*Publ. Math. Debrecen*
**59 / 3-4** (2001), 327–338

# On submanifolds of Lorentzian almost paracontact manifolds

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**Abstract.** Lorentzian almost paracontact structures are constructed from Riemannian almost paracontact structures on a differentiable manifold. Some properties of submanifolds of a Lorentzian $s$-paracontact manifold are presented. Non-existence of any anti-invariant distribution $\mathcal{A}$ on a submanifold tangent to the structure vector field of an $LP$-Sasakian manifold such that the structure vector field is orthogonal to $\mathcal{A}$ is proved. This non-existence implies that an $LP$-Sasakian or $LSP$-Sasakian manifold does not admit any proper $CR$, generalized $CR$, semi-invariant or almost semi-invariant submanifold. It is also proved that a Lorentzian $s$-paracontact manifold can not admit any proper mixed foliated semi-invariant submanifold.

## 1. Introduction

K. Matsumoto introduced [9] the notion of a Lorentzian almost paracontact manifold. Later on several authors studied Lorentzian almost paracontact manifolds including those of [5], [10]–[13], [16]. Different types of submanifolds of Lorentzian almost paracontact manifold have been studied in [3], [4], [6], [7], [15], [17]–[19], [21], [23], [24].

In this paper, we study submanifolds of Lorentzian almost paracontact manifolds. The paper is organized as follows. Section 2 is devoted to preliminaries. In Section 3, we prove a theorem which interrelates the Riemannian and Lorentzian almost paracontact structures on a differentiable manifold. In this way, we find a general method for constructing

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*Mathematics Subject Classification*: 53C25, 53C40.
*Key words and phrases*: $CR$, generalized $CR$, semi-invariant and almost semi-invariant submanifold of Lorentzian almost para-contact manifolds.
The second author is thankful to C.S.I.R., New Delhi, for Junior Research Fellowship # 9/107(206)/98-EMR-1.

Lorentzian almost paracontact structures from Riemannian almost paracontact structures on a differentiable manifold. In Section 4, some properties of submanifolds of a Lorentzian $s$-paracontact manifold are presented. In Section 5, we mainly prove the non-existence of any anti-invariant distribution $\mathcal{A}$ on a submanifold tangent to the structure vector field of an $LP$–Sasakian manifold such that the structure vector field is orthogonal to $\mathcal{A}$. Therefore, we are able to state that a $LP$-Sasakian or $LSP$-Sasakian manifold can not admit any proper $CR$, generalized $CR$, semi-invariant or almost semi-invariant submanifold. In the last section, we prove that a Lorentzian $s$-paracontact manifold can not admit any proper mixed foliated semi-invariant submanifold.

## 2. Preliminaries

A differentiable manifold $\overline{M}$ is said to admit an *almost paracontact Riemannian structure* $(\phi, \xi, \eta, g)$, where $\phi$ is a $(1,1)$ tensor field, $\xi$ is a vector field, $\eta$ is a 1-form and $g$ is a Riemannian metric on $\overline{M}$ such that

$$\phi^2 = I - \eta \otimes \xi, \quad \eta(\xi) = 1, \tag{1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2}$$

for all vector fields $X$ and $Y$ on $\overline{M}$ (see [20]).

On the other hand, $\overline{M}$ is said to admit a *Lorentzian almost paracontact structure* $(\phi, \xi, \eta, g)$, if $\phi$ is a $(1,1)$ tensor field, $\xi$ is a vector field, $\eta$ is a 1-form and $g$ is a Lorentzian metric on $\overline{M}$, which makes $\xi$ a timelike unit vector field, such that

$$\phi^2 = I + \eta \otimes \xi, \quad \eta(\xi) = -1, \tag{3}$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y) \tag{4}$$

for all vector fields $X$ and $Y$ on $\overline{M}$ (see [9], [10]).

For both the structures mentioned above, it follows that

$$\eta \circ \phi = 0, \quad \phi\xi = 0, \tag{5}$$

$$g(\xi, X) = \eta(X), \tag{6}$$

$$g(\phi X, Y) = g(X, \phi Y). \tag{7}$$

A Lorentzian almost paracontact manifold is called

1. *Lorentzian para Sasakian* (in brief, *LP-Sasakian*) manifold [9] if

$$(\overline{\nabla}_X\phi)Y = g(\phi X, \phi Y)\xi + \eta(Y)\phi^2 X, \tag{8}$$

where $\overline{\nabla}$ is the covariant differentiation with respect to $g$,

2. *Lorentzian special para Sasakian* (in brief, *LSP-Sasakian*) manifold [9] if

$$\Phi(X, Y) = \varepsilon g(\phi X, \phi Y), \quad \varepsilon^2 = 1. \tag{9}$$

An *LSP*-Sasakian manifold is always *LP*-Sasakian [9].

Let $M$ be a submanifold of a Lorentzian almost paracontact manifold $\overline{M}$ with Lorentzian almost paracontact structure $(\phi, \xi, \eta, g)$. Let the induced metric on $M$ also be denoted by $g$. Then Gauss and Weingarten formulae are given respectively by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y), \qquad X, Y \in TM, \tag{10}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^{\perp}_X N, \qquad N \in T^{\perp}M, \tag{11}$$

where $\nabla$ is the induced connection on $M$, $h$ is the second fundamental form of the immersion, and $-A_N X$ and $\nabla^{\perp}_X N$ are the tangential and normal parts of $\overline{\nabla}_X N$. From (10) and (11) one gets

$$g(h(X, Y), N) = g(A_N X, Y). \tag{12}$$

Moreover, we have

$$\begin{aligned}(\overline{\nabla}_X\phi)Y = {}&((\nabla_X P)Y - A_{FY}X - th(X, Y)) \\ &+ ((\nabla_X F)Y + h(X, PY) - fh(X, Y)),\end{aligned} \tag{13}$$

where

$$\phi X \equiv PX + FX, \qquad PX \in TM, \quad FX \in T^{\perp}M, \tag{14}$$

$$\phi N \equiv tN + fN, \qquad tN \in TM, \quad fN \in T^{\perp}M, \tag{15}$$

$$(\nabla_X P)Y \equiv \nabla_X PY - P\nabla_X Y, \qquad (\nabla_X F)Y \equiv \nabla^{\perp}_X FY - F\nabla_X Y. \tag{16}$$

Let $\xi \in TM$. We write $TM = \{\xi\} \oplus \{\xi\}^{\perp}$, where $\{\xi\}$ is the distribution spanned by $\xi$ and $\{\xi\}^{\perp}$ is the complementary orthogonal distribution of $\{\xi\}$ in $M$. Then we get

$$P\xi = 0 = F\xi, \qquad \eta \circ P = 0 = \eta \circ F, \tag{17}$$

$$P^2 + tF = I + \eta \otimes \xi, \qquad FP + fF = 0, \tag{18}$$

$$f^2 + Ft = I, \qquad tf + Pt = 0. \tag{19}$$

A submanifold $M$ of a Lorentzian almost paracontact manifold $\overline{M}$ with $\xi \in TM$ is an *almost semi-invariant submanifold* of $\overline{M}$ if $TM$ can be decomposed as a direct sum of mutually orthogonal differentiable distributions

$$TM = \mathcal{D}^1 \oplus \mathcal{D}^0 \oplus \mathcal{D} \oplus \{\xi\},$$

where $\mathcal{D}^1 = TM \cap \phi(TM)$, $\mathcal{D}^0 = TM \cap \phi(T^{\perp}M)$ (see [7]). A submanifold $M$ of a Lorentzian almost paracontact manifold $\overline{M}$ with $\xi \in TM$ is a *generalized CR-submanifold* [21] of $\overline{M}$ if $TM$ can be decomposed as a direct sum of mutually orthogonal differentiable distributions $TM = \mathcal{D}^0 \oplus \mathcal{D} \oplus \{\xi\}$, where $\mathcal{D}^0 = TM \cap \phi(T^{\perp}M)$.

A submanifold $M$ of a Lorentzian almost paracontact manifold $\overline{M}$ is an *invariant* (resp. *anti-invariant*) *submanifold* of $\overline{M}$ if $\phi(TM) \subset TM$ (resp. $\phi(TM) \subset T^{\perp}M$). An almost semi-invariant submanifold of a Lorentzian almost paracontact manifold is a *semi-invariant submanifold* if $\mathcal{D} = \{0\}$. A semi-invariant submanifold of a Lorentzian almost paracontact manifold becomes an invariant submanifold (resp. anti-invariant submanifold) if $\mathcal{D}^0 = \{0\}$ (resp. $\mathcal{D}^1 = \{0\}$). An almost semi-invariant submanifold is *proper* if none of the distributions $\mathcal{D}^1$, $\mathcal{D}^0$ and $\mathcal{D}$ is zero. A semi-invariant submanifold is *proper* if $\mathcal{D}^0 \neq \{0\} \neq \mathcal{D}^1$.

## 3. Riemannian and Lorentzian almost paracontact structures

An example of a 5-dimensional *LP*-Sasakian manifold is given as follows.

*Example 3.1* (K. Matsumoto, I. Mihai and R. Rosca [11]). Let $\mathbb{R}^5$ be the 5-dimensional real number space with a coordinate system $(x, y, z, t, s)$. Defining

$$\eta = ds - ydx - tdz \ , \qquad \xi = \frac{\partial}{\partial s},$$

$$g = \eta \otimes \eta - (dx)^2 - (dy)^2 - (dz)^2 - (dt)^2 \ ,$$

$$\phi\left(\frac{\partial}{\partial x}\right) = -\frac{\partial}{\partial x} - y\frac{\partial}{\partial s} \ , \qquad \phi\left(\frac{\partial}{\partial y}\right) = -\frac{\partial}{\partial y},$$

$$\phi\left(\frac{\partial}{\partial z}\right) = -\frac{\partial}{\partial z} - t\frac{\partial}{\partial s} \ , \qquad \phi\left(\frac{\partial}{\partial t}\right) = -\frac{\partial}{\partial t}, \qquad \phi\left(\frac{\partial}{\partial s}\right) = 0,$$

the structure $(\phi, \xi, \eta, g)$ becomes an $LP$-Sasakian structure in $\mathbb{R}^5$.

Now, we prove the following theorem which interrelates the Riemannian and Lorentzian almost paracontact structures on a differentiable manifold.

**Theorem 3.2.** *A differentiable manifold $\overline{M}$ admits an almost paracontact Riemannian structure if and only if it admits a Lorentzian almost paracontact structure.*

Proof. Let $\overline{M}$ admit an almost paracontact Riemannian structure $(\phi, \xi, \eta, g)$. We define a 1-form $\beta$ by

$$\beta(X) \equiv -\eta(X) \tag{20}$$

and a $(0,2)$ tensor field $\gamma$ (see page 148, B. O'Neill [14]) by

$$\gamma(X, Y) \equiv g(X, Y) - 2\eta(X)\eta(Y). \tag{21}$$

From (20), it is clear that

$$\beta(X)\,\beta(Y) = \eta(X)\eta(Y)$$

and hence (21) becomes equivalent to

$$g(X, Y) = \gamma(X, Y) + 2\beta(X)\,\beta(Y)\,. \tag{22}$$

In view of (20), (1) transforms to

$$\phi^2 = I + \beta \otimes \xi, \quad \beta(\xi) = -1.$$

From (21), it is clear that $\gamma$ is symmetric. Moreover,

$$\begin{aligned}\gamma(\phi X, \phi Y) &= g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \\ &= \gamma(X, Y) + \eta(X)\eta(Y) = \gamma(X, Y) + \beta(X)\,\beta(Y),\end{aligned}$$

where (21), (5), (2) and (20) have been used. Consequently,

$$\gamma(\xi, X) = \beta(X) = -\eta(X) = -g(\xi, X).$$

Thus $X$ is orthogonal to $\xi$ with respect to $g$ if and only if $X$ is orthogonal to $\xi$ with respect to $\gamma$. From the last equation we see that $\gamma(\xi, \xi) = -1$, that is, $\gamma$ makes $\xi$ a timelike unit vector field. If $X$ and $Y$ are orthogonal to $\xi$ with respect to $\gamma$, then $\gamma(X, Y) = g(X, Y)$, that is, $X$ and $Y$ are spacelike. Thus the metric $\gamma$ is a Lorentzian metric associated with the structure $(\phi, \xi, \beta)$. This proves the necessary part.

Conversely, let $(\phi, \xi, \beta, \gamma)$ be a Lorentzian almost paracontact structure on $\overline{M}$. Then it is easy to check that $(\phi, \xi, \eta, g)$ is an almost paracontact Riemannian structure on $\overline{M}$, where $\eta$ and $g$ are defined by (20) and (22) respectively. □

*Remark 3.3.* In view of the preceding theorem, it is now easy to construct a Lorentzian almost paracontact structure by an almost paracontact Riemannian structure and vice-versa.

## 4. Submanifolds of Lorentzian $s$-paracontact manifolds

We begin this section with the following definition, which is analogous to the definition of special paracontact Riemannian manifolds.

*Definition 4.1.* We call a Lorentzian almost paracontact manifold as a *Lorentzian $s$-paracontact manifold* if

$$\phi X = \overline{\nabla}_X \xi. \tag{23}$$

It can be verified that an $LP$-Sasakian manifold is always a Lorentzian $s$-paracontact manifold.

*Definition 4.2.* The distribution $\{\xi\}^{\perp}$ in a Lorentzian almost paracontact manifold will be called the *paracontact distribution.*

First, we prove

**Theorem 4.3.** *On a Lorentzian s-paracontact manifold $\overline{M}$ the paracontact distribution $\{\xi\}^{\perp}$ is integrable.*

Proof. Let $X, Y \in \{\xi\}^{\perp}$. Then $\eta(X) = 0 = \eta(Y)$ and consequently, in view of (23) and (7), it follows that $\eta[X,Y] = 0$, $X, Y \in \{\xi\}^{\perp}$. □

In view of Definition 4.2 and Theorem 4.3, we can state the following theorem.

**Theorem 4.4.** *Let $M$ be a submanifold of a Lorentzian s-paracontact manifold such that $\xi$ is tangential to $M$. Then the paracontact distribution $\{\xi\}^{\perp}$ on $M$ is integrable.*

In view of the above theorem we have the following corollary.

**Corollary 4.5.** *Let $M$ be either a semi-invariant or an almost semi-invariant or a generalized CR-submanifold of a Lorentzian s-paracontact manifold. Then the paracontact distribution $\{\xi\}^{\perp}$ on $M$ is integrable.*

Now, we prove a lemma.

**Lemma 4.6.** *For a submanifold $M$ of a Lorentzian s-paracontact manifold, we have*

$$\phi X = \nabla_X \xi + h\left(X, \xi\right), \qquad \xi \in TM, \tag{24}$$

$$\phi X = -A_\xi X + \nabla^{\perp}_X \xi, \qquad \xi \in T^{\perp}M, \tag{25}$$

$$\eta\left(A_N X\right) = 0, \qquad \xi \in T^{\perp}M, \tag{26}$$

$$\eta\left(A_N X\right) = g\left(\phi X, N\right), \qquad \xi \in TM \tag{27}$$

*for all $X \in TM$ and $N \in T^{\perp}M$.*

Proof. From (23) and Gauss formula, we get (24) and (25). In view of (6), we get (26). In last, for $\xi \in TM$, we have

$$\eta\left(A_N X\right) = g\left(\xi, A_N X\right) = -g\left(\xi, \overline{\nabla}_X N\right) = g\left(\overline{\nabla}_X \xi, N\right) = g\left(\phi X, N\right),$$

where (6), (11) and (23) have been used. □

In view of (24), we can state the following

**Theorem 4.7.** *Let $M$ be a submanifold of a Lorentzian $s$-paracontact manifold such that $\xi$ is tangential to $M$. Then $M$ is an invariant submanifold if and only if $h(X, \xi) = 0$, and $M$ is an anti-invariant submanifold if and only if $\nabla_X \xi = 0$.*

Now, we prove the following

**Theorem 4.8.** *If $M$ is a totally umbilical submanifold of a Lorentzian $s$-paracontact manifold such that $\xi$ is tangential to $M$, then*

(a) *$M$ is necessarily minimal and consequently totally geodesic, and*

(b) *$M$ is an invariant submanifold and $\nabla_X \xi \neq 0$ for any non-zero $X$ that is not in $\{\xi\}$.*

PROOF. Let $M$ be totally umbilical. Using (24), $\phi\xi = 0$ and (6) we get

$$0 = h(\xi, \xi) = g(\xi, \xi) H = H, \qquad H \equiv \operatorname{trace}(h) / \dim(M),$$

hence we have (a). The second part is obvious from Theorem 4.7 and Theorem 4.8 (a). □

Next, we prove the following

**Theorem 4.9.** *A submanifold $M$ of a Lorentzian $s$-paracontact manifold such that $\xi$ is normal to $M$ is an anti-invariant submanifold if and only if $A_\xi X = 0$. Consequently, if $M$ is totally geodesic then it is anti-invariant.*

PROOF. Since, $\xi$ is normal to $M$, therefore in view of (25) and (12), we obtain

$$g(\phi X, Y) = -g(A_\xi X, Y), \quad X, Y \in TM,$$

which proves the theorem. □

We complete this section by the following

*Remark 4.10.* Since $LP$-Sasakian and $LSP$-Sasakian manifolds are Lorentzian $s$-paracontact manifolds, therefore the results of this section are valid for the submanifolds of $LP$-Sasakian and $LSP$-Sasakian manifolds.

## 5. Nonexistence of an anti-invariant distribution

Let $M$ be a submanifold of a Lorentzian $s$-paracontact manifold $\overline{M}$ with $\xi \in TM$. Then in view of (27) and (14) we get

$$\eta(A_N X) = g(FX, N), \qquad X \in TM,\ N \in T^{\perp}M. \tag{28}$$

Moreover, if $\overline{M}$ is $LP$-Sasakian, then in view of (8) and (13) we get

$$(\nabla_X P)Y - A_{FY}X - th(X,Y) = g(\phi X, \phi Y)\xi + \eta(Y)\phi^2 X. \tag{29}$$

Now, we prove the following

**Theorem 5.1.** *Let $M$ be a submanifold of a $LP$-Sasakian manifold $\overline{M}$ with $\xi \in TM$. Then there does not exist any anti-invariant distribution $\mathcal{A}$ such that $\mathcal{A} \perp \{\xi\}$.*

Proof. We shall prove that $\mathcal{A} = \{0\}$. Let $X \in \mathcal{A}$ and $Y \in TM$. We get

$$\begin{aligned} g(A_{FX}X, Y) &= g(h(Y,X), FX) = g(th(Y,X), X) \\ &= g(\nabla_Y PX - P\nabla_Y X - A_{FX}Y - g(\phi Y, \phi X)\xi - \eta(X)\phi^2 Y, X) \\ &= -g(\nabla_Y X, PX) - g(A_{FX}Y, X) = -g(A_{FX}X, Y), \end{aligned}$$

which implies that

$$A_{FX}X = 0, \quad X \in \mathcal{A}$$

and consequently

$$0 = \eta(A_{FX}X) = g(FX, FX) = g(\phi X, \phi X) = g(X, X),$$

that is, $\mathcal{A} = \{0\}$. □

Since an $LSP$-Sasakian manifold is $LP$-Sasakian, therefore we can state the following

**Corollary 5.2.** *Let $M$ be a submanifold of a $LSP$-Sasakian manifold $\overline{M}$ with $\xi \in TM$. Then there does not exist any anti-invariant distribution $\mathcal{A}$ such that $\mathcal{A} \perp \{\xi\}$.*

In view of the definitions of *CR* [3], [17], generalized *CR* [21], semi-invariant [6] and almost semi-invariant [7] submanifolds of Lorentzian almost paracontact manifolds and in view of Theorem 5.1 and Corollary 5.2 we have the following theorem.

**Theorem 5.3.** *An LP–Sasakian or LSP–Sasakian manifold does not admit any proper CR, generalized CR, semi-invariant or almost semi-invariant submanifold. In fact, in these cases the anti-invariant distribution* $\mathcal{D}^0$ *becomes* $\{0\}$.

*Remark 5.4.* The geometry of *CR*-submanifolds of Kaehler manifolds was initiated by A. Bejancu in 1978 (see A. Bejancu [2] and K. Yano & M. Kon [25]). The definition of *CR*-submanifolds of Lorentzian almost paracontact manifolds resembles with the definition of semi-invariant submanifolds. However, the name *CR*-submanifold does not seem to be appropriate as possibility of getting a *CR*-structure on the so called *CR*-submanifold of a *LAP*-manifold is very far from reality. In [4], [23] it is proved that a Lorentzian para-Sasakian manifold does not admit proper semi-invariant submanifold. In [6], [18] it is claimed that the distributions $\mathcal{D}^0$ and $\mathcal{D}^0 \oplus \{\xi\}$ are never integrable on semi-invariant submanifolds of *LP*-Sasakian manifolds. The same result is claimed in [7] for almost semi-invariant submanifolds of *LP*-Sasakian manifolds. Contrary to these claims, the authors of [21] claim that the distribution $\mathcal{D}^0$ is always integrable on generalized *CR*-submanifolds of *LP*-Sasakian manifolds. However, in view of the above theorem, in these cases the anti-invariant distribution $\mathcal{D}^0$ becomes $\{0\}$, which makes a number of results of [3], [6], [7], [17], [18], [21] redundant.

## 6. Nonexistence of proper mixed foliated semi-invariant submanifolds

In [8], a semi-invariant submanifold is said to be *mixed foliated* if $\mathcal{D}^1 \oplus \{\xi\}$ is integrable and $h(Z+\xi, X) = 0$ for all $Z \in \mathcal{D}^1$ and $X \in \mathcal{D}^0$. In [22], the first author of this paper proved that a Sasakian manifold does not admit any proper mixed foliated semi-invariant submanifold.

In similar manner, we prove the following

**Theorem 6.1.** *A Lorentzian s-paracontact manifold can not admit any proper mixed foliated semi-invariant submanifold.*

Proof. For a submanifold $M$ of a Lorentzian $s$-paracontact manifold, it follows that

$$\phi X = \overline{\nabla}_X \xi = \nabla_X \xi + h(X, \xi), \quad \xi, X \in TM.$$

If $M$ is semi-invariant, then for $X \in \mathcal{D}^0$ we get $\nabla_X \xi = 0$ and $\phi X = h(X, \xi)$. Moreover, if $M$ is assumed to be mixed foliated, then for $X \in \mathcal{D}^0$ we get $\phi X = 0$, that is, $\mathcal{D}^0 = \{0\}$. $\square$

*Remark 6.2.* The authors of [4] prove that $LP$-Sasakian manifolds do not admit proper mixed foliated semi-invariant submanifolds. But in view of Theorem 5.3, $LP$-Sasakian manifolds do not admit even proper semi-invariant submanifolds. Therefore, that result of [4] is redundant.

*Acknowledgement.* The authors are thankful to referees for their comments towards the improvement of the paper.

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*(Received October 6, 2000; revised March 3, 2001)*