Uniform measures on vector-valued functions

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Abstract. It is proved that $(C_{ub}(X, E), \beta_u)$ is strongly Mackey and its dual is weak* sequentially complete. Also for special Banach spaces E, it is proved that $(C_{ub}(X, E), \beta_u)$ has D-P property.

1. Introduction and notations

In this paper, X is a uniform space, K be the field of real or complex numbers (called the scalar field), E a Banach space over K, $C_b(X, E)$ $(C_{ub}(X, E))$ the space of all E-valued, bounded continuous (uniformly continuous) on X (we denote them by $C_b(X)$ $(C_{ub}(X))$ if $E = K$). If $f : X \to E$, then $\|f\| : X \to R$ is defined by $\|f\|(x) = \|f(x)\|$. All vector spaces are taken over the field K. Uniform measures, on uniform spaces, are defined and studied in ([3], [4], [5], [7], [11], [13]). All uniform spaces are assumed to be generated by a filtering upwards family of pseudo-metrics (see [9]). For a uniform space X, let $\mathcal{H}$ be the collection of all uniformly bounded and uniformly equicontinuous subsets of $C_{ub}(X)$ (these sets will be called ueb sets); $\mathcal{H}$ is closed under taking, in pointwise topology, the closed absolutely convex hull and, in this topology, elements of $\mathcal{H}$ are relatively compact. The finest locally convex topology on $C_{ub}(X)$, agreeing with pointwise topology on every $H \in \mathcal{H}$, is denoted by β_u [8]; in this topology, bounded sets are norm-bounded. The dual of $(C_{ub}(X), \beta_u)$ is denoted by $M_u(X)$ and its elements are called uniform measures ([7], [13]). If d is a

uniformly continuous pseudo-metric on X, the corresponding metric space is denoted by X_d. The canonical mapping $\varphi : X \to X_d$ gives rise to the uniformly continuous mapping $(C_{ub}(X_d), \beta_u) \to (C_{ub}(X), \beta_u)$, $f \to f \circ \varphi$. The space X, with weakest uniformity for which each f in $C_{ub}(X)$ is uniformly continuous, is precompact and its completion, denoted by $\check{X}$, is called the Samuel compactification of X ([13]); the completion of X is denoted by $\hat{X}$. For uniform measures, we will be using the notations and results from ([13], [7], [11]). It is proved in ([7]) that when a uniform measure is tight, it can be uniquely extended to a tight measure on $C_b(X)$, and on a complete metric space, a uniform measure is always tight; also μ uniform implies $|\mu|$ uniform. The spaces $(C_{ub}(X), \beta_u)$ and $(C_{ub}(\hat{X}), \beta_u)$ are easily seen to be isomorphic $(f \to \hat{f})$.

The definition of uniform measure easily extends to $(C_{ub}(X, E))$ when E is a Banach space, the space $\mathcal{H}$ is the collection of all E-valued ueb sets in $(C_{ub}(X, E)$ (this is done in [11]); the elements in $\mathcal{H}$ are not necessarily relatively pointwise compact. The dual of the $(C_{ub}(X, E), \beta_u)$ is denoted by $M_u(X, E')$ and has the property that μ is in the dual if and only if $|\mu|$ is a uniform measure, where, for any f in $C_{ub}(X)$, $f \geq 0$

$$|\mu|(f) = \sup\{|\mu(g)| : g \in C_{ub}(X, E),\ \|g\| \leq f\}.$$

Let X be a complete metric space $\mu \in M_u(X, E')$. Thus $|\mu|$ is a tight measure in $C_b(X)$. In order to extend μ to a bigger class of functions we define (as in [10]) $|\mu|^*$:

$\mathcal{U} = \{g : X \to [0, \infty]$, there exists a net $g_\alpha \in C_b(X),\ \geq 0,\ g = \sup g_\alpha\}$.

For an $f \in \mathcal{U}$, define

$$|\mu|^*(f) = \sup\{\mu(g) : g \in C_b(X),\ 0 \leq g \leq f\}.$$

For any $h : X \to [0, \infty]$, define

$$|\mu|^*(h) = \inf\{|\mu|^*(g) : g \in \mathcal{U},\ g \geq h\}.$$

Let $g_i : X \to [0, \infty]$, $i = 1, 2$; the following properties are easily verified:

$$(i) \quad |\mu|^*(g_1 + g_2) \leq |\mu|^*(g_1) + |\mu|^*(g_2)$$

$$(ii) \quad g_1 \leq g_2 \Rightarrow |\mu|^*(g_1) \leq |\mu|^*(g_2)$$

$$(iii) \quad |\mu|^*(\alpha g_1) = \alpha|\mu|^*(g_1), \quad \alpha > 0.$$

Let $P = \{f : X \to E, |\mu|^*(\|f\|) < \infty\}$. P is a vector space and p, $p(f) = |\mu|^*(\|f\|)$ is a semi-norm on P. Denote by $\mathcal{L}_1(\mu, X, E)$ the closure of $C_{ub}(X, E)$ in the semi-normed space (P, p). As in [10], $|\mu(g)| \leq |\mu|(\|g\|)$ $\forall g \in \mathcal{L}_1(\mu, X, E)$ and g in $\mathcal{L}_1(\mu, X, E)$ implies that $\|g\| \in \mathcal{L}_1(|\mu|)$. Using the extension theorem in [1], it is easily shown that $C_b(X, E)$ is in $\mathcal{L}_1(\mu, X, E)$. Also if g is in $\mathcal{L}_1(\mu, X, E)$ then for any bounded h in $\mathcal{L}_1(|\mu|)$, hg is in $\mathcal{L}_1(\mu, X, E)$.

For locally convex spaces, the notations and results from [15] are used. For measures we will follow the notations of and results from [16], [10]. Let F be a lcs (locally conves space) and F' its dual. It is called strongly Mackey if every relatively $\sigma(F', F)$ countably compact subset of F' is equicontinuous. N will denote the set of natural numbers. If X is a metric space and $A \subset X$, $\epsilon > 0$, $B(A, \epsilon) = \{x \in X : d(x, A) < \epsilon\}$. We need the following lemmas:

Lemma 1. Let $\lambda_n : 2^N \to K$ be a sequence of countably additive functions (on all subsets of N). If $\lim \lambda_n(A)$ exists for all subsets $A \subset N$, then the convergence is uniform on 2^N.

PROOF. This is proved in [10, p. 199]; it follows from Vitali–Hahn–Saks theorem and also from classical Phillips' lemma ([6]). $\qquad\square$

Lemma 2. Let T be a Hausdorff topological space having a σ-compact dense subset, $C(T)$ all scalar-valued continuous functions on T with the topology of pointwise convergence, and $A \subset C(T)$ such that every sequence in A has a cluster point in $C(T)$. Let g be in the closure of A in K^T. Then there exists a sequence in A which converges to g pointwise.

PROOF. This is proved in [14]. $\qquad\square$

It is proved in ([13], [5]) that $(C_{ub}(X), \beta_u)$ is Mackey and its dual is weak* sequentially complete; the proofs are quite technical and they do not naturally extend to the vector case. Even for the real case, our proof is very different and it easily extends to the vector case.

2. Main results

Lemma 3. Assume (X, d) to be a complete metric space. Let μ_n be a sequence of uniform measures in $(C_{ub}(X, E), \beta_u)'$ such that either $\lim \mu_n(f)$ exists for every f in $C_{ub}(X, E)$ or the sequence is relatively

countably compact in $(M_u(X, E'), \sigma(M_u(X, E'), C_{ub}(X, E))$, X_0 a Borel subset of X with the property that $|\mu_n|(X \setminus X_0) = 0$, $\forall n$, f_n a ueb sequence in $C_{ub}(X, E)$ such that $|f_n(x) - f_n(y)| \leq d(x, y)$, $\forall n$ and $f_n \to 0$, pointwise on X_0. Then $\mu_n(f_n) \to 0$.

PROOF. Suppose this is not true. We can assume $|\mu_n| \leq 1$, and $\|f_n\| \leq 1$. By taking subsequences, if necessary, we assume that $|\mu_n(f_n)| > 64\epsilon$, for some $\epsilon > 0$, $\forall n$. We claim that given a compact subset K of X_0 and $n_0 \in N$, there exists an $n_1 > n_0$ such that

$$\int_{B(K, 4\epsilon)} \|f_n\| d|\mu_n| \leq 9\epsilon, \quad \forall n > n_1.$$

Take a finite number of points $\{x_i : 1 \leq i \leq p\}$ in K so that $\bigcup_{i=1}^{p} B(x_i, 8\epsilon)$ covers $B(K, 4\epsilon)$. Since $f_n \to 0$ uniformly on K, choose an $n_1 > n_0$ such that $\|f_n\| \leq \epsilon$ $\forall n > n_1$, on K. Put $Y_1 = B(x_1, 8\epsilon) \cap B(K, 4\epsilon) \cap X_0$. For $2 \leq j \leq p$

$$Y_j = (B(x_j, 8\epsilon) \setminus \cup_{i=1}^{j-1} B(x_i, 8\epsilon)) \cap B(K, 4\epsilon) \cap X_0.$$

For any $n \geq n_1$ we have

$$\left| \int_{B(K, 4\epsilon)} f_n d\mu_n \right| \leq \sum_{i=1}^{p} \int_{Y_i} \|f_n\| d|\mu_n| \leq \sum_{i=1}^{p} \int_{Y_i} (\|f_n(x_i)\| + 8\epsilon) d|\mu_n| \leq 9\epsilon.$$

So the claim is proved. Inductively we construct below a sequence K_n of compact subsets of X_0, a ueb sequence h_n such that:

$$(i) \quad d(K_i, K_j) \geq 4\epsilon, \ \forall i \neq j;$$

$$(ii) \quad h_i = 0 \text{ outside } B(K_i, \epsilon), \ \forall i;$$

$$(iii) \quad \left| \int f_i h_i d\mu_i \right| > \epsilon, \ \forall i.$$

Take a compact set $K_1 \subset X_0$, with $|\mu_1|(X \setminus K_1) < \epsilon$. Put $h_1(x) = \inf \left(\frac{1}{\epsilon} d(B(K_1, \epsilon)', x), 1 \right)$. It is immediate that $\epsilon |(h_1(x) - h_1(y)| \leq d(x, y)$. Also $|\int f_1 d\mu_1 - \int_{K_1} f_1 d\mu_1| \leq \epsilon$. Further $|\int h_1 f_1 d\mu_1 - \int_{K_1} f_1 d\mu_1| \leq \int |h_1 - \chi_{K_1}| d|\mu_1| \leq \epsilon$. From these inequalities and from $|\int f_1 d\mu_1| > 64\epsilon$ it follows that $|\int h_1 f_1 d\mu_1| > \epsilon$.

Having determined K_1, we take $K = K_1$ in the claim proved above and find an n_2, such that

$$\int_{B(K,4\epsilon)} \|f_n\| \, d|\mu_n| \le 9\epsilon, \quad \forall n \ge n_2.$$

Since $|\mu_n(f_n)| > 64\epsilon$, $\forall n$, we get $B(K,4\epsilon) \cap X_0 \subsetneq X_0$. For notational convenience, we denote n_2 by 2. Thus $\int_{B(K,4\epsilon)} \|f_2\| \, d|\mu_2| \le 9\epsilon$. Take a compact $K_2 \subset B(K,4\epsilon)' \cap X_0$ such that $|\mu_2|(B(K,4\epsilon)' \setminus K_2) < \epsilon$. As was done for h_1, put $h_2(x) = \inf\left(\frac{1}{\epsilon}d(B(K_2,\epsilon)',x),1\right)$. From

$$\int_{B(K_2,\epsilon)\setminus K_2} \|f_2\| d|\mu_2|$$

$$\le \int_{B(K_2,\epsilon)\cap B(K_1,4\epsilon)} \|f_2\| d|\mu_2| + \int_{B(K_1,4\epsilon)'\setminus K_2} \|f_2\| d|\mu_2|$$

$$\le 9\epsilon + \epsilon = 10\epsilon$$

and

$$\left|\int h_2 f_2 d\mu_2\right| \ge \left|\int_{K_2} f_2 d\mu_2\right| - \int |h_2 - \chi_{K_2}| \, \|f_2\| d|\mu_2|$$

$$\ge \left|\int_{K_2} f_2 d\mu_2\right| - 10\epsilon$$

(note $|h_2 - \chi_{K_2}| \le \chi_{B(K_2,\epsilon)\setminus K_2}$), and

$$\left|\int_{K_2} f_2 d\mu_2\right| \ge \left|\int f_2 d\mu_2\right| - \left|\int_{X\setminus K_2} f_2 d\mu_2\right|$$

$$\ge \left|\int f_2 d\mu_2\right| - \left|\int_{B(K_1,4\epsilon)'\setminus K_2} f_2 d\mu_2\right| - \left|\int_{B(K_1,4\epsilon)} f_2 d\mu_2\right| \ge 32\epsilon - \epsilon - 9\epsilon = 22\epsilon$$

we get

$$\left|\int h_2 f_2 d\mu_2\right| \ge 22\epsilon - 10\epsilon > \epsilon.$$

To get h_3, we apply the claim to the compact set $K_1 \cup K_2$ and proceed as above. So, by induction, the sequences mentioned above get determined.

Suppose X is compact. Take a sequence $\{x_n\}$ such that $x_n \in K_n$, $\forall n$. This means $d(x_i, x_j) \geq 4\epsilon$ for $i \neq j$; since X is compact this is a contradiction. So, in case X is compact, the lemma is proved.

Now suppose X is not compact. It is easily verified that for any $M \subset N$, $\sum_{i \in M} h_i f_i \in C_{ub}(X, E)$. If μ_n does not converge pointwise on $C_{ub}(X, E)$, then, by the countable compactness of this sequence and Lemma 2, a subsequence of μ_n converges to some $\mu \in M_u(X, E')$, pointwise on $\sum_{i \in M} h_i f_i \in C_{ub}(X, E)$, as M varies as a subset of N. By Lemma 1, $\int f_i h_i d\mu_i \to 0$ gives a contradiction.

Now we come to the Mackey property.

Theorem 1. *Let Q be a countably compact subset of $(M_u(X), \sigma(M_u(X), C_{ub}(X)))$. Then Q is β_u-equicontinuous (thus $(C_{ub}(X), \beta_u)$ is strongly Mackey, [13], [15]).*

PROOF. Since Q is norm-bounded, we can assume that $|\mu| \leq 1$, $\forall \mu \in Q$. Let H be a ueb set, uniformly bounded by 1 and pointwise closed (note H is compact in pointwise topology). Defining the metric

$$d(x, y) = \sup\{|h(x) - h(y)| : h \in H\},$$

we can assume that X is a complete metric space. Let $\mathcal{A}$ be the uniformity, on H, of uniform convergence on all finite subsets of X and on Q. It is easily seen that H is complete in this uniformity. It is enough to prove that H is compact in this uniformity. To prove this, it is enough to prove that every sequence has a cluster point ([12], p. 37). So we take a sequence $\{f_n\}$ in H with 0 as a cluster point, in the pointwise convergence on X, and assume that 0 is not a cluster point in uniform convergence on Q. By taking subsequences, if necessary, there exist an $\epsilon > 0$ and a sequence $\{\mu_n\}$ in Q such that

$$|\mu_n(f_n)| > \epsilon, \quad \forall n,$$

0 still being a cluster point of the sequence $\{f_n\}$ in the pointwise topology. Take an increasing sequence $\{C_i\}$ of compact subsets of X, such that

$$|\mu_n|\left(X \setminus \bigcup_{i=1}^{\infty} C_i\right) = 0, \quad \forall n.$$

Since $\{f_n\}$ is a ueb set, by taking subsequences, if necessary, we assume that

$$f_n \to 0, \quad \text{pointwise on } X_0 = \bigcup_{i=1}^{\infty} C_i \text{ and } |\mu_n(f_n)| > \epsilon, \quad \forall n.$$

By Lemma 3, this is a contradiction. $\qquad\square$

Now we come to the vector case.

Theorem 2. *Let E be a Banach space and Q be a norm-bounded and countably compact subset of $(M_u(X, E'), \sigma(M_u(X, E'), C_{ub}(X, E)))$. Then Q is β_u-equicontinuous (this means $(C_{ub}(X, E), \beta_u)$ is strongly Mackey).*

PROOF. Take H to be a ueb set. We can assume that (X, d) is a complete metric space, where $d(x, y) = \sup\{\|f(x) - f(y)\| : f \in H\}$, and H and Q are uniformly bounded by 1. Suppose a net $f_\alpha \to 0$ in H, pointwise on X but not uniformly on Q. By taking subnets if necessary, take a net μ_α in Q such that $|\mu_\alpha(f_\alpha)| > \epsilon$, $\forall \alpha$, for some $\epsilon > 0$. Take a $\mu_{\alpha(1)}$ and a compact C_1 such that $|\mu_{\alpha(1)}|(X \setminus C_1) < 1/2$; take an $\alpha(2) > \alpha(1)$ such that $|f_{\alpha(2)|C_1}| < 1/2$; take a compact $C_2 \supset C_1$ such that $(|\mu_{\alpha(1)}| + |\mu_{\alpha(2)}|) \times (X \setminus C_2) < 1/3$; take an $\alpha(3) > \alpha(2)$ such that $|f_{\alpha(3)|C_2}| < 1/3$; continuing like this we get a sequence $\{f_n\}$ in H, a sequence $\{C_n\}$ of compact subsets of X, and a sequence $\{\mu_n\}$ in Q, such that

$$|\mu_n|\left(X \setminus \bigcup_{i=1}^{\infty} C_i\right) = 0, \quad \forall n.$$

$f_n \to 0$ on $\bigcup_{i=1}^{\infty} C_i$, and

$$|\mu_n(f_n)| > \epsilon, \quad \forall n.$$

By Lemma 3, this is a contradiction. $\qquad\square$

Theorem 3. $M_u(X, E')$, $\sigma(M_u(X, E'), C_{ub}(X, E))$ *is σ-complete.*

PROOF. Suppose a sequence $\mu_n \in M_u(X, E')$ pointwise converges on $C_{ub}(X, E)$ and let $\mu(g) = \lim \mu_n(g)$, $\forall g \in C_{ub}(X, E))$. Let H be a ueb set, uniformly bounded by 1, and a net $f_\alpha \to 0$ in H, pointwise on X. Suppose $|\mu(f_\alpha)| > \epsilon$, $\forall \alpha$ for some $\epsilon > 0$. We can assume that (X, d) is a

complete metric space, where $d(x,y) = \sup\{\|f(x) - f(y)\| : f \in H\}$. Take an increasing sequence $\{C_i\}$ of compact subsets of X, such that

$$|\mu_n|\left(X \setminus \bigcup_{i=1}^{\infty} C_i\right) = 0, \quad \forall n.$$

From the net f_α select a sequence f_n such that $f_n \to 0$, pointwise on $\bigcup_{i=1}^{\infty} C_i$. Take subsequences of μ_n, f_n, if necessary, such that $|\mu_n(f_n)| > \epsilon$, $\forall n$. By Lemma 3, this is a contradiction. $\qquad\square$

A locally convex space F, with F', F'' its dual and bidual, has Dunford–Pettis (D-P) property if every absolutely convex $\sigma(F, F')$-compact $\subset F$, is also compact in the topology of uniform convergence on all equicontinuous, absolutely convex and $\sigma(F', F'')$-compact $\subset F'$. It is proved in ([2]) that $(C(Y, L_1(\lambda)), \|\,.\,\|)$, λ, being a finite measure and Y a compact Hausdorff space, has D-P property.

Theorem 4. *Let E be a Banach space with the property that for any compact Hausdorff space Y, $(C(Y, E), \|\,.\,\|)$ has D-P property. Then $(C_{ub}(X, E), \beta_u)$ has D-P property.*

PROOF. Let $F = (C_{ub}(X, E), \beta_u)$. Since β_u-bounded sets are norm-bounded, we have $(F', \beta(F', F)) = (F', \|\,.\,\|)$. Let P be an absolutely convex $\sigma(F, F')$-compact subset of F and Q a weakly compact set in the Banach space $(F', \|\,.\,\|)$. We can assume that the elements of P and Q are uniformly bounded in norm by 1. Let $\mathcal{U}$ be the uniformity, on P, of pointwise convergence on F' and uniform convergence on Q. P is complete in this uniformity. Let $\{f_n\}$ be a sequence in P with 0 as a cluster point in $\sigma(F, F')$. Since $\{f_n\}$ is uniformly bounded the pseudo-metric $d(x,y) = \sum_{i=1}^{\infty} \frac{1}{2^n} \|(f_n(x) - f_n(y))\|$ is uniformly continuous. So we assume X to be a complete metric space. Now for every $\mu \in Q$, $|\mu|$ is tight. We claim that the measures $\{|\mu| : \mu \in Q\}$ are uniformly inner regular by the compact subsets of X. If this is not true, there exists a sequence μ_n in Q, a sequence $\{C_n\}$ of disjoint compact subsets of X, such that $|\mu_n|(C_n) > \epsilon$ for some $\epsilon > 0$. By [10], for each n, there exists a finite disjoint collection $\{C_{n,i} : 1 \leq i \leq p(n)\}$ of compact subsets of $\{C_n\}$, and elements $\{e_{n,i} : 1 \leq i \leq p(n)\}$ in the unit ball of E such that $|\mu_n(\sum_{i=1}^{p(n)} \chi_{C_{n,i}} \otimes e_{n,i})| > \epsilon$, $\forall n$. Denoting the countable collection $\{\chi_{C_{n,i} \otimes e_{n,i}}\}$ by $\{g_j\}$ ($1 \leq j < \infty$), we first prove that for any subset M of N, $\sum_{j \in M} g_j$ is in F''. Now from the

definition of g_j, $\|(\sum_{j \in M} g_j\| \leq 1$ and so for any $\mu \in F'$, $\|\mu\| \leq 1$, we have $|\mu(\sum_{j \in M} g_j)| \leq 1$ ([10]). This proves $(\sum_{j \in M} g_j) \in F''$. The mapping

$$\lambda_n : 2^N \to K, \quad \lambda_n(M) = \mu_n\left(\sum_{j \in M} g_j\right)$$

satisfies the conditions of Lemma 1 (note $|\mu_n|$ are tight). Thus we get a contradiction and so the claim is proved.

Fix K, a compact subset of X. The mappings $(C_{ub}(X, E), \beta_u) \to (C(K, E), \| \, . \, \|)$, $f \to f_{|K}$ and $(M_u(X, E'), \| \, . \, \|) \to (M(K, E'), \| \, . \, \|)$, $\mu \to \mu_{|K}$ are easily seen to be continuous (note $\mu_{|K}(g) = \int_K g_0 \, d\mu$, g_0 being any continuous extension of g to $X \to E$ [1]. Thus $P_{|K}$ is weakly compact in $(C(K, E), \| \, . \, \|)$. Also $Q_{|K}$ is weakly compact in $(C(K, E), \| \, . \, \|)'$. Fix $\epsilon > 0$ and take a compact subset K of X such that $|\mu|(X \setminus K) < \epsilon, \forall \mu \in Q$. Suppose a net f_α, taken out of $\{f_n\}$, converges to 0 in P. Take a $\mu \in Q$. Since $(C(K, E), \| \, . \, \|)$ has D-P property and $|\int f_\alpha d\mu| \leq |\int_K f_\alpha d\mu| + \epsilon$, $\forall \mu \in Q$, the result follows.

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