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Solubility of finite groups admitting a fixed-point-free operator group

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Abstract. The main purpose of this paper is to prove the following result: Let G be an S_3-free finite group. Let A be an operator group of G. Denote $\pi = \pi(G)$. Suppose that $A \in E_\pi^n$ and A is π-nilpotent. If $C_G(A) = 1$, then G is soluble. With the help of CFSG we also prove a result enabling us to drop the S_3-free assumption and to replace $A \in E_\pi^n$ by $A \in E_\pi^c$. This result may be viewed as a unified generalization of two well-known theorems on fixed-point-free automorphism groups which are consequences of the classification theorem of finite simple groups.

§1. Introduction

It is of considerable interest to investigate the solubility of a finite group G which admits a fixed-point-free automorphism group A. When A is cyclic or $(|G|, |A|) = 1$, people have given a positive answer by using the classification theorem of finite simple groups. (Refer to [W-C] and [G].) In [G], Gorenstein stated without proof that, as a consequence of the classification theorem of finite simple groups, a finite group with a fixed-point-free automorphism is soluble. Recently Rowley gave a short proof to confirm the result [Ro]. In fact, by using CFSG, people can ask more general questiones. In [W-C], we proved that fixed-point-freeness is too strong a restriction. What we have obtained shows that if G admits a coprime order operator group with fixed point subgroup S_3, A_4 and $Sz(2)$ free (i.e. no subgroup of its quotient group isomorphic to these groups),

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then G is soluble. In particular, if the fixed point subgroup $C_G(A)$ is either nilpotent or is of odd order then G is soluble. The result is best possible since we can find simple groups which admit a coprime order operator group with fixed point subgroup isomorphic to each of these 3 groups. Recently BELTRAN gave another proof for the case in which $C_G(A)$ is nilpotent [B]. However, the cyclic case has no significant improvement. The restriction of "fixed-point-freeness" plays an essential role in the induction of ROWLEY's recent proof [Ro]. Meanwhile some people also try to drop, or at least modify, the hypotheses of coprime order or cyclicity to get similar results in more general cases [P].

All the groups in this paper are finite. The symbols and terms are standard (refer to [H] or [R]).

We denote by $\pi(G)$ the set of prime divisors of $|G|$.

We write $G \in E_\pi$ if there exists a Hall π-subgroup of G, $G \in C_\pi$ if any two Hall π-subgroups of G are conjugate in G; $G \in D_\pi$ if every π-subgroup of G is contained in a a given Hall π-subgroup of G up to conjugation, i.e. $G \in C_\pi$ and every π-subgroup of G contained in some Hall π-subgroup of G.

We write $G \in E_\pi^n$ if G has a nilpotent Hall π-subgroup; $G \in E_\pi^c$ if G has a cyclic Hall π-subgroup.

In this paper, we consider the following more general.

Hypothesis $(*)$**.** *Let G be a finite group. Let A be an operator group of G with $C_G(A) = 1$. Denote $\pi = \pi(G)$. Suppose that $A \in E_\pi^c$ and A is π-nilpotent.*

Our purpose is to prove the following

Theorem (Theorem 2.4)**.** *If (G, A) satisfies the hypothesis $(*)$, then G is soluble.*

Remark 1. The following two important special examples which satisfy hypothesis $(*)$ are of special interest for many scholars.

(1) A is cyclic (with natural π-Hall subgroup and normal complement);

(2) $|G|, |A|) = 1$ (with identity Hall $\pi(G)$-subgroup).

Hence, the hypothesis $(*)$ may be viewed as a natural and unified generalization of these two special cases.

Our main theorem is a consequence of the classification theorem of finite simple groups.

Without CFSG, we use Glauberman's result on S_3-free simple groups to give a generalization of Parrott's main theorem and we try to get best possible hypotheses on the operator group. From our proof, it is reasonable to conjecture that we may replace $A \in E^c_{\pi(G)}$ by $A \in E^n_{\pi(G)}$. The main idea in this paper is to find suitable hypotheses which allow us to use induction and to reduce the operator group to the easy case of automorphism of a simple group. It seems that the $\pi(G)$-part of A plays the most important role in controlling the solubility of the groups.

§2. Theorems and proofs

Lemma 2.1. *Let N be a p-group, A, A_1 be two p-nilpotent operator groups of N with $N \cap A = 1$. If $N \rtimes A = G = N \rtimes A_1$ and $C_N(A) = 1 = C_N(A_1)$, then there exists $n \in N$, such that $A_1 = A^n$.*

Proof. We prove the lemma by induction on $|G|$. Obviously, we may assume that $N \neq 1$.

Since A is p-nilpotent, there exists a p-subgroup $A_p \in \mathrm{Syl}_p(A)$ and a normal Hall p'-subgroup $A_{p'}$ of A such that $A_{p'} \trianglelefteq A$ and $A = A_p A_{p'}$.

Firstly, we prove that $N_G(A_{p'}) < G$.

In fact, if $N_G(A_{p'}) = G$, then $[N, A_{p'}] \leq N \cap A_{p'} = 1$, i.e. $N \leq C_G(A_{p'})$. Now, the p-group A_p acts on the nontrivial p-subgroup N, by the orbit formula, it is easy to see that $C_N(A_p) \neq 1$. Hence, $1 \neq C_N(A_p) \leq C_N(A) = 1$, a contradiction.

Denote $G_1 = N_G(A_{p'})$. Then $G > G_1 \geq A$. Let $(A_1)_{p'}$ be a Hall p'-subgroup of A_1. Since $A \cong G/N \cong A_1$ and both of them are p-nilpotent, we have that both $A_{p'}$ and $(A_1)_{p'}$ are Hall p'-subgroups of G. Hence $(A_1)_{p'}N/N = A_{p'}N/N \lhd G/N$, that is $(A_1)_{p'}N = (A)_{p'}N$. By the Schur–Zassenhaus Theorem [R] 9.1.2, there exists $n_1 \in N$ such that $(A_1)^n_{p'} = A_{p'}$. It is easy to show that $G_1 = N_G(A_{p'}) = N_G((A_1)^{n_1}_{p'}) = (N_G((A_1)_{p'}))^{n_1} \geq A_1^{n_1}$. Now Dedekind's Law implies that $G_1 = G_1 \cap (NA) = (G_1 \cap N)A$ and $G_1 = G_1 \cap (NA_1^{n_1}) = (G_1 \cap N)A_1^{n_1}$. Since $G_1 < G$ and G_1 satisfies the hypothesis in the lemma, by induction there exists $n_2 \in N$ such that $A_1^{n_1 n_2} = A$, i.e. $A_1^n = A$ where $n = n_1 n_2 \in N$. This completes our proof. $\square$

Theorem 2.2. *Let G be a finite group and suppose that G is S_3-free. Let A be an operator group of G. Suppose that $A \in E^n_{\pi(G)}$ and A is π-nilpotent. If $C_G(A) = 1$, then G is soluble.*

Proof. Assume that the theorem is false and choose G to be a counterexample with $|G| + |A|$ minimal. Then

(1) For every nontrivial A-invariant proper subgroup M of G, M is soluble and $A \leq \mathrm{Aut}(G)$.

Let $M < G$ and M is A-invariant. We have that $C_M(A) \leq C_G(A) = 1$ and M is S_3-free. Furthermore, $\pi(M) \subseteq \pi(G)$. Since $A \in E^n_{\pi(G)}$, by definition we have that $A \in E^n_{\pi(M)}$. A is π-nilpotent hence A has a normal $\pi(G)$-complement $A_{\pi(G)'} = K$. Since A has a nilpotent Hall $\pi(G)$-subgroup $A_{\pi(G)}$, we have that $A_{\pi(G)} = A_{\pi(M)} \times A_{\pi(G)-\pi(M)}$. $(A_{\pi(G)-\pi(M)})K$ is the normal $\pi(M)$-complement of A. Hence (M, A) satisfies the hypotheses of (G, A). The minimal choice of G implies that M is soluble. A similar proof yields that $A \leq \mathrm{Aut}(G)$.

Now we assume that $A \leq \mathrm{Aut}(G)$ and $GA = G \rtimes A$ and we denote $\pi = \pi(G)$.

(2) There is no nontrivial A-invariant normal subgroup N of G.

Let N be a minimal normal A-invariant subgroup of G, by (1) N is soluble. Therefore N is an elementary abelian p-group for a prime $p \in \pi(N)$. Consider $\overline{G} = G/N$. Certainly $\overline{G}$ is S_3-free, since $\pi(\overline{G}) \subseteq \pi(G)$. The same argument as in (1) implies that $A \in E^n_{\pi(\overline{G})}$ and A has a normal $\pi(\overline{G})$-complement. In order to show that $(\overline{G}, A)$ satisfies the hypotheses of the theorem, we only need to show that $C_{\overline{G}}(A) = \bar{1}$.

Let $\bar{g} \in C_{\overline{G}}(A)$. Then $[g, A] \subseteq N$. Hence, $\forall x \in N$, $a \in A$, and we have $xa^g = xg^{-1}aga^{-1}a \in NA$, i.e. $NA^g \leq NA$. Since $|NA^g| = |N^g A^g| = |NA|$, we have $NA = NA^g$. Furthermore, by hypothesis, $A = A_\pi A_{\pi'}$ $A_{\pi'} \trianglelefteq A$ and A_π is nilpotent. Since $p \in \pi(N) \subseteq \pi$, we have that $O_{p'}(A_\pi)A_{\pi'}$ is the normal p-complement of A. It is clear that $C_N(A) \leq C_G(A) = 1$ and $C_N(A^g) = (C_N(A))^g = 1$. Now the hypotheses of Lemma 2.1 are satisfied for (N, A, A^g). By Lemma 2.1, there exists $n \in N$ such that $A^g = A^n$, i.e. $gn^{-1} \in N_G(A)$. Hence $[gn^{-1}, A] \leq G \cap A = 1$, that is $gn^{-1} \in C_G(A) = 1$. This yields that $g = n \in N$ and so $\bar{g} = \bar{1}$.

Now $(G/N, A)$ satisfies the hypotheses of (G, A). The minimal choice of G implies that G/N is soluble. Now both N and G/N are soluble and so is G, a contradiction.

(3) G is a nonsoluble simple group.

By (2), G has no nontrivial A-invariant normal proper subgroup and so G is characteristic simple. Since G is a nonsoluble simple group, we have that $G = G_1 \times \cdots \times G_k$ is a direct product of isomorphic non-abelian simple groups. $k \geq 1$, G_1 is a minimal normal subgroup of G.

Now we prove that $k = 1$.

Suppose it is false, i.e. $k > 1$. Then $N_A(G_1) = A_1 < A$ by (1). Since $A \in E^n_{\pi(G)}$, the Wielandt Theorem [R] 9.1.10 yields that $A \in D^n_{\pi(G)}$. A has a normal $\pi(G)$-complement hence A_1 has a normal $\pi(G)$-complement. By the Schur–Zassenhaus Theorem [R] 9.1.2, there exists a Hall $\pi(G)$-subgroup $(A_1)_{\pi(G)}$ of A_1. Since $A \in D^n_{\pi(G)}$ and $(A_1)_{\pi(G)}$ is nilpotent, we have that $A_1 \in E^n_{\pi(G)}$. Now $\pi(G_1) \subseteq \pi(G)$ implies that $A_1 \in E^c_{\pi(G_1)}$. It is clear that the product of the normal $\pi(G)$-complement of A_1 and the normal $\pi(G_1)$-complement of $(A_1)_{\pi(G)}$ is exactly the normal $\pi(G_1)$-complement of A_1. If $C_{G_1}(A_1) = 1$, then (G_1, A_1) satisfies the hypothesis of the theorem. Since $k > 1$, we have that $G_1 < G$). By the minimal choice of G we have that G_1 is soluble and hence G is soluble, a contradiction. Therefore $C_{G_1}(A_1) \neq 1$. Let $A = A_1 + A_1a_2 + \cdots + A_1a_n$, $\{1 = a_1, a_2, \ldots, a_n\}$ be the coset transversal of A_1 in A. Since G_i is a minimal normal subgroup of G, $G_1^{a_i} = G_1^{a_j}$ if and only if $a_i = a_j$. Since G has no nontrivial A-invariant normal proper subgroup, we have that $1 \neq \langle G_1^a : a \in A \rangle = G$. Note that each $G_1^{a_i}$ is a minimal normal subgroup of G and each of them is a nonabelian simple group. By induction on n, (see [H] Satz I.9.12), we have that $G = G_1 \times \cdots \times G_k = G_1 \times G_1^{a_2} \times \cdots \times G_1^{a_n}$. Hence $n = k$. Since $C_{G_1}(A_1) \neq 1$, there exists $1 \neq g_1 \in C_{G_1}(A_1)$. It is easy to show that $1 \neq g = g_1 g_1^{a_2} \cdots g_1^{a_n} \in C_G(A) = 1$, in contradiction to our hypothesis. Hence $k = 1$.

(4) $A \leq \mathrm{Out}(G)$.

By (1), we have $A \leq \mathrm{Aut}(G)$. It is a basic fact that $\mathrm{Inn}(G) \trianglelefteq \mathrm{Aut}(G)$. By (3), G is a nonabelian simple group and so $\mathrm{Inn}(G) \cong G/Z(G) \cong G$. Let $I_g \in \mathrm{Inn}(G)$ with $g \in G$. By definition, $\forall\, x \in G$, we have $I_g : x \to x^g$. Let $\mathrm{Inn}(G) \cap A = A_1$, then $A_1 \trianglelefteq A$. Since A_1 is a $\pi(G)$-subgroup of A and $A \in E^n_{\pi(G)}$ and hence $A \in D^n_{\pi(G)}$, we have that $A_1 \leq A_{\pi(G)}$, where

$A_{\pi(G)}$ is a Hall $\pi(G)$-subgroup of A. Suppose $A_1 \neq 1$. Since $A_1 \trianglelefteq A$ and A has a normal $\pi(G)$-complement $A_{(\pi(G))'}$, we have that $Z(A_1)A_{(\pi(G))'} = Z(A_1) \times A_{(\pi(G))'}$. Since $A_\pi(G)$ is nilpotent, we have that $1 \neq B_1 = A_1 \cap Z(A_\pi(G)) \leq Z(A_1)$. It follows that $B_1 \leq Z(A)$. Let $1 \neq I_g \in B_1$ with $1 \neq g \in G$. For any $h \in A$, we have $h^{-1}I_g h = I_g$, that is, $\forall\ x \in G$, $h^{-1}I_g h : x \to (x^{h^{-1}})I_g h = (g^{-1})x^{h^{-1}}g)^h = (g^{-1})^h x g^h = x^{g^a}$, i.e. $I_{g^h} = I_g$. Hence $g^{-1}g^h \in Z(G) = 1$. It follows that $g = g^h$ for every $h \in A$, i.e. $1 \neq g \in C_G(A) = 1$, a contradiction. Hence $A \cap \mathrm{Inn}(G) = A_1 = 1$. $A \cong A/A \cap \mathrm{Inn}(G) \cong A\,\mathrm{Inn}(G)/\,\mathrm{Inn}(G) \leq \mathrm{Aut}(G)/\,\mathrm{Inn}(G) = \mathrm{Out}(G)$.

Since G is an S_3-free nonabelian simple group, by a result of Glauberman ([Gl] Corollary 7.3) it follows that G is isomorphic to either $S_z(2^{2n+1})$ or $PSL(2, 3^{2n+1})$.

The structure of the outer automorphism groups of the above groups is well known. Assume that G is $S_z(2^{2n+1})$. By [S], $\mathrm{Out}(G)$ is isomorphic to a field automorphism group F. In this case, F is cyclic. Let p be a prime divisor of $2m+1$. If $p < 2n+1$, then $S_z(2^p)$ is an A-invariant subgroup since $A \leq F$. By (1), $S_z(2^p)$ is soluble, a contradiction. Hence $2n+1 = p$ is a prime. That means A is a fixed point free automorphism group of prime order. A well known theorem of Thompson [T] implies that G is soluble, a contradiction.

Now we assume that G is $PSL(2, 3^{2n+1})$. By [C] XVI, we know that $\mathrm{Out}(G) = DF$ because the graph automorphism is the identity group. Furthermore we know that $|D| = 2$ and $|F| = 2n+1$. Hence $F \triangleleft DF$ since F has index 2. By [G] p. 303, F normalizes D, and we have that $DF = D \times F$. Since the field automorphism group is a cyclic of odd order, we have that DF is a cyclic group in this case. It follows that A is cyclic fixed point free automorphism group of G. Refering to Rowley's proof of the theorem (4) [Ro], we get a contradiction. □

Corollary 2.3. *Let G be a finite group. Let A be an operator group of G. Suppose that $C_G(A) = 1$ and G is S_3-free. Then G is soluble provided that one of the following conditions holds:*

(a) *A is nilpotent;* (b) *A is S_3;* (c) $(|G|, |A|) = 1$.

This corollary implies the main theorems of [P] and [W].

If we appeal to CFSG, we can use a similar proof to prove our main theorem.

Theorem 2.4. *Let G be a finite group. Let A be an operator group of G. Suppose that $A \in E^c_{\pi(G)}$ and A is π-nilpotent. If $C_G(A) = 1$, then G is soluble.*

Proof. Suppose that this is false and consider the counterexample G with $|G| + |A|$ minimal. Then (1) to (4) of the proof of Theorem 2.2 are valid for our proof. We have that G is a simple group with every proper A-invariant ubgroup soluble and $A \leq \mathrm{Out}(G)$. Furthermore we have that A is cyclic.

If $A_{(\pi(G))'} = 1$, then A is a π-group. $A \in E^c_{\pi(G)}$ implies that A is cyclic.

Now we assume that the normal $\pi(G)$-complement $A_{(\pi(G))'}$ of A is not the identity group. By [G] 4.239, $|\mathrm{Out}(G)/G| \leq 4$ when G is one of the sporadic simple groups, alternative groups or the Tits simple group. So we know that G is not isomorphic to any of the above simple groups. By the classification theorem of finite simple groups, we see that G is a simple group of Chevalley type. By [G] p. 303, $\mathrm{Out}(G) \cong DFM$, where D, F, M are the diagonal, the field and the graph automorphism of G respectively. Since $\pi(A_{(\pi(G))'}) \cap \pi(G)$ is empty, by [C] p. XV, $|\mathrm{Out}(G)| = dfg$, $f = |F|$. By [C] page XVI Table 5 and Table 6, $(q-1)q(q+1) \mid |G|$. Hence $\pi(d) \cup \pi(g) \subseteq \pi(G)$. So $\pi(A_{(\pi(G))'}) \subseteq \pi(F)$. Since F is cyclic, there is a Hall-$\pi(A_{(\pi(G))'})$-subgroup F_1 of F. By our hypothesis, F_1 is also a Hall-$\pi(A_{(\pi(G))'})$-subgroup of $\mathrm{Out}(G)$. By Wielandt's Theorem [R] 9.1.10, $A^x_{(\pi(G))'} \leq F_1$ for some $x \in \mathrm{Out}(G)$. Without loss of generality, we can assume that $A_{(\pi(G))'} \in F_1 \leq F$. (We refer to the proof of [W-C] Theorem 2 (2).) Let $1 \neq \langle\alpha\rangle = A_{(\pi(G))'}$. Take a prime order element β of $\langle\alpha\rangle$, say $|\beta| = p$. Since $1 \neq \langle\beta\rangle \,\mathrm{char} \langle\alpha\rangle = A_{(\pi(G))'} \trianglelefteq A$, $C_G(\beta)$ is an A-invariant proper subgroup of G. By (1), $C_G(\beta)$ is soluble. By [G-L] 1.2 (5), G is isomorphic to one of the following simple groups: $A_1(2^p)$, $A_1(3^p)$, ${}^2A_2(2)$, $p \neq 2, 3$, a prime, or ${}^2B(2^p)$, $p \neq 2, 5$, a prime.

Now we see that $A_{(\pi(G))'}$ is a cyclic group of order p. By [C] p. XVI Table 5 and Table 6, $d = g = 1$ when G is $A_1(2^p)$ or $S_z(2^p) = {}^2B(2^p)$. In these cases, $1 < |A| \leq |\mathrm{Out}(G)| = p$ is cyclic. If $G = A_1(3^p)$, then $g = 1$, $d = 2$, $f = p$. Since $A_{\pi'} \neq 1$, $p \notin \pi(G)$, $|A_{\pi(G)}| = 1$ or 2. If $|A_{\pi(G)}| = 2$, then $A_{\pi(G)} = D$. By [G] p. 303, $A_{(\pi(G))'}$ normalizes D.

Hence $A = A_{\pi(G)}A_{(\pi(G))'} = A_{\pi(G)} \times A_{(\pi(G))'}$ is a cyclic group of order $2p$. If $|A_{\pi(G)}| = 1$, then A is a cyclic group of order p.

Now the only case left is $G = {}^2A_2(2^p)$. In this case, $d = 3$ or 1 and $f = 2p$. If $d = 1$ then A is cyclic so we assume that $d = 3$. Since $\{2, 3\} \subset \pi(G)$ in this case, we know that $p \neq 2, 3$. By [G] p. 303, D is normalized by F and hence D is the unique nontrivial 3-subgroup of DF. If $3 \notin \pi(A)$, then $A \leq F$ by conjugation and hence A is cyclic. Now assume that $3 \in \pi(A)$ and hence $D \lhd A$. Since A has normal $\pi(G)$-complement and cyclic $\pi(G)$-subgroup, we have that $A_{(\pi(G))'}$ commutes elementwise with D and F, and hence $A = A_{\pi(G)}A_{(\pi(G))'} = A_{\pi(G)} \times A_{(\pi(G))'}$ is a cyclic group.

Now by the same proofs of Rowley [Ro] it follows that G is soluble and the final contradiction completes our proof. $\square$

According to the proof of Theoremes 2.2 and 2.4, we would like to pose the following

Conjecture. *Let G be a finite group. Let A be an operator group of G with $C_G(A) = 1$. Suppose that $A \in E^n_{\pi(G)}$ and A is $\pi(G)$-nilpotent. Then G is soluble.*

Remark 2. We have an example to show that the hypothesis of $A \in E^n_{\pi(G)}$ is necessary. Let $A_5 = G$ and $A_3 = A$. Consider A acts on G by conjugate. Then we have that $C_G(A) = 1$ but G is non-abelian simple.

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References

[B] A. Beltran, Actions with nilpotent fixed point subgroups, *Arch. Math.* **69** (1997), 177–184.

[C] J. Conway *et al.*, ATLAS of Finite Groups, *Clarendon Press*, *Oxford*, 1985.

[G] D. Gorenstein, Finite Simple Groups, *Plenum Press*, *New York*, 1982.

[G-L] D. Gorenstein and R. Lyons, Nonsolvable signilizer functor revisited, *J. of Algebra* **133** (1990), 446–466.

[Gl] G. Glauberman, Local Analysis of finite groups, CBMS Monograph 33, *Amer. Math. Soc.*, 1977.

[H] B. Huppert, Endlich Gruppen, *Springer-Verlag*, 1967.

[P] D. Parrott, Finite groups which admit a fixed-point-free automorphism group isomorphic to S_3, *J. Austral. Math. Soc.* (Series A) **48** (1990), 384–396.

[R] D. Robinson, A Course of Groups Theory, *Springer-Verlag*, *Berlin, New York*, 1982.

[Ro] P. Rowley, Finite groups admitting a fixed-point-free automorphism group, *J. of Algebra* **174** (1995), 724–728.

[S] M. Suzuki, On a class of double transitive groups, *Ann. of Math.* **75** (2) (1962), 105–145.

[T] J. Thompson, Finite groups admitting a fixed-point-free automorphism of prime order, *Proc. Nat. Acad. Sci. USA* **45** (1959), 578–581.

[W] Y. Wang, Solubility of finite groups admitting a fixed-point-free automorphism group, *Math. Res. Expo.* **1** (1990), 78–81.

[W-C] Y. Wang and Z. Chen, Solubility of finite groups admitting a coprime order operator group, *Boll. Union. Ital. Mat.* **(7) A** (1993), 84–88.

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