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On the (H,L) typeness of the maximal function of Cesàro means of two-parameter integrable functions on bounded Vilenkin groups

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Abstract. In this paper we prove that the operator $\sigma^* f := \sup_{n\in\mathbb{N}^2} |\sigma_n f|$ from the Hardy space H to $L^1(G_m \times G_{\widetilde{m}})$ is bounded, where the quotient of the coordinates of n is bounded. In other words σ^* is of type (H,L).

1. Introduction

Let us briefly introduce the necessary definitions and notations. Let $m := (m_0, m_1, \dots)$ denote a sequence of positive integers not less than 2. Denote by $Z_{m_j} := \{0, 1, \dots, m_j - 1\}$ the additive group of integers modulo m_j $(j \in \mathbb{N})$. Z_{m_j} is endowed by the discrete topology, i.e. every subset of Z_{m_j} is open.

Define the group G_m as the cartesian product of the discrete cyclic groups Z_{m_j},

$$G_m := \mathop{\times}_{j=0}^{\infty} Z_{m_j}.$$

The elements of G_m can be represented by sequences $x := (x_0, x_1, \dots, x_j, \dots)$ $(x_j \in Z_{m_j},\ j \in \mathbb{N})$. The group operation on $G_m(+)$ is the coordinate-wise additon, the inverse operation is $-$. The topology on G_m is the product topology, and G_m is a compact Abelian

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group. A base $I_n(x)$, $n = 0, 1, \dots$ for the topology of G_m can be given in the following way. Set

$$I_0(x) := G_m, \quad I_n(x) := \{y = (y_i, i \in \mathbb{N}) \in G_m : y_i = x_i \text{ for } i < n\}$$

for $x \in G_m$, $n \in \mathbb{P} := \mathbb{N} \setminus \{0\}$. The element $(0, 0, \dots)$ is the nullelement of G_m it will be denoted by 0. Put $I_n = I_n(0)$ $(n \in \mathbb{N})$, and observe that I_n is a subgroup of G_m $(n \in \mathbb{N})$. The direct product μ of the measures

$$\mu_k(\{j\}) := \frac{1}{m_k} \quad (j \in Z_{m_k},\ k \in \mathbb{N})$$

is the Haar measure on G_m with $\mu(G_m) = 1$.

If $M_0 = 1$, $M_{k+1} = m_k M_k$ $(k \in \mathbb{N})$, then every $n \in \mathbb{N}$ can be uniquely expressed as $n = \sum_{j=0}^{\infty} n_j M_j$, where $n_j \in Z_{m_j}$ $(j \in \mathbb{N})$ and only a finitely many of n_j differ from zero.

The group G_m is metrizable. Define the distance between the elements $x \in G_m$ and $y \in G_m$ by

$$d(x, y) := \sum_{k=0}^{\infty} \frac{|x_k - y_k|}{M_{k+1}}.$$

The topology induced by this metric coincides with that of G_m. Denote by $L^p(G_m)$ the usual Lebesgue space with the corresponding norm.

$$\|f\|_p := \left(\int_{G_m} |f|^p \right)^{\frac{1}{p}} \quad (f \in L^p(G_m)), \quad 1 \le p < \infty),$$

$$\|f\|_\infty := \inf\{r : \ r > 0, \ \ \mu(|f| > r) = 0\} \quad (f \in L^\infty(G_m)).$$

Let $\widehat{G}_m := \{\psi_n : n \in \mathbb{N}\}$ denote the character group of G_m. We enumerate the elements as follows. Denote on G_m the *generalized Rademacher functions* by

$$r_k(x) := \exp \frac{2\pi \imath x_k}{m_k} \quad (\imath^2 := -1,\ x \in G_m,\ k \in \mathbb{N}).$$

It is known that the functions

$$\psi_n(x) := \prod_{k=0}^{\infty} r_k^{n_k}(x) \quad (n \in \mathbb{N})$$

on G_m are elements of the character group of G_m, and all the elements of the character group are of this form.

The system $(\psi_n : n \in \mathbb{N})$ is called a *Vilenkin system* and G_m a *Vilenkin group*.

Let $\mathcal{A}_n$ be the σ-algebra generated by the cosets $I_n(z)$, where $n \in \mathbb{N}$, $z \in G_m$. Let α_j^k, α_n $(k, j, n \in \mathbb{N})$ be functions satisfying the following conditions:

(i) $\alpha_j^k : G_m \to \mathbb{C}$ is $\mathcal{A}_j$-measurable $(k, j \in \mathbb{N})$,

(ii) $|\alpha_j^k| := \alpha_0^k := \alpha_j^0 := \alpha_j^k(0) := 1$ $(k, j \in \mathbb{N})$,

(iii) $\alpha_n := \prod_{j=0}^{\infty} \alpha_j^{n^{(j)}} \Big(n \in \mathbb{N},\ n^{(j)} := \sum_{k=j}^{\infty} n_k M_k\Big)$.

Let $\chi_n = \psi_n \alpha_n$ $(n \in \mathbb{N})$. A function system $\{\chi_n : n \in \mathbb{N}\}$ of this type is called a $\psi\alpha$ *(Vilenkin-like) system* on the Vilenkin group G_m. [Gát1, 2]

In [Gát2] it is proved that a Vilenkin-like system is orthonormal and complete in $L^1(G_m)$.

We mention some examples.

1. If $\alpha_j^k = 1$ for each $k, j \in \mathbb{N}$, then we obtain the "ordinary" Vilenkin systems [Vil].

2. If $m_j = 2$ for all $j \in \mathbb{N}$ and $\alpha_j^{n^{(j)}} = (\beta_j)^{n_j}$, where

$$\beta_j(x) = \exp\left(2\pi\imath\left(\frac{x_{j-1}}{2^2} + \cdots + \frac{x_0}{2^{j+1}}\right)\right) \quad (n, j \in \mathbb{N},\ x \in G_m),$$

then we get the character system of the group of 2-adic integers (see e.g. [Tai, HR, Gát7]).

The (about) 25 years old conjecture of M. H. Taibleson [Tai], namely that $\sigma_n f \to f$ $(f \in L^1)$ a.e. (with respect to this system) was proved by Gát [Gát7]. In [BG] the two-dimensional version of this question was also solved.

3. If

$$t_n(x) := \exp\left(2\pi\imath\left(\sum_{j=0}^{\infty} \frac{n_j}{M_{j+1}}\right) \sum_{j=0}^{\infty} x_j M_j\right) \quad (x \in G_m, n \in \mathbb{N}),$$

then we have a Vilenkin-like system which is a useful tool in the approximation theory of limit periodic, almost even arithmetical functions (cf. [Gát3]).

Define the Fourier coefficients, the partial sums of the Fourier series and the Fejér means with respect to the Vilenkin-like system χ in the following way:

$$\widehat{f}(n) := \int_{G_m} f\bar{\chi}_n, \quad S_n f := \sum_{k=0}^{n-1} \widehat{f}^{\chi}(k)\chi_k, \quad \sigma_n f := \frac{1}{n}\sum_{k=0}^{n-1} S_k^{\chi} f$$
$$\left(n \in \mathbb{P},\ \widehat{f}^{\chi}(0) := \int_{G_m} f,\ f \in L^1(G_m)\right).$$

For more details about Vilenkin and Vilenkin-like systems see [SWS, AVD, Vil, Gát1-Gát5].

For $(n_1, n_2) = n \in \mathbb{N}^2$ set $\vee n := \max(n_1, n_2)$, $\wedge n := \min(n_1, n_2)$. Let $\widetilde{m}$ also a sequence of integers with the same properties as m. In the sequel the **bounded**ness of the **Vilenkin groups** G_m, $G_{\widetilde{m}}$ is supposed. Denote the $L^p(G_m \times G_{\widetilde{m}})$ norm of any function f by $\|f\|_p$ $(1 \le p \le \infty)$.

Let $\widetilde{I}$ denote the n-th interval generated by $\widetilde{m}$. Define $\widetilde{n} = \widetilde{n}(n) := \min(l \in \mathbb{N} : M_n \le \widetilde{M_l})$ $(n \in \mathbb{N})$. Then there exists a constant c for which $M_n \le \widetilde{M_{\widetilde{n}}} < cM_n$ for all $n \in \mathbb{N}$ (c does not depend on n, but depends on $\max_{j \in \mathbb{N}} m_j$ and $\max j \in \mathbb{N} \to \max\ \widetilde{m}_j$).

So, from now on we are going to use cM_n instead of $\widetilde{M_{\widetilde{n}}}$ (in some inequalities below).

The Kronecker product $\{\chi_{n,m} : n, m \in \mathbb{N}\}$ of two Vilenkin-like systems $\{\chi_n : n \in \mathbb{N}\}$ and $\{\widetilde{\chi}_n : n \in \mathbb{N}\}$ is said to be a two-dimensional (or double) Vilenkin-like system. Thus

$$\chi_{n,m}(x, y) := \chi_n(x)\widetilde{\chi}_m(y),$$

where $x \in G_m$, $y \in G_{\widetilde{m}}$.

If $f \in L^1(G_m \times G_{\widetilde{m}})$ then the (n, k)-th Fourier coefficients, the (n, k)-th partial sum of Fourier series and the Fejér means of order (n, k) of double Fourier series are the following

$$\widehat{f}(n, k) := \int_{G_m \times G_{\widetilde{m}}} f\overline{\chi}_{n,k}, \quad S_{n,k} f := \sum_{j=0}^{n-1}\sum_{l=0}^{k-1} \widehat{f}(j, l)\chi_{j,l},$$

$$\sigma_{n,k} f := \frac{1}{nk}\sum_{j=0}^{n-1}\sum_{l=0}^{k-1} S_{j,l} f.$$

For each $f \in L^1(G_m \times G_{\widetilde{m}})$ let the maximal function and the diagonal maximal function be defined by

$$f^\star := \sup_{n_1, n_2 \in \mathbb{N}} \left| S_{M_{n_1}, \widetilde{M}_{n_2}} f \right| \qquad f^\circ := \sup_{n \in \mathbb{N}} \left| S_{M_n, \widetilde{M}_{\widetilde{n}}} f \right|$$

and

$$\|f\|_H := \|f^\star\|_1, \quad \|f\|_{H^\circ} := \|f^\circ\|_1.$$

Define the Hardy space H to be the collection of functions $f \in L_1(G_m \times G_{\widetilde{m}})$ such that $\|f\|_H < \infty$ i.e.

$$H := \{f \in L_1(G_m \times G_{\widetilde{m}}) : \|f\|_H < \infty\},$$

and the diagonal Hardy space H° as

$$H^\circ := \{f \in L_1(G_m \times G_{\widetilde{m}}) : \|f\|_{H^\circ} < \infty\}.$$

Since $\|f\|_{H^\circ} \leq \|f\|_H$, thus $H^\circ \subset H$.

The atomic decomposition is a useful characterisation of Hardy spaces. In order to show this, we introduce the concept of an atom.

We say that $a \in L^\infty(G_m \times G_{\widetilde{m}})$ is a H° atom if

(i) there exist $x = (x_1, x_2)$ and $k \in \mathbb{N}$ such that $\operatorname{supp} a \subset I_k(x_1) \times \widetilde{I}_{\widetilde{k}}(x_2)$,

(ii) $\|a\|_\infty \leq M_k \widetilde{M}_{\widetilde{k}}$,

(iii) $(E_k a)(y) := (E_{k,\widetilde{k}} a)(y) = 0$ for all $y \in G_m \times G_{\widetilde{m}}$ $(E_{r,s} f)(y) = M_r \widetilde{M}_s \int_{I_n(y_1) \times \widetilde{I}_s(y_2)} f$.

We will use the following decomposition theorem (similar and others can be found in [Wei1]).

Theorem 1. *A function $f \in L^1(G_m \times G_{\widetilde{m}})$ is in the diagonal Hardy space H° if and only if there exist a sequence $(a_k : k \in \mathbb{N})$ of H°-atoms and a sequence $(\lambda_k : k \in \mathbb{N})$ of real numbers such that*

$$f = \sum_{k=0}^{\infty} \lambda_k a_k \tag{1}$$

and

$$\sum_{k=0}^{\infty} |\lambda_k| < \infty. \tag{2}$$

Moreover, $\|f\|_{H^\circ}$ *and* $\inf(\sum_{k=0}^{\infty}|\lambda_k|)$ *are equivalent norms, where the infimum is taken over all decompositions of* f*.*

To prove Theorem 1 recall the first steps of the decomposition algorithm of Calderon–Zygmund (for the original version see [CZ], [SWS]). For any $f\in L^1(G_m\times G_{\widetilde{m}})$ and $\lambda>0$ set

$$\Omega_0:=\begin{cases} I_0(x_1^0)\times \widetilde{I}_0(x_2^0) & \text{if } |E_0 f(\mathbf{x}^0)|>\lambda\\ \emptyset & \text{otherwise}\end{cases}$$

and

$$\Omega_k:=\begin{cases} \bigcup_i I_k(x_1^i)\times \widetilde{I}_{\widetilde{k}}(x_2^i) & \text{if } |E_k f(\boldsymbol{x}^i)|>\lambda,\\ & \quad \Big(I_k(x_1^i)\times \widetilde{I}_{\widetilde{k}}(x_2^i)\Big)\cap\Big(\bigcup_{j=0}^{k-1}=\Omega_j\Big)=\emptyset\\ \emptyset & \text{otherwise,}\end{cases}$$

where $\boldsymbol{x}^i=(x_1^i,x_2^i)\in G_m\times G_{\widetilde{m}}$, and $I_k(x_1^i)\times \widetilde{I}_{\widetilde{k}}(x_2^i)$ denotes the ith rectangle defined in the course of procedure (start the counting from zero).

For the elements of Ω_k we introduce the following notation $I_k^i:=I_k(x_1^i)\times \widetilde{I}_{\widetilde{k}}(x_2^i)$, where $i\in\mathbb{N}$. Let

$$f_k^i:=f1_{I_k^i}-\left(M_k\widetilde{M}_{\widetilde{k}}\int_{I_k^i} f\right)1_{I_k^i}$$

$$f_\lambda:=f1_{\overline{\cup\Omega_k}}+\sum_{i,k}\left(M_k\widetilde{M}_{\widetilde{k}}\int_{I_k^i} f\right)1_{I_k^i},$$

where $1_{I_k^i}$ is the characteristic function of I_k^i. Thus $f=f_\lambda+\sum_{i,k}f_k^i$ and the number of elements of Ω_k is finite. These facts imply a modyfied version of the Calderon–Zygmund decomposition lemma, (see [CZ], [SW]):

Lemma 2. *Let* $f\in L^1(G_m\times G_{\widetilde{m}})$, $\lambda>\|f\|_1$*. Then* $f=f_\lambda+\sum_{n=1}^{\infty}f_n$*, where* $\|f_\lambda\|_\infty<c\lambda$, $\operatorname{supp} f_n\subseteq I_{k_n}^n$ *for some rectangles* ($x_1^n\in G_m$, $x_2^n\in G_{\widetilde{m}}$, $k_n\in\mathbb{N}$), $\int_{I_{k_n}^n} f_n=0$, $\|f_n\|_1\le c\lambda\mu(I_{k_n}^n)$ ($n\in\mathbb{P}$)*. The rectangles* $I_{k_n}^n$ *are disjoint, furthermore* $\mu(\Omega)=\mu\left(\bigcup_{n\in\mathbb{P}}I_{k_n}^n\right)\le c\frac{\|f\|_1}{\lambda}$*.*

Proof of Theorem 1. Throughout this paper c will denote a constant which may vary at different occurances and may depend only on β (definied later), $\sup m_n$ and $\sup\widetilde{m}_n$.

For a H°-atom a we have

$$\|a\|_{H^\circ} = \left\|\sup_{n\in\mathbb{N}} |E_n a|\right\|_1 \leq \left\|\sup_{n\in\mathbb{N}} E_n \left(M_k \widetilde{M}_{\widetilde{k}} 1_{I_k(x_1)\times \widetilde{I}_{\widetilde{k}}(x_2)}\right)\right\|_1 \leq 1.$$

Consequently, if a function f in $L^1(G_m \times G_{\widetilde{m}})$ is of the form $f = \sum_{i=0}^\infty \lambda_i a_i$ with (2) then

$$\|f\|_{H^\circ} \leq \sum_{i=0}^\infty |\lambda_i| \|a_i\|_{H^\circ} \leq \sum_{i=0}^\infty |\lambda_i| < \infty,$$

that is, f is in H°. The first part of the proof is complete.

Conversely, use the Calderon–Zygmund decomposition lemma with $\lambda = 2^q$ and $q \in \mathbb{Z}$. We have

$$f = f_{2^q} + \sum_{n=1}^\infty f_{2^q,n},$$

where $\|f_{2^q}\|_\infty \leq c2^q$, and for a fixed $q \in \mathbb{Z}$ the sets $\operatorname{supp} f_{2^q,n} = I^n_{k_{q,n}}$ are pairwise disjoint. Moreover, from the decomposition algorithm we obtain $2^q < |E_q f_{2^q,i}| < c2^q$, and $|E_j f_{2^q,n}| \leq 2^q$ for any $j = 0, 1, \ldots, q-1$. However, $f_{2^q} \to f$ and $f_{-2^q} \to 0$ as $q \to \infty$, thus

$$f = \sum_{q=-\infty}^\infty (f_{2^{q+1}} - f_{2^q}) = \sum_{q=-\infty}^\infty \left(\sum_{n=1}^\infty f_{2^q,n} - \sum_{j=1}^\infty f_{2^{q+1},j}\right)$$

can be written. There exist for a $j \in \mathbb{P}$ and $n \in \mathbb{P}$ such that $\operatorname{supp} f_{2^{q+1},j} \subset \operatorname{supp} f_{2^q,n}$. Let

$$b_{q,n} := f_{2^q,n} - \sum_{j\colon \operatorname{supp} f_{2^{q+1},j} \subset \operatorname{supp} f_{2^q,n}} f_{2^{q+1},j}.$$

For every element $\boldsymbol{x}$ of $\operatorname{supp} f_{2^q,n}$ the equation $b_{q,n}(\boldsymbol{x}) = f_{2^{q+1}}(\boldsymbol{x}) - f_{2^q}(\boldsymbol{x})$ holds, consequently $|b_{q,n}| \leq c2^q$. Let $a_{q,n} := M_q \widetilde{M}_{\widetilde{q}} \frac{1}{c2^q} b_{q,n}$, and $\lambda_{q,n} := c2^q \frac{1}{M_q \widetilde{M}_{\widetilde{q}}}$, $q \in \mathbb{Z}$, $n \in \mathbb{P}$ then

$$f = \sum_{q=-\infty}^\infty \sum_{n=1}^\infty \lambda_{q,n} a_{q,n},$$

where the functions $a_{q,n}$ are H°-atoms with supports $I^n_{k_{q,n}}$.

$$\sum_{q=-\infty}^{\infty}\sum_{n=1}^{\infty}|\lambda_{q,n}| = c\sum_{q=-\infty}^{\infty}2^q\sum_{n=1}^{\infty}\mu(I^n_{k_{q,n}})$$
$$\le c\sum_{q=-\infty}^{\infty}2^q\mu(f^\circ > 2^q) \le c\|f^\circ\|_1 = c\|f\|_{H^\circ}.$$

This completes the proof. □

2. The (H, L) typeness of the maximal operator

Define the maximal operator of Cesàro means of two parameter integrable functions on bounded Vilenkin groups as follows

$$\sigma^* f := \sup_{\substack{(n_1,n_2)\in\mathbb{N}^2\\ \beta^{-1}\le\frac{n_1}{n_2}\le\beta}} |\sigma_n f| \quad (n = (n_1, n_2)),$$

where $\beta > 1$ is some fixed parameter.

In order to prove the main theorem we need the corollary of the following lemma (see [BG], and for the Walsh case see [Gát6]).

Lemma 3. *Let $A, k \in \mathbb{N}$ be fixed, $A > k - c$. Set $\operatorname{supp} f \subseteq I_k(x_1) \times \widetilde{I}_{\widetilde{k}}(x_2)$, $f \in L^1(G_m \times G_{\widetilde{m}})$ $((x_1, x_2) \in G_m \times G_{\widetilde{m}})$. Then we have*

$$\int_{(G_m\times G_{\widetilde{m}})\setminus I_k(x_1)\times\widetilde{I}_{\widetilde{k}}(x_2)} \sup\left\{|\sigma_n f| : \mathbf{n}\in\mathbb{P}^2,\ \wedge n \ge M_A,\ \beta^{-1}\le n_1/n_2\le\beta\right\}$$
$$\le c\left(\frac{M_k}{M_A}\right)^{\frac{1}{2}}\|f\|_1.$$

Corollary 4. *Suppose that $\operatorname{supp} f \subseteq I_k(x_1)\times\widetilde{I}_{\widetilde{k}}(x_2)$ and $f \in L^1(G_m \times G_{\widetilde{m}})$ $(k \in \mathbb{N},\ (x_1, x_2) \in G_m \times G_{\widetilde{m}})$. Then we have*

$$\int_{(G_m\times G_{\widetilde{m}})\setminus I_k(x_1)\times\widetilde{I}_{\widetilde{k}}(x_2)} \sigma^* f \le c\|f\|_1.$$

Theorem 5. *The operator σ^* is of type (H°, L) i.e.*

$$\|\sigma^* f\|_1 \le c\|f\|_{H^\circ} \quad (f \in H).$$

Proof of Theorem 5. Recall that the operator σ^* from $L^p(G_m \times G_{\widetilde{m}})$ to $L^p(G_m \times G_{\widetilde{m}})$ is bounded for $p > 1$, (because of the week (1,1) and (∞, ∞) typeness of the operator σ^* (see [BG])). Suppose that a is an H°-atom with support $I_k(x_1) \times \widetilde{I}_{\widetilde{k}}(x_2)$. Using Corollary 4, Hölder's inequality and the boundedness of σ^*, we get

$$\begin{aligned}
\int_{G_m \times G_{\widetilde{m}}} \sigma^* a &= \int_{I_k(x_1) \times \widetilde{I}_{\widetilde{k}}(x_2)} \sigma^* a + \int_{(G_m \times G_{\widetilde{m}}) \setminus (I_k(x_1) \times \widetilde{I}_{\widetilde{k}}(x_2))} \sigma^* a \\
&\le \left(\int_{G_m \times G_{\widetilde{m}}} (\sigma^* a)^p \right)^{\frac{1}{p}} \mu(I_k(x_1) \times \widetilde{I}_{\widetilde{k}}(x_2))^{1-\frac{1}{p}} + c\|a\|_1 \\
&\le c_p \left(\int_{I_k(x_1) \times \widetilde{I}_{\widetilde{k}}(x_2)} |a|^p \right)^{\frac{1}{p}} \mu(I_k(x_1) \times \widetilde{I}_{\widetilde{k}}(x_2))^{1-\frac{1}{p}} + c \\
&\le c_p \left((M_k \widetilde{M}_{\widetilde{k}})^p \frac{1}{M_k \widetilde{M}_{\widetilde{k}}} \right)^{\frac{1}{p}} \left(\frac{1}{M_k \widetilde{M}_{\widetilde{k}}} \right)^{1-\frac{1}{p}} + c \le c.
\end{aligned}$$

Using the atomic decomposition of the function f and the σ-sublinearity of the operator σ^*,

$$\|\sigma^* f\|_1 = \left\| \sigma^* \left(\sum_{i=1}^{\infty} \lambda_i a_i \right) \right\|_1 \le \sum_{i=1}^{\infty} |\lambda_i| \|\sigma^* a_i\|_1 \le c \sum_{i=1}^{\infty} |\lambda_i| \le c\|f\|_{H^\circ}.$$

This completes the proof. □

Since $\|\,.\,\|_{H^\circ} \le \|\,.\|_H$, we have $H \subset H^\circ$. Consequently, we also have

Corollary 6. *The operator σ^* is of type (H, L).*

Corollary 6 with respect to the ordinary Vilenkin systems (bounded case) can be found [Wei2].

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