ir}\Gamma^{s}_{pj}, \tag{16}
$$

$$
\underset{2}{R}{}^{s}{}_{ijr} = \Gamma^{s}_{ji,r} - \Gamma^{s}_{ri,j} + \Gamma^{p}_{ji}\Gamma^{s}_{rp} - \Gamma^{p}_{ri}\Gamma^{s}_{jp} \tag{17}
$$

*are the 1st and the 2nd kind curvature tensors of the $GR_N$.*

Proof. Applying the Ricci-type identities (6), (11) from [5], we have

$$
\lambda_{(h)i\underset{\theta}{|}jr} - \lambda_{(h)i\underset{\theta}{|}rj} = -\underset{\theta}{R}{}^{s}{}_{ijr}\lambda_{(h)s} + 2(-1)^{\theta}\Gamma^{s}_{\underset{\smile}{jr}}\lambda_{(h)i\underset{\theta}{|}s}, \quad \theta = 1, 2.
$$

By repeated differentiation of (13) one can form the difference on the left side of this equation, and then (15) easily follows.

**Theorem 7.** *The third integrability condition of the equation (13) in a $GR_N$ is*

$$
\begin{aligned}
&-\underset{3}{R}{}^{s}{}_{ijr}\lambda_{(h)s}\lambda^{i}_{(p)}\lambda^{j}_{(q)}\lambda^{r}_{(t)} = \underset{1}{\gamma}_{(hpq),j}\lambda^{j}_{(t)} - \underset{2}{\gamma}_{(hpt),j}\lambda^{j}_{(q)} + \\
&+ \sum_{k=1}^{N} e_{(k)}\big[\underset{1}{\gamma}_{(hkq)}\underset{2}{\gamma}_{(kpt)} - \underset{2}{\gamma}_{(hkt)}\underset{1}{\gamma}_{(kpq)} + \underset{1}{\gamma}_{(hpk)}\underset{2}{\gamma}_{(kqt)} - \underset{2}{\gamma}_{(hpk)}\underset{1}{\gamma}_{(ktq)}\big],
\end{aligned}
\tag{18}
$$

*where*

$$
\underset{3}{R}{}^{s}{}_{ijr} = \Gamma^{s}_{ij,r} - \Gamma^{s}_{ri,j} + \Gamma^{p}_{ij}\Gamma^{s}_{rp} - \Gamma^{p}_{ri}\Gamma^{s}_{pj} + \Gamma^{p}_{rj}(\Gamma^{s}_{pi} - \Gamma^{s}_{ip}) \tag{19}
$$

*is the 3rd kind curvature tensor of the $GR_N$.*

Proof. Applying the corresponding identity from [5] we get

$$
\lambda_{(h)i\underset{1}{|}j\underset{2}{|}r} - \lambda_{(h)i\underset{2}{|}r\underset{1}{|}j} = -\underset{3}{R}{}^{s}{}_{ijr}\lambda_{(h)s}.
$$

Further, use (13) to form the difference on the left side.

**Theorem 8.** *The fourth integrability condition of the equation (13) in a $GR_N$ is*

$$
\begin{aligned}
&-2\underset{3}{\tilde{R}}{}^{s}{}_{ijr}\lambda_{(h)s}\lambda^{i}_{(p)}\lambda^{j}_{(q)}\lambda^{r}_{(t)} =\\
&=\underset{1}{\gamma}_{(hpq),j}\lambda^{j}_{(t)} - \underset{1}{\gamma}_{(hpt),j}\lambda^{j}_{(q)} + \underset{2}{\gamma}_{(hpq),j}\lambda^{j}_{(t)} - \underset{2}{\gamma}_{(hpt),j}\lambda^{j}_{(q)} +\\
&\quad+\sum_{k=1}^{N} e_{(k)} \Big[ \underset{1}{\gamma}_{(hkq)}\underset{1}{\gamma}_{(kpt)} - \underset{1}{\gamma}_{(hkt)}\underset{2}{\gamma}_{(kpq)} + \underset{2}{\gamma}_{(hkq)}\underset{2}{\gamma}_{(kpt)} -\\
&\quad- \underset{2}{\gamma}_{(hkt)}\underset{1}{\gamma}_{(kpq)} + \underset{1}{\gamma}_{(hpk)}\underset{1}{\gamma}_{(kqt)} - \underset{1}{\gamma}_{(hpk)}\underset{2}{\gamma}_{(ktq)} + \underset{2}{\gamma}_{(hpk)}\underset{2}{\gamma}_{(kqt)} -\\
&\quad- \underset{2}{\gamma}_{(hpk)}\underset{1}{\gamma}_{(ktq)} \Big],
\end{aligned}
\tag{20}
$$

*where*

$$
\underset{3}{\tilde{R}}{}^{s}{}_{ijr} = \Gamma^{s}_{\underline{ij},r} - \Gamma^{s}_{\underline{ir},j} + \frac{1}{2}(\Gamma^{p}_{ij}\Gamma^{s}_{rp} + \Gamma^{p}_{ji}\Gamma^{s}_{pr} - \Gamma^{p}_{ir}\Gamma^{s}_{pj} - \Gamma^{p}_{ri}\Gamma^{s}_{jp})
\tag{21}
$$

*is the "derived" 3rd kind curvature tensor of the $GR_N$.*

Proof. By virtue of (48'), (51), (35), (37) in [6], we have

$$
\lambda_{(h)i\underset{1}{|}jr} - \lambda_{(h)i\underset{1}{|}r\underset{2}{|}j} + \lambda_{(h)i\underset{2}{|}jr} - \lambda_{(h)i\underset{2}{|}r\underset{1}{|}j} = -2\underset{3}{\tilde{R}}{}^{s}{}_{ijr}\ \lambda_{(h)s}
$$

and the use of (13) yields the integrability condition (20).

**Theorem 9.** *The fifth integrability condition of the equation (13) is*

$$
\begin{aligned}
&\Big[ -3\underset{4}{\tilde{R}}{}^{s}{}_{ijr}\lambda_{(h)s} + \Gamma^{s}_{\underset{\smile}{jr}} \Big( \lambda_{(h)i\underset{1}{|}s} + \lambda_{(h)i\underset{2}{|}s} \Big)\Big] \lambda^{i}_{(p)}\lambda^{j}_{(q)}\lambda^{r}_{(t)} =\\
&= 2\underset{2}{\gamma}_{(hpq),j}\lambda^{j}_{(t)} - 2\underset{2}{\gamma}_{(hpt),j}\lambda^{j}_{(q)} + \underset{1}{\gamma}_{(hpq),j}\lambda^{j}_{(t)} - \underset{1}{\gamma}_{(hpt),j}\lambda^{j}_{(q)} +\\
&+ \sum_{k} e_{(k)} \Big[\underset{2}{\gamma}_{(hkq)}\underset{2}{\gamma}_{(kpt)} - \underset{1}{\gamma}_{(hkt)}\underset{2}{\gamma}_{(kpq)} + \underset{1}{\gamma}_{(hkq)}\underset{2}{\gamma}_{(kpt)} - \underset{2}{\gamma}_{(hkt)}\underset{1}{\gamma}_{(kpq)} +\\
&+ \underset{2}{\gamma}_{(hkq)}\underset{1}{\gamma}_{(kpt)} - \underset{2}{\gamma}_{(hkt)}\underset{2}{\gamma}_{(kpq)} + \underset{2}{\gamma}_{(hpk)}\underset{2}{\gamma}_{(kqt)} - \underset{1}{\gamma}_{(hpk)}\underset{2}{\gamma}_{(ktq)} +\\
&+ \underset{1}{\gamma}_{(hpk)}\underset{2}{\gamma}_{(kqt)} - \underset{2}{\gamma}_{(hpk)}\underset{1}{\gamma}_{(ktq)} + \underset{2}{\gamma}_{(hpk)}\underset{1}{\gamma}_{(kqt)} - \underset{2}{\gamma}_{(hpk)}\underset{2}{\gamma}_{(ktq)} \Big],
\end{aligned}
\tag{22}
$$

*where*

$$
\begin{aligned}
\underset{4}{\tilde{R}}{}^{s}{}_{ijr} =& \frac{1}{3}(\Gamma^{s}_{ij,r} - \Gamma^{s}_{ir,j} + 2\Gamma^{s}_{ji,r} - 2\Gamma^{s}_{ri,j} + \Gamma^{p}_{ij}\Gamma^{s}_{rp} + \Gamma^{p}_{ji}\Gamma^{s}_{pr} -\\
&- \Gamma^{p}_{ri}\Gamma^{s}_{pj} - \Gamma^{p}_{ri}\Gamma^{s}_{jp} + \Gamma^{p}_{ji}\Gamma^{s}_{rp} - \Gamma^{p}_{ir}\Gamma^{s}_{jp} - 2\Gamma^{s}_{\underset{\smile}{pi}}\Gamma^{p}_{\underset{\smile}{jr}})
\end{aligned}
\tag{23}
$$

*is the "derived" fourth kind curvature tensor of the* $GR_N$.

PROOF. We use that Ricci-type identity in which the curvature tensor $\underset{4}{\tilde{R}}$ appears. From (52'), (56') and (62) in [6], we have

$$\lambda_{(h)i\underset{2}{|}jr} - \lambda_{(h)i\underset{1}{|}r\underset{2}{|}j} + \lambda_{(h)i\underset{1}{|}j\underset{2}{|}r} - \lambda_{(h)i\underset{2}{|}r\underset{1}{|}j} + \lambda_{(h)i\underset{2}{|}j\underset{1}{|}r} - \lambda_{(h)i\underset{2}{|}rj} =$$

$$= -3\underset{4}{\tilde{R}}{}^s{}_{ijr}\lambda_{(h)s} + \Gamma^s_{\underset{\smile}{jr}}(\lambda_{(h)i\underset{1}{|}s} + \lambda_{(h)i\underset{2}{|}s}),$$

and then use (13).

## References

[1] L. P. EISENHART, Generalized Riemannian spaces I, *Proc. Nat. Acad. Sci. USA* **37** (1951), 311–315.
[2] L. . EISENHART, Generalized Riemannian spaces II, *Proc. Nat. Acad. Sci. USA* **38** (1952), 505–508.
[3] T. P. ANDELIĆ, Tenzorski račun, *Beograd.*
[4] C. E. WEATHERBURN, Riemannian Geometry and the Tensor Calculus, *Cambridge U.P.* (1950).
[5] S. M. MINČIĆ, Ricci identities in the space of non-symmetric affine connexion, *Matematički vesnik* **10(25),sv.2** (1973), 161–172.
[6] S. M. MINČIĆ, Curvature tensors of the space of non-symmetric affine connexion, obtained from the curvature pseudotensors, *Matematički vesnik* **13(28)** (1976), 421–435.
[7] S. M. MINČIĆ, Independent curvature tensors and pseudotensors of spaces with non-symmetric affine connexion, *Colloquia Math. Societas Janos Bolyai, 31. Diff. Geometry* (1979), *Budapest (Hungary).*

SVETISLAV M. MINČIĆ
UNIVERSITY OF NIŠ
PHILOSOPHICAL FACULTY
NIŠ, YUGOSLAVIA

*(Received April 27, 1990)*