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Automatic continuity of homomorphisms into normed quadratic algebras

By ANTON CEDILNIK (Ljubljana)
and ANGEL RODRÍGUEZ PALACIOS (Granada)

Abstract. Let B be a real or complex complete normed quadratic algebra. All homomorphisms from arbitrary (possibly non associative) complete normed algebras into B are continuous if and only if B has no non-zero element with zero square.

1. Introduction

A classical topic in the theory of automatic continuity is that of determining those normed algebras B which satisfy Property *ACHR* which follows.

ACHR (**A**utomatic **C**ontinuity of **H**omomorphisms into the **R**ight side of the arrow). *For every complete normed algebra A, each homomorphism $\varphi : A \to B$ is continuous.*

Usually, Property *ACHR* is considered in an associative context, so that the normed algebra B is assumed to be associative, and Property *ACHR* for B means that, for every associative complete normed algebra A each homomorphism $\varphi : A \to B$ is continuous. For an approach to results in this direction, the reader is referred to the survey paper of H. G. Dales [4]. In this paper we are interested in the natural non-associative meaning of Property *ACHR*. Then the normed algebra B can

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be or not associative, and, even if B is associative, Property *ACHR* for B has the stronger sense that, for every possibly non associative complete normed algebra A, each homomorphism $\varphi : A \to B$ is continuous. In this new setting, we know that real or complex absolute-valued algebras, as well as complete normed complex algebras with no non-zero one-sided topological divisors of zero, have property *ACHR* (see [9] and [11], respectively).

As main result, we show that real or complex complete normed quadratic algebras have property *ACHR* if and only if they have no non-zero element with zero square (Theorem 1). We note that quadratic algebras are important not only from an algebraic point of view [12], but also in connection with the analysis. For instance, the structure theory of JB-algebras [5] would not be understood without the consideration of the so-called spin JBW-factors, which are relevant examples of quadratic algebras. We note also that smooth normed non-associative algebras and normed non-commutative Jordan division algebras are quadratic algebras (see [2] and [7], and [6], respectively). By the way, we remark that, as a consequence of our results, spin JBW-factors, as well as smooth normed algebras and complete normed non-commutative Jordan division algebras, are *ACHR*-algebras.

2. The results

Algebras arising throughout this paper are not assumed to be associative. Let B be an algebra. A (two-sided) ideal M of B is said to be modular if there exists some element e in B such that $y - ye \in M$ and $y - ey \in M$ for every y in B. The strong radical of B is defined as the intersection of all modular maximal ideals of B. We say that B is strongly semisimple whenever the strong radical of B is equal to zero. For y in B, we define the sequence $\{y^n\}_{n\geq 1}$, of left powers of y, by $y^1 = y$ and $y^{n+1} = yy^n$. By $\mathbb{F}$ we mean the field of real or complex numbers. Our argument begins with the following variant of [1; Proposition 25.10].

Lemma 1. *Let A be a complete normed algebra over $\mathbb{F}$, B a strongly semisimple algebra over $\mathbb{F}$, and $\varphi : A \to B$ a surjective homomorphism. Then $\mathrm{Ker}(\varphi)$ is closed in A.*

Proof. Take a modular maximal ideal M of B, and denote by π the quotient mapping $B \to B/M$. Since B/M is a simple algebra with a unit (say $\mathbf{1}$), and the homomorphism $\pi \circ \varphi : A \to B/M$ is surjective, $(\pi \circ \varphi)(\overline{\mathrm{Ker}(\varphi)})$ must be equal to either B/M or zero. Assume that the first possibility happens. Then we have $\mathbf{1} = (\pi \circ \varphi)(x)$ for some x in $\overline{\mathrm{Ker}(\varphi)}$. Taking y in $\mathrm{Ker}(\varphi)$ with $\|x - y\| < 1$, putting $z := x - y$, and considering the element t of A given by $t := \sum_{n=1}^{\infty} z^n$, we obtain $\mathbf{1} = (\pi \circ \varphi)(z)$ and $zt = t - z$, leading the contradiction $(\pi \circ \varphi)(t) = (\pi \circ \varphi)(t) - \mathbf{1}$. It follows $(\pi \circ \varphi)(\overline{\mathrm{Ker}(\varphi)}) = 0$, or equivalently $\varphi(\overline{\mathrm{Ker}(\varphi)}) \subseteq M$. Since M is an arbitrary modular maximal ideal of B, and B is strongly semisimple, we finally deduce $\varphi(\overline{\mathrm{Ker}(\varphi)}) = 0$. □

The above proof actually shows that, if A is a complete normed algebra over $\mathbb{F}$, if B is any algebra over $\mathbb{F}$, and if $\varphi : A \to B$ is a surjective homomorphism, then $\varphi(\overline{\mathrm{Ker}(\varphi)})$ is contained in the strong radical of B.

Let B be an algebra over $\mathbb{F}$. We say that B is algebraic (respectively, power-associative) if every one-generated subalgebra of B is finite-dimensional (respectively, associative). Assume that B is associative, and let y be an element of B. We say that y is quasi-invertible in B if there exists z in B satisfying $yz = zy = y + z$. It is easy to see that y is quasi-invertible in B if an only if $\mathbf{1} - y$ is invertible in the formal unital hull of B. For $\mathbb{F}$ equal to $\mathbb{C}$ and $\mathbb{R}$, we define the spectrum $\mathrm{sp}(B, y)$ of y relative to B by the equalities

$$\mathrm{sp}(B, y) := \{0\} \cup \{\lambda \in \mathbb{C}\backslash\{0\} : \lambda^{-1} y \text{ is not quasi-invertible in } B\}$$

and

$$\mathrm{sp}(B, y) := \mathrm{sp}(B_{\mathbb{C}}, y) \text{ (where } B_{\mathbb{C}} \text{ stands for the complexification of } B),$$

respectively.

Now let B be an algebraic power-associative algebra over $\mathbb{F}$, and y an element of B. An elemental spectral calculus shows that, for every associative subalgebra C of B containing y, $\mathrm{sp}(C, y)$ does not depend on C. This allows us to define the spectrum of y relative to B, $\mathrm{sp}(B, y)$, by means of the equality $\mathrm{sp}(B, y) := \mathrm{sp}(C, y)$ for some C as above. Actually $\mathrm{sp}(B, y)$ is nothing but the set of (possibly complex) roots of a one-indeterminate polynomial p over $\mathbb{F}$ without constant term and which has minimum degree among those satisfying $p(y) = 0$. We define the algebraic spectral radius $r(B, y)$ of y relative to B by the equality $r(B, y) := \max\{|\lambda| : \lambda \in \mathrm{sp}(B, y)\}$.

Lemma 2. *Let A be a complete normed algebra over $\mathbb{F}$, B a strongly semisimple algebraic power-associative algebra over $\mathbb{F}$, and $\varphi : A \to B$ a surjective homomorphism. Then, for every x in A, the inequality $r(B, \varphi(x)) \leq \|x\|$ holds.*

PROOF. By Lemma 1, $\operatorname{Ker}(\varphi)$ is a closed ideal of A, and therefore $A/\operatorname{Ker}(\varphi)$ is a complete normed algebra over $\mathbb{F}$ in a natural way. Let $\phi : A/\operatorname{Ker}(\varphi) \to B$ be the unique surjective isomorphism such that $\phi \circ \pi = \varphi$ (where $\pi : A \to A/\operatorname{Ker}(\varphi)$ denotes quotient mapping). Then the function $y \to |y| := \|\phi^{-1}(y)\|$ from B to $\mathbb{R}$ is an algebra norm on B. Since B is algebraic, such a norm is complete on each one-generated subalgebra of B, and therefore, by power-associativity and standard Banach algebra theory, we have $r(B, y) \leq |y|$ for every y in B. Finally, for x in A we obtain

$$r(B, \varphi(x)) \leq |\varphi(x)| = \|\phi^{-1}(\varphi(x))\| = \|\pi(x)\| \leq \|x\|. \qquad \square$$

An element y of an algebra is said to be isotropic if $y \neq 0 = y^2$. As we show in the next remark, the presence of isotropic elements in a normed algebra B becomes a serious handicap for the automatic continuity of homomorphisms from normed algebras into B.

Remark 1. Let A be a normed algebra over $\mathbb{F}$ such that A^2 is contained in some non-closed hyperplan, and B a normed algebra over $\mathbb{F}$ possessing isotropic elements. Then there exist discontinuous homomorphisms from A into B. Indeed, we have $A^2 \subseteq \operatorname{Ker}(\psi)$ for a suitable discontinuous linear functional ψ on A, so that, by choosing an isotropic element z in B, the mapping $\varphi : x \to \psi(x)z$ from A to B becomes a discontinuous homomorphism. We note that the normed algebra A above can be chosen complete, associative, and commutative. Indeed, starting from an arbitrary infinite-dimensional Banach space X over $\mathbb{F}$, and considering the normed algebra A over $\mathbb{F}$ whose vector space is $\mathbb{F} \times X$ and whose product and norm are given by $(\lambda, \xi)(\mu, \eta) := (\lambda\mu, 0)$ and $\|(\lambda, \xi)\| := |\lambda| + \|\xi\|$, respectively, A becomes a complete normed, associative, and commutative algebra with A^2 contained in some non closed hyperplan. In the case that B has a unit, an alternative method, to construct discontinuous homomorphisms from (better) complete normed associative and commutative algebras into B, can be derived from [4; Proposition 6.6 and Example 6.5].

Following [12; pp. 49–50], we say that a non-zero algebra B over $\mathbb{F}$ is quadratic if it has a unit $\mathbf{1}$, and for each y in B there exist elements $\tau(y)$

and $n(y)$ in $\mathbb{F}$ such that $y^2 - 2\tau(y)y + n(y)\mathbf{1} = 0$. Actually, in the definition of [12] the fact that $B \neq \mathbb{F}\mathbf{1}$ is additionally required, but such a requirement is unnecessary for our development. If B is a quadratic algebra over $\mathbb{F}$, then, for y in $B \backslash \mathbb{F}\mathbf{1}$, the scalars $\tau(y)$ and $n(y)$ are uniquely determined, so that, choosing $\tau(\alpha\mathbf{1}) := \alpha$ and $n(\alpha\mathbf{1}) := \alpha^2$ ($\alpha \in \mathbb{F}$), we obtain mappings τ and n (called the trace form and the norm form, respectively) from B to $\mathbb{F}$, which are linear and quadratic, respectively (see again [12; pp. 49–50]). We note that quadratic algebras are algebraic and power-associative.

Lemma 3. *Let B be a quadratic algebra over $\mathbb{F}$ without isotropic elements, and C a non-zero subalgebra of B. Then C is either simple with a unit or isomorphic to $\mathbb{F} \oplus \mathbb{F}$.*

Proof. First assume that C does not contain the unit $\mathbf{1}$ of B. Then for every y in C we have $y^2 = 2\tau(y)y$ and hence, since B has no isotropic element, the restriction of τ to C is an injective linear from. It follows $C = \mathbb{F}f$ for some f in C satisfying $2\tau(f) = 1$, so that the mapping $\lambda \to \lambda f$ is an isomorphism from $\mathbb{F}$ onto C, and the proof is concluded in this case. Now assume that $\mathbf{1} \in C$. Then C is a quadratic algebra over $\mathbb{F}$ without isotropic elements, so that there is no restriction in taking $C = B$. Assume additionally that B is not simple. Then we can choose an ideal M of B with $0 \neq M$ and $\mathbf{1} \notin M$. By the first part of the proof, we have $M = \mathbb{F}f$ for some idempotent f in B with $0 \neq f \neq \mathbf{1}$: Let $B = B_1 \oplus B_{1/2} \oplus B_0$ be the Peirce decomposition of B relative to f [12; pp. 130–131]. The definition of B_1 and $B_{1/2}$, together with the fact that $\mathbb{F}f$ is an ideal of B, directly leads to $B_1 = \mathbb{F}f$ and $B_{1/2} = 0$, so that $B = \mathbb{F}f \oplus B_0$. By the definition of B_0, we have $fB_0 = 0$, and hence $\mathbf{1} \notin B_0$. But B_0 is a subalgebra of the algebra B^+ obtained by symmetrization of the product of B (see again [12; p. 131]). Since B^+ has the same properties as B, it follows from the first part of the proof that $B_0 = \mathbb{F}g$ for some non-zero idempotent g in B. Now we have $B = \mathbb{F}f \oplus \mathbb{F}g$ with f and g non-zero idempotents satisfying $fg = gf = 0$, and hence $(\lambda, \mu) \to \lambda f + \mu g$ is an isomorphism from $\mathbb{F} \oplus \mathbb{F}$ onto B. □

Remark 2. It is worth mentioning that, if $\mathbb{F} = \mathbb{C}$, then in fact the algebra B in the above lemma is isomorphic to either $\mathbb{C}$ or $\mathbb{C} \oplus \mathbb{C}$. Indeed, if the dimension of B is ≥ 3, then the restriction of the norm form n of B to $\mathrm{Ker}(\tau)$ (where τ denotes the trace form on B) is a quadratic form on a complex vector space of dimension ≥ 2, and hence there exists a non-zero $y \in \mathrm{Ker}(\tau)$ such that $n(y) = 0$, leading to the contradiction $y \neq 0 = y^2$.

Now we are ready to formulate and prove the main tool in our argument.

Proposition 1. *Let A be a complete normed algebra over $\mathbb{F}$, B a quadratic algebra over $\mathbb{F}$ without isotropic elements, and $\varphi : A \to B$ a homomorphism. Then, for every x in A, the inequality $r(B, \varphi(x)) \leq \|x\|$ holds.*

PROOF. We can assume $\varphi \neq 0$, so that $\varphi(A)$ is a non-zero subalgebra of B. By Lemma 3, $\varphi(A)$ is strongly semisimple. Therefore we can see φ as a homomorphism from A onto $\varphi(A)$, and apply Lemma 2 to obtain $r(\varphi(A), \varphi(x)) \leq \|x\|$ for every x in A. Finally note that, for y in $\varphi(A)$, the equality $r(\varphi(A), y) = r(B, y)$ holds. □

We note that, if B is an algebraic power-associative algebra over $\mathbb{F}$, and if $r(B, y) \neq 0$ for every non-zero $y \in B$, then B has no isotropic element. Keeping in mind this fact, the next result follows straightforwardly from the above proposition.

Corollary 1. *Let B be a normed quadratic algebra over $\mathbb{F}$. Assume that there exists a positive constant M such that the inequality $\|y\| \leq M r(B, y)$ holds for every y in B. Then all homomorphisms from complete normed algebras over $\mathbb{F}$ into B are continuous.*

To illustrate the field of applicability of Corollary 1, let us recall some facts about smooth normed algebras. Smooth normed algebras over $\mathbb{F}$ are defined as those normed algebras B over $\mathbb{F}$ having a norm-one unit which is a smooth point of the closed unit ball of A. It is well-known that $\mathbb{C}$ is the unique smooth normed complex algebra (see for example [10; Corollary 1.6]). In the real setting, things are not so easy. For instance, every non-zero real pre-Hilbert space H can be converted into a commutative smooth normed real algebra by fixing a norm-one element $\mathbf{1}$ in H, and defining the product by means of the equality $xy := (x \,|\, \mathbf{1})y + (y \,|\, \mathbf{1})x - (x \,|\, y)\mathbf{1}$ [10; Observation 1.3]. More generally, if H is any (possibly equal to zero) real pre-Hilbert space, and if $\wedge$ is an anticommutative product on H satisfying $\|\xi \wedge \eta\| \leq \|\xi\| \, \|\eta\|$ and $(\xi \wedge \eta \mid \vartheta) = (\xi \mid \eta \wedge \vartheta)$ for all ξ, η, ϑ in H, then the pre-Hilbert space $(\mathbb{R} \oplus H)_{\ell_2}$ becomes a smooth normed real algebra relative to the product

$$(\lambda, \xi)(\mu, \eta) := (\lambda\mu - (\xi \mid \eta), \lambda\eta + \mu\xi + \xi \wedge \eta)$$

[7; Proposition 24]. Moreover, all smooth normed real algebras come from the construction just described (see [2], [7; Theorem 27], and [8; Section 2]). It follows that, if B is a smooth normed algebra over $\mathbb{F}$, then B is quadratic, and the equality $\|y\| = r(B, y)$ holds for every y in B. In this way the next result is a particular case of Corollary 1.

Corollary 2 [3]. *Homomorphisms from complete normed algebras over $\mathbb{F}$ into smooth normed algebras over $\mathbb{F}$ are continuous.*

Another method to build normed quadratic real algebras from real pre-Hilbert spaces is the following. If H is a real pre-Hilbert space, then the normed space $B := (\mathbb{R} \oplus H)_{\ell_1}$, endowed with the product

$$(\lambda, \xi)(\mu, \eta) := (\lambda\mu + (\xi \mid \eta), \lambda\eta + \mu\xi),$$

becomes a normed quadratic commutative real algebra whose elements y satisfy the equality $\|y\| = r(B, y)$. We note that, when H is actually a Hilbert space of dimension ≥ 2, the above procedure gives rise to the so-called spin JBW-factors [5; Chapter 6], which are of capital relevance in the structure theory of JB-algebras. Now Corollary 1 applies, leading the next result.

Corollary 3. *Homomorphisms from complete normed real algebras into spin JBW-factors are continuous.*

Spin JBW-factors, as well as smooth normed commutative algebras, are particular relevant cases of the so-called Jordan algebras of a bilinear form, whose construction is recalled in the sequel. Given a vector space X over $\mathbb{F}$ and a symmetric bilinear form $f : X \times X \to \mathbb{F}$, we can consider the algebra over $\mathbb{F}$ with vector space equal to $\mathbb{F} \oplus X$ and product given by

$$(\lambda, \xi)(\mu, \eta) := (\lambda\mu + f(\xi, \eta), \lambda\eta + \mu\xi).$$

Such an algebra is commutative and quadratic, is called the Jordan algebra of the bilinear form f, and is denoted by $J(X, f)$. Jordan algebras of bilinear forms are more than examples of quadratic commutative algebra. Indeed, it follows easily from the properties of the functions τ and n on quadratic algebras that every quadratic commutative algebra over $\mathbb{F}$ is isomorphic to $J(X, f)$ for a suitable couple (X, f) as above.

Now we are ready to formulate and prove the main result of the paper.

Theorem 1. *Let B be a complete normed quadratic algebra over $\mathbb{F}$. Then the following assertions are equivalent:*

1. *Homomorphisms from complete normed algebras over $\mathbb{F}$ into B are continuous.*
2. *Homomorphisms from complete normed, associative, and commutative algebras over $\mathbb{F}$ into B are continuous.*
3. *B has no isotropic element.*

Proof. The implication $1 \Rightarrow 2$ is clear, whereas the one $2 \Rightarrow 3$ follows from Remark 1.

Assume that Assertion 3 holds. Then we claim that the mapping $y \to r(B, y)$ is a vector space norm on B. If $\mathbb{F} = \mathbb{C}$, then the claim is easily verified by keeping in mind Remark 2. If $\mathbb{F} = \mathbb{R}$, then, by passing to B^+, we can assume that B is commutative, and hence we have $B = J(X, f)$ for some real vector space X and some symmetric bilinear form f on X. Now, the absence of isotropic elements in B implies $f(\xi, \xi) \neq 0$ for every non-zero element ξ in X, and therefore f is of the form $\epsilon(. | .)$, where $\epsilon = \pm 1$ and $(. | .)$ is a suitable inner product on X. Then a straightforward computation shows that, for $y = (\lambda, \xi)$ in $B = J(X, f)$, we have

$$r(B, y) = |\lambda| + (\xi \mid \xi)^{1/2}, \qquad \text{if } \epsilon = 1,$$

and

$$r(B, y) = (|\lambda|^2 + (\xi \mid \xi))^{1/2}, \qquad \text{if } \epsilon = -1.$$

Now that the claim is proved, we show that Assertion 1 holds by means of a standard closed graph argument. Let A be a complete normed algebra over $\mathbb{F}$, and $\varphi : A \to B$ a homomorphisms. If $x_n \to 0$ in A with $\varphi(x_n) \to y \in B$, then, by the claim and Proposition 1, we have

$$r(B, y) \leq r(B, y - \varphi(x_n)) + r(B, \varphi(x_n)) \leq \|y - \varphi(x_n)\| + \|x_n\| \to 0,$$

and hence $y = 0$. $\square$

Remark 3. i) We show, by means of an easy example, that the assumption of completeness of B in Theorem 1 cannot be removed. Take A equal to an arbitrary infinite-dimensional spin JBW-factor (i.e., $A = J(X, f)$, where X is an infinite-dimensional real Hilbert space and f is the inner product $(.\mid .)$ of X, with norm defined by $\|(\lambda, \xi)\| := |\lambda| + (\xi \mid \xi)^{1/2}$. Now, choose a discontinuous linear functional h on X, and consider $B = J(X, f)$ with the norm $|||.|||$ given by

$$|||(\lambda, \xi)||| := \|(\lambda, \xi)\| + |h(\xi)|.$$

Then A is a complete normed algebra, B is a normed quadratic algebra without isotropic elements, and the identity mapping $A \to B$ is a discontinuous homomorphism.

ii) Let B be a normed quadratic algebra. Consider the condition on B given by:

$(*)$ *There exists* $M > 0$ *satisfying* $\|y\| \leq Mr(B, y)$ *for every* y *in* B.

Corollary 1 shows that Condition $(*)$ is sufficient to ensure that B is an *ACHR*-algebra. On the other hand, we have seen in the proof of Theorem 1 that, if B has no isotropic element, then the function $r(B, .)$ is a norm on B. It follows from Remark 1 that, if B is finite-dimensional, then Condition $(*)$ is also necessary for B to enjoy Property *ACHR*. Now we can realize that, in general, Condition $(*)$ need not be necessary for B to have Property *ACHR*. Take $B = J(X, f)$, where X is the Banach space of all real-valued continuous functions on the closed real interval $[0, 1]$, and f is the symmetric bilinear form on X defined by

$$f(\xi, \eta) := \int_0^1 \xi(t)\eta(t)dt,$$

and endow B with the norm

$$\|(\lambda, \xi)\| := |\lambda| + \max\left\{|\xi(t)| : t \in [0, 1]\right\}.$$

Then B is a complete normed quadratic algebra without isotropic elements, and hence, by Theorem 1, has Property *ACHR*. However, it is easily seen that, for such a choice of B, there is no positive constant M satisfying $\|y\| \leq Mr(B, y)$ for every y in B.

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ANTON CEDILNIK
INSTITUTE OF MATHEMATICS, PHYSICS, AND MECHANICS
UNIVERSITY OF LJUBLJANA
1111 LJUBLJANA, JADRANSKA 19
SLOVENIA

E-mail: anton.cedilnik@uni-lj.si

ANGEL RODRÍGUEZ PALACIOS
UNIVERSIDAD DE GRANADA
FACULTAD DE CIENCIAS
DEPARTAMENTO DE ANÁLISIS MATEMÁTICO
18071 GRANADA
SPAIN

E-mail: apalacio@goliat.ugr.es

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