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On incidence algebras associated with regular cell decomposition of $\mathbb{S}^n$

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Abstract. The minimal projective resolution of a simple module over the incidence algebra associated with a regular cell decomposition of $\mathbb{S}^n$ is constructed. As a corollary we prove that any such algebra is Koszul. We also calculate the global dimensions of certain quotients of such algebras.

1. Introduction

There are several papers where the problem of constructing a projective resolution for a simple module over an incidence algebra is investigated, see for example [9], [7], [11], [8], [16] and references therein. Although different powerful methods are known ([1], [2]) there are no general machinery for constructing the minimal resolution (see also [14] for the injective analogue). Only some restrictive conditions under which the classical resolution is minimal ([9], [16]) are known. Moreover, usually the answer is given in terms of some geometrical realization of a path algebra ([10], [11], [14], [3], [16]) leading to complicated calculations in non-trivial cases.

In the paper we study a class of incidence algebras obtained from the poset of cells of a regular cell decomposition of $\mathbb{S}^n$ (i.e. a cell decomposition of $\mathbb{S}^n$ such that the closure of each cell is homeomorphic to a closed ball of the corresponding dimension). We call them RC-algebras. Classical results lead to difficult technical calculations in order to obtain a projective resolution of a simple module over an RC-algebra. Nevertheless, we

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succeed to obtain the minimal projective resolution for a simple module using some geometrical properties of regular cell decomposition. We also show that all RC-algebras are Koszul ([9], [4], [12]). Our construction is inspired by the well-known BGG-resolution of a simple finite-dimensional module over a simple finite-dimensional complex Lie algebra ([5]).

The paper is organized in the following way: In Section 2 we collect all necessary preliminaries on incidence algebras. In Section 3 we define our main object – the RC-algebras. In Section 4 we construct the minimal projective resolution for a simple module over an RC-algebra and prove that RC-algebras are Koszul. In Section 5 we calculate global dimension of some zero relation quotients of an RC-algebra. Finally, in Section 6 we illustrate our results by examples.

2. Preliminaries

Let $\mathbb{F}$ denote a fixed field of arbitrary characteristics. Consider a finite partially ordered set (poset) $(P,<)$. In a natural way we associate with P its oriented quiver (i.e. a finite oriented graph) Q in the following way: the set Q_0 of vertices (points) of Q coincides with P and there is an arrow from x to y, $x, y \in Q_0$ if and only if $y < x$ and there are no $u \in Q_0$ such that $y < u < x$ ([6]). We will denote by Q_1 the set of all arrows of Q.

A path of length m in Q is a sequence of arrows $\alpha_1, \dots, \alpha_m$ written $p = \alpha_m\alpha_{m-1}\dots\alpha_1$, such that the initial point of α_{i+1} coincides with the terminal point of α_i for all $1 \leq i \leq m-1$. We assume that to each $x \in Q_0$ corresponds the "trivial path" e_x of length 0 with x as the initial and the terminal point. We will denote by B the set of all paths in Q.

Consider a path algebra $\mathbb{F}Q$. This is an $\mathbb{F}$-algebra with the base B and a multiplication defined as follows:

1. $e_x^2 = e_x$ for any $x \in Q_0$;
2. $e_x \cdot p = p \cdot e_y = p$ where y is the initial point of p and x is the terminal point of p;
3. $p \cdot g = 0$ if the initial point of p does not coincide with the terminal point of g;
4. $p \cdot g = pg$ if the initial point of p coincides with the terminal point of g.

Let I be a two-sided ideal in $\mathbb{F}Q$ generated by all differences $p - g$ where p, g are paths in B having common initial and common terminal points. The quotient algebra $\mathcal{A} = \mathbb{F}Q/I$ is called the incidence algebra of P ([6]).

For $x, y \in P$ we will write $[x, y]$ for the set $\{z \in P \mid x < z < y\} \cup \{x, y\}$ in the case $x < y$ or $x = y$ and we set $[x, y] = \varnothing$ otherwise. We will also denote by $[\cdot, x]$ the set $\{z \in P \mid z < x\} \cup \{x\}$.

By an $\mathcal{A}$-module we will mean a finite-dimensional $\mathcal{A}$-module. It is well-known (se for example [13]) that any module V over $\mathcal{A}$ can be considered as a set of $\mathbb{F}$-spaces V_x, $x \in Q_0$ and a set of linear operators $C(V, f)$, $f \in Q_1$ that satisfy all necessary relations caused by the ideal I. For an $\mathcal{A}$-module V we will denote by $\operatorname{Supp} V$ the set $\{x \in Q_0 \mid V_x \neq 0\}$. Clearly, each $C(V, e_x)$ is the identity operator on V_x.

Simple modules $S(x)$ over $\mathcal{A}$ are in 1–1 correspondence with $x \in Q_0$ and have rather simple structure: $\operatorname{Supp} S(x) = \{x\}$ and $\dim(S(x))_x = 1$. Thus $C(S(x), f) = 0$ for any $e_x \neq f \in B$ and $C(S(x), e_x)$ is the identity map.

There is a 1–1 correspondence between indecomposable projectives $P(x)$ and $x \in Q_0$ also. The structure of $P(x)$ is again quite simple: $\operatorname{Supp} P(x) = [\cdot, x]$ and $\dim(P(x))_y = 1$ for any $y \in [\cdot, x]$. Fixing a base in each $(P(x))_y$ we set an operator $C(P(x), f)$ to be the unit matrix if and only if initial and terminal points of f belong to $\operatorname{Supp} P(x)$. $C(P(x), f)$ is zero otherwise.

Clearly, any path a with the initial point x and the terminal point y defines a linear operator $a(V) : V_x \to V_y$ in any $\mathcal{A}$-module V. For two $\mathcal{A}$-modules V and W a homomorphism $\varphi : V \to W$ is a collection of linear operators $\varphi_x : V_x \to W_x$, $x \in P$ such that $\varphi_y a(V) = a(W)\varphi_x$ holds for any path a with the initial point x and the terminal point y.

We will also need some topological notations. For a subset A of a topological space we will denote by $\overline{A}$ the closure of A, by $\operatorname{int}(A)$ the interior of A and by $\tilde{A} = \overline{A} \setminus \operatorname{int}(A)$ the boundary of A.

3. Definition of RC-algebras

Let K be a regular cell decomposition of the sphere $\mathbb{S}^n$, $n > 0$ ([15]). By K_i, $0 \leq i \leq n$ we will denote the set of i-cells in K. Consider the poset $P = P(K)$ containing the elements $\varnothing$, L, $e \in K_i$, $0 \leq i \leq n$ with the order $<$ defined as follows:

1. $\varnothing < x$ for any $\varnothing \neq x \in P$;
2. $x < L$ for any $L \neq x \in P$;
3. $x < y$, $x, y \in K$ if and only if $x \in \bar{y}$.

For the rest of the paper $\mathcal{A} = \mathcal{A}(K)$ will denote the incidence algebra of P. We will call $\mathcal{A}$ regular cell algebra (RC-algebra) associated with K.

Since $\varnothing$ is the minimal and L is the maximal elements in P we have $[\cdot, x] = [e, x]$ for any $x \in P$ and $[e, L] = P$. For $0 \leq i \leq n$ we set $P_i = K_i$, $P_{-1} = \{\varnothing\}$ and $P_{n+1} = \{L\}$.

4. Projective resolution and global dimension

The aim of this section is to construct the minimal projective resolution of a simple $\mathcal{A}$-module and to calculate the global dimension of $\mathcal{A}$. As a corollary we will obtain that $\mathcal{A}$ is Koszul.

Clearly, the algebra $\mathcal{A}_x = \mathcal{A} \sum_{y \leq x} e_y$ is an RC-algebra for any $x \in P$. $\mathcal{A}_x$ is trivial for $x = \varnothing$ and coincides with $\mathcal{A}$ for $x = L$. For $x \in K$ the algebra $\mathcal{A}_x$ is the RC-algebra associated with the natural regular cell decomposition of $\tilde{x}$. Thus, it is sufficient to construct the minimal projective resolution for $S(L)$ only.

Lemma 1. *Let $x, y \in P$, $x < y$ then*

$$\dim \operatorname{Hom}_{\mathcal{A}}(P(x), P(y)) = 1$$

and each non-zero homomorphism from the above space is a monomorphism.

Proof. Let v_x (v_y) denote a generator of $P(x)$ $(P(y))$ i.e. $0 \neq v_x \in P(x)_x$ $(0 \neq v_y \in P(y))$. Let a be a path with the initial point y and the terminal point x. It exists since $x < y$.

Let $\varphi \in \operatorname{Hom}_{\mathcal{A}}(P(x), P(y))$. Then $\varphi(v_x) \in P(y)_x$ and thus $\varphi(v_x) = cav_y$ for some $c \in \mathbb{F}$. Since v_x generates $P(x)$, each $c \in \mathbb{F}$ defines the unique homomorphism $\varphi_c \in \operatorname{Hom}(P(x), P(y))$ by $\varphi_c(v_x) = cav_y$. Thus we obtain that $\dim \operatorname{Hom}(P(x), P(y)) = 1$. The rest follows directly from the construction of φ_c. $\square$

Fix a generator v_x of $P(x)$ for each $x \in P$. Set $C_i = \bigoplus_{x \in P_i} P(x)$, $-1 \leq i \leq n+1$. By Lemma 1 any homomorphism $\varphi : C_i \to C_{i+1}$, $-1 \leq i \leq n$ is uniquely defined by a complex matrix

$$d = (d_{yx})_{y \in P_{i+1}}^{x \in P_i}.$$

It is convenient to extend our cell-complex K to the cell-decomposed ball K' by adding one $n+1$-cell L in a natural way. Fix an orientation in each $x \in K'$ ([15]) and set $[y : x]$ to be the incidence numbers with respect to this choice of orientations. Denote by

$$d_i = (d_{yx})_{y \in P_{i+1}}^{x \in P_i}, \quad 0 \leq i \leq n$$

the complex matrix such that $d_{yx} = [y : x]$. We also set

$$d_{-1} = (d_{x\varnothing})_{x \in P_0}, \quad d_{x\varnothing} = 1 \text{ for all } x \in P_0.$$

As it was mentioned, the matrices d_i, $-1 \leq i \leq n$ define homomorphisms $d_i : C_i \to C_{i+1}$. Let $p : C_{n+1} \to S(L)$ be a canonical projection.

Theorem 1. *The sequence*

$$(1) \quad 0 \to C_{-1} \xrightarrow{d_{-1}} C_0 \xrightarrow{d_0} C_1 \xrightarrow{d_1} \dots \xrightarrow{d_{n-1}} C_n \xrightarrow{d_n} C_{n+1} \xrightarrow{p} S(L) \to 0$$

is the minimal projective resolution of $S(L)$.

We divide the proof of this theorem into several steps.

Lemma 2. *The sequence* (1) *is a complex (i.e. $d_{i+1} \circ d_i = 0$ for $-1 \leq i \leq n-1$ and $p \circ d_n = 0$).*

Proof. Clearly, $p \circ d_n = 0$ by the definition of p. By [15, Theorem 6.2] for any $x \in P_i$, $y \in P_{i+2}$ the number of $z \in P_{i+1}$ such that $x < z < y$ is either 2 or 0 for each $0 \leq i \leq n-1$. By [15, Theorem 6.6], $[y : z_1][z_1 : x] + [y : z_2][z_2 : x] = 0$ holds for any $0 \leq i \leq n-1$ and any $x \in P_i$, $y \in P_{i+2}$ such that there exist two distinct z_1, $z_2 \in P_{i+1}$ with $x < z_j < y$, $j = 1, 2$. At the same time $[x : z_1] + [x : z_2] = 0$ holds for any $x \in P_1$ with faces $z_1, z_2 \in P_0$ by [15, Theorem 6.6]. Hence $d_{i+1} d_i = 0$ for all $-1 \leq i \leq n$. This completes the proof. □

Lemma 3. *For any $x \in K_i$, $0 \leq i \leq n-1$ the poset $[x, L]$ is isomorphic to $P(K_1)$ where K_1 is a cell decomposition of the sphere $\mathbb{S}^{n-i-1}$.*

Proof. Clearly, it is enough to prove this statement for $i = 0$. Thus we will assume that x is a point on $\mathbb{S}^n$. Since K is regular we can find a closed ball T containing x such that for each $y \in K$ the following holds: $T \cap y \neq \varnothing$ if and only if $x < y$. Consider the n-dimensional ball $T_1 = T \cap \mathbb{S}^n$ and let T_2 be the quotient $T_1 / \tilde{T}_1$. Clearly, T_2 is homeomorphic to an n-dimensional sphere and possess a natural regular cell decomposition induced from the original $\mathbb{S}^n$. Crossing it with a hyperplane that does not contain any zero-dimensional cell we easily get the required regular cell decomposition of $\mathbb{S}^{n-1}$ with the corresponding poset isomorphic to $[x, L]$. $\square$

Proposition 1. *The sequence* (1) *is exact.*

Proof. Clearly, it is sufficient to show that for any fixed $x \in P$ the induced sequence

$$(2) \qquad 0 \to (C_{-1})_x \to (C_0)_x \to \cdots \to (C_n)_x \to (C_{n+1})_x \to S(L)_x \to 0$$

is exact. Obviously, this is true for $x = L$.

To prove the statement we will construct a new complex. Suppose that $x \in P_k$ and $x \neq L$. For $i \geq k$ set A_i to be a free abelian group with the base a_y, $y \in P_i$, $x < y$. Let $\delta_i : A_i \to A_{i+1}$, $k \leq i \leq n$ be a homomorphism defined by the matrix

$$\delta_i = ([y : z])_{y \in P_{i+1}, x<y}^{z \in P_i, x<z} .$$

One can see that the complex (2) is exact if and only if the following complex:

$$(3) \qquad 0 \longrightarrow A_k \xrightarrow{\delta_k} A_{k+1} \xrightarrow{\delta_{k+1}} \dots \xrightarrow{\delta_{n-1}} A_n \xrightarrow{\delta_n} A_{n+1} \longrightarrow 0$$

is exact.

Using Lemma 3 we see that the homology of the sequence (3) coincide with the homology of the regular cell complex $\mathbb{S}^{n-k-1}$ with the poset isomorphic to $[x, L]$. Thus they coincide with the reduced homology of the closed ball. Since ball is homotopic to a one-point space we conclude that the sequence (3) and thus the sequence (1) is exact. This completes the proof. $\square$

Proof of Theorem 1. Clearly, each C_i is projective for $-1 \leq i \leq n+1$. By Lemma 2 and Proposition 1 the sequence (1) is a projective resolution of $S(L)$. Its minimality follows easily from the fact that $\operatorname{Im} d_i \subset \operatorname{Rad} C_{i+1}$, $-1 \leq i \leq n$. $\square$

Corollary 1. $\mathrm{Gl.\,dim}\,\mathcal{A} = n+2$.

Proof. Follows from the minimality of the resolution (1). □

Let A be a path algebra of some finite quiver Q and I be an ideal of A generated by all arrows. Set $A' = A/I$. A' is a semi-simple algebra isomorphic to a finite product of copies of $\mathbb{F}$, one for each vertex. Define an Ext-algebra $E(A)$ by

$$E(A) = \sum_{i\geq 0} \mathrm{Ext}^i_A(A', A').$$

An algebra A is called Koszul if the Ext-algebra $E(A)$ is generated by elements of degree 0 and 1 (here we mean the standard grading). For details on Koszul algebras see [12], [4].

Corollary 2. *$\mathcal{A}$ is Koszul.*

Proof. Clearly, $\mathcal{A}$ is quadratic. By [9], $\mathcal{A}$ is Koszul if the i-th component in the minimal projective resolution of $\mathcal{A}'$ is generated by elements of degree i. This follows immediately from Theorem 1. □

5. Global dimension of some quotient algebras

In this section we calculate the global dimension of the quotients of $\mathcal{A}$ defined by one zero relation. Analogous results for trees were obtained in [8].

Consider an algebra $\mathcal{A}$ defined in Section 3. Let a be a path with the initial point x and the terminal point y for some $x, y \in P$. Let $I = \mathcal{A}a\mathcal{A}$ and set $\mathcal{A}_a = \mathcal{A}/I$. We note the following difference with [8]: The case when $a \in Q_1$ is not trivial (as it was it [8]) since $\mathcal{A}$ is not a path algebra of Q but its quotient by some commutativity relations.

Theorem 2. $\mathrm{Gl.\,dim}\,\mathcal{A}_a = n+2$.

To prove this we need some auxiliary lemmas. We recall that all the modules considered in this section are $\mathcal{A}_a$-modules.

Lemma 4. *For $z > x$ or $z = x$, $z \in P$ holds*

$$\mathrm{Supp}\,P(z) = \{u \in P \mid u < z, u \not< y\}.$$

Proof. Follows from the fact that the path a is zero in $\mathcal{A}_a$. □

Lemma 5. *For $z \not> x$, $z \in P$ holds* $\operatorname{Supp} P(z) = [\varnothing, z]$.

Proof. Since I is generated by a it follows that $u \notin \operatorname{Supp} P(z)$ if and only if $u \not< z$ or there exists a path with the initial point z and with the terminal point u containing a as a subpath. Thus $z > x$ which contradicts to our assumption. □

Proof of Theorem 2. We will consider four different cases.

Case 1. Assume that $x = L$ and the length of a is greater than 1. Then for any $z \in P$, $z \neq x$ a projective resolution of $S(z)$ can be constructed using Theorem 1 and thus $\operatorname{Gl.dim} \mathcal{A}_a \geq n+1$. We will construct a projective resolution for $S(L)$.

Let C_i, $-1 \leq i \leq n+1$, $i \neq n$ and d_i, $1 \leq i \leq n-2$ be as in Section 4. Set

$$C'_n = \bigoplus_{z \in P_n} P(z), \quad C'_{n-1} = P(y) \oplus \bigoplus_{z \in P_{n-1}} P(z).$$

Denote by d_{n-1} the map from C_{n-1} to C_n defined by the matrix

$$([u:z])_{u \in P_n}^{z \in P_{n-1}}$$

and by d'_{n-2} the map from C_{n-2} to C'_{n-1} defined in the following way: the matrix defines a map from $P(z)$ to $P(u)$, $z \in P_{n-2}$, $u \in P_{n-1}$ and the map $P(z) \to P(y)$ is leave as printed for all $z \in P_{n-2}$. As usual, we denote by $p : P(L) \to S(L)$ the canonical projection.

Consider the following sequence:

$$(4) \qquad 0 \longrightarrow C_{-1} \xrightarrow{d_{-1}} C_0 \xrightarrow{d_0} \dots \xrightarrow{d_n} C_{n+1} \xrightarrow{p} S(L) \longrightarrow 0.$$

One can see that the (4) is a complex. Now we will try to find the homology of this complex. As it was done in Proposition 1 for any $z \in P$ we consider the corresponding complex

$$(5) \quad 0 \to (C_{-1})_z \to (C_0)_z \to \dots \to (C_n)_z \to (C_{n+1})_z \to (S(L))_z \to 0.$$

For $z \not< y$ it follows from the proof of Proposition 1 that the complex (5) is exact. For $z < y$ one obtains that the only $\operatorname{Ker} d_n / \operatorname{Im} d_{n-1}$ is one-dimensional.

Thus there exists a homomorphism $d'_n : C'_n \to C_{n+1}$ and $d'_{n-1} : C'_{n-1} \to C'_n$ such that the complex

$$0 \longrightarrow C_{-1} \xrightarrow{d_{-1}} \dots \xrightarrow{d'_{n-2}} C'_{n-1} \xrightarrow{d'_{n-1}} C'_n \xrightarrow{d'_n} C_{n+1} \xrightarrow{p} S(L) \longrightarrow 0$$

is exact. Clearly, this gives us the minimal resolution for $S(L)$ and Case 1 follows.

Case 2. Let $x = L$ and $y \in P_n$. The proof is analogues to that of Case 1 with the following substitution:

$$C_n = \bigoplus_{z \in P_n, z \neq y} P(z).$$

Case 3. Assume that $x \neq L$ and a is a path of length at least 2. We will prove that the sequence (1) constructed in Section 4 is the minimal projective resolution for $S(L)$. Since the length of a is greater than 1 all the maps d_i, $-1 \leq i \leq n$ are well-defined. For the exactness it is sufficient to prove that the sequence (2) is exact. If $z \not< y$ this follows from Proposition 1. Let X denote the cell subcomplex of K formed by union of all u, $u > z$ and $u \not> y$. Set $Y = X/z$. One can see that the exactness of sequence (2) is equivalent to vanishing of the reduced simplicial homology of Y. But the last is true since Y is homotopic to a one-point space. The minimality is straightforward.

Case 4. Finally, assume that $x \neq L$, say $x \in P_k$, and a is an arrow. Consider the sequence (1) with the only difference that $d_{xy} = 0$. At the same way as in the Case 3 one obtains that this sequence is exact and minimal. $\square$

6. Examples

In this section we propose an example of a non-trivial RC-algebra and an example of an algebra constructed from the Möbius strip. In the first case we explain our results and in the second case we show that the analogue of the main theorem is not true.

6.1. An RC-algebra for $n = 2$

Let K be a cell decomposition of S^2 shown on Figure 1. Then P has a form shown on Figure 2. Fixing an orientation we can choose the following d_i:

$$d_{-1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad d_0 = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad d_1 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad d_2 = \begin{pmatrix} 1 & -1 \end{pmatrix}.$$

Thus we obtain the following resolution for $S(L)$:

$$0 \to P(\varnothing) \to P(x) \oplus P(y) \to P(z) \oplus P(u) \to P(I) \oplus P(II) \to P(L) \to S(L) \to 0$$

and conclude that $\mathrm{Gl.\,dim}\,\mathcal{A} = 4$.

Since any RC-algebra associated with S^2 is a minimal incidence algebra of global dimension 4 it seems to be impossible to give complete combinatorical description of such algebras as it was done in [11] for algebras of global dimension 3.

Figure 1. *Figure 2.*

6.2. Example of a non RC-algebra

Consider a cell decomposition of the Möbius strip shown on Figure 3. The corresponding P is shown on Figure 4. Unfortunately, one obtains the following resolution:

$$0 \to P(\varnothing) \oplus P(\varnothing) \to P(A) \oplus P(A) \oplus P(B)$$
$$\oplus P(B) \to P(l) \oplus P(m) \oplus P(n) \to P(L) \to S(L) \to 0.$$

Thus $\mathrm{Gl.\,dim}\,\mathcal{A} = 3$, although the resolution is more complicated.

Figure 3. *Figure 4.*

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