Publ. Math. Debrecen
63/1-2 (2003), 5–17

The mathematics of Zoltán T. Balogh

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1. Introduction

Zoli's research was in set-theoretic topology. (The authors of this article were friends as well as professional colleagues of Zoltán Balogh. So we would like to call him Zoli, as we did throughout his life.) Deep infinitary combinatorics lie at the heart of many of the problems in this field, and thus their solutions frequently make use of the tools of modern set theory, e.g., special axioms such as the continuum hypothesis (CH) or Martin's Axiom (MA), or building models of set theory by Cohen's method of forcing. Statements shown to be true using special axioms or models are thereby proven *consistent* with the usual axioms of ZFC (the Zermelo–Fraenkel axioms plus the axiom of choice). Sometimes the negation of a consistent statement is also shown to be consistent, and hence the statement is *independent* of ZFC. The statement of Souslin's problem, for example, is a well-known independent statement. A problem in set-theoretic topology is not considered "solved" until either its statement is proven independent, or a positive or negative answer in ZFC is found. Some of Zoli's very best results were finding ZFC solutions to problems for which previously only a consistent answer was known.

Zoli's research spans 25 years or so, and includes many significant contributions in diverse areas within set-theoretic topology. To help organize

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Mathematics Subject Classification: 01A70.
This discussion of Zoltán Balogh's mathematics will also appear, possibly in an expanded form, on the Topology Atlas website and in a future issue of Topology Proceedings.

our discussion, we have divided much of Zoli's work into five "themes" which ran through many of his papers. It is not surprising that a mathematician as strong and broad as Zoli would also have much work that cannot be conveniently classified, so there is also a relatively large "miscellaneous" category.

What makes Zoli's research especially stand out are a series of solutions to several long-standing problems in the field, which he obtained at an amazing pace starting in the mid-1980's, continuing essentially until his death. We have singled out for special discussion what in our opinion are six of his most remarkable results, which for easy identification we call his "greatest hits".

Our discussion of Zoli's research will be roughly chronological within themes. The References section consists of a complete list of Zoli's publications.

2. Early work: relative compactness and hereditarily nice spaces

The 1960's were a kind of golden age for so-called "generalized" metrizable spaces. A. V. Arhangel'skiĭ defined p-spaces, K. Nagami defined Σ-spaces, K. Morita M-spaces, and so on. Investigations of these classes, sometimes with an eye toward generalizing what were by then "classical" results in the area, were still going strong in the mid 1970's, when Zoli came on the scene. R. Hodel had generalized some metrization results to higher cardinals by defining "metrizability degree" and put them in the language of cardinal function theory[1], a hot topic at that time. Another topic of interest was: what can be said of the whole space if one knows that every subspace is "nice" in the sense of belonging to a certain class of generalized metric spaces?

[1]Typical cardinal functions that appear in our discussion are the weight $w(X)$, the character $\chi(X)$, and the Lindelöf degree $L(X)$, which are, respectively, the least cardinal of a base, the least cardinal such that each point has a local base not greater than that cardinality, and the least cardinal such that every open cover of X has a subcover of that cardinality or less.

Zoli's first contributions of his career were in this area. Recall that a space X is a paracompact p-space if there is a perfect (i.e., closed with compact fibers) map f from X to a metrizable space Y. Let τ be the topology on X and τ' the weaker topology on X obtained by pulling back the metrizable topology on Y by the function f. Then if a filter on X has a cluster point in the topology τ', it is easy to see, using the perfectness of the map f, that the filter also has a cluster point in τ. Zoli's nice idea [Ba76], [Ba78a], [Ba79a] was to study exactly this relationship between topologies, calling τ *relatively compact* to τ' if they satisfy the above filter convergence condition. There was also a countably compact analogue [Ba79c], defined in terms of filters having a countable base. Zoli noticed that in many cases, especially when the space had a point-separating open cover of some sort, or when every subspace was "nice" in the sense of being relatively compact to the topology τ, various cardinal functions on τ' were a bound for those of τ. General results of this form for relative compactness, and the similar notion of relative countable compactness, had many corollaries which superseded classical results and answered questions of Arhangel'skiĭ, Hodel, and others. The following example gives the flavor:

Theorem 2.1 ([Ba76])**.** *Suppose (X, τ) is compact relative to a weaker topology τ' with metrizability degree $\leq \kappa$. If (X, τ) has a point $\leq \kappa$, T_1-separating open cover, then the metrizability degree of (X, τ) is $\leq \kappa$.*

Taking $\kappa = \omega$ and $\tau = \tau'$ gets J. Nagata's classical result that a paracompact p-space with a point-countable T_1-separating open cover is metrizable. A similar theorem with metrizability degree replaced by weight has cardinal function results of Hodel as corollaries, and answers a question of Arhangel'skiĭ on spaces whose every subspace is a paracompact p-space.

The proofs of the these early results of Zoli already showed the style which he became well-known for later, involving heavy use of complicated combinatorics of sets and collections of sets, the arguments slowly but steadily making their way towards the final conclusion. The power of his mind was evident from the beginning!

3. Q-set spaces

Zoli had a long-standing interest in *Q-sets*, i.e., uncountable subsets X of the real line (or separable metric spaces) in which every subset of X is a relative G_δ-set. Such sets, which do not exist in all models of set theory, had been shown to be relevant to the famous normal Moore space problem. The appropriate generalization of Q-set to arbitrary spaces is that of a *Q-set space* X, which means that every subset of X is a G_δ-set in X, yet X is not σ-discrete (i.e., not a countable union of discrete subspaces). It was not known for a long time if this more general kind of Q-set existed in ZFC.

The first paper of Zoli's to mention Q-set spaces was [Ba78b], where he showed that if a member of a certain class of spaces was non-metrizable, it was because it either contained the one-point compactification of an uncountable discrete space, or the so-called "Alexandrov duplicate" of a metric Q-set space (this was the relation to the Normal Moore Space Conjecture he refers to in the title of [Ba78b]).

Zoli's first paper studying a kind of Q-set type of space for itself was with H. Junnila in 1983 [BJ83], where the authors consider "totally analytic" spaces, i.e., spaces in which every subset is analytic. In this paper they show that under Gödel's axiom of constructibility $\mathsf{V}=\mathsf{L}$, every totally analytic space of character $\leq \aleph_1$ is σ-discrete.

Of course, this left open the problem if there could be a totally analytic non-σ-discrete space, or even a Q-set space, in ZFC. Zoli finally settled this [Ba91b] by constructing a ZFC example of a Q-set space of cardinality $\mathfrak{c}$ and character $2^{\mathfrak{c}}$. This was his first use of a technique of M. E. Rudin which he later went on to develop into an amazing example-constructing machine. More on this in Section 6. Later [Ba98a], he saw how to obtain a paracompact Q-set space in ZFC, and in a handwritten note, unpublished at the time of his death, he obtained a Lindelöf Q-set space.

4. Locally nice spaces

Zoli also had a long-standing interest in spaces that are "locally nice", usually in the sense of being locally compact, sometimes also locally connected or even a manifold. His earliest paper in this area is "Locally nice spaces under Martin's Axiom" [Ba83]. Zoli's results here are fundamental structural results which had many important corollaries, for example, Rudin's result that under $\mathsf{MA} + \neg\mathsf{CH}$, all perfectly normal Hausdorff manifolds are metrizable. The results of this paper are still finding important uses, e.g., in the recent work of P. Larson and F. Tall where a long-standing problem of S. Watson is solved by proving that, consistently, all perfectly normal locally compact spaces are paracompact.

In 1986, Zoli published two more papers on the theme of paracompactness in locally nice spaces. In [Ba86a], he answers a question of G. Gruenhage by showing that normal, locally connected, rim-compact, metalindelöf[2] spaces are paracompact. The paper [Ba86b] starts with answers to questions of Tall and Watson by showing:

Theorem 4.1. *Normal locally compact (or more generally, locally Lindelöf) screenable spaces are paracompact.*

Theorem 4.2 ($(\mathsf{V{=}L})$)**.** *Normal, locally compact, metalindelöf spaces are paracompact.*

The first result is a pretty partial result on a famous problem of Nagami, to be discussed in Section 6, whether the statement is true without the "locally compact" assumption (see that section for the definition of "screenable"). The second extends a result of Watson in which "metacompact" replaces "metalindelöf."

The first of Zoli's "greatest hits" that we get to in this article happens to be in the locally nice theme. The Normal Moore Space Conjecture had been shown to be essentially equivalent to the question whether normal first-countable spaces must be collectionwise normal[3]. It had been known for some time that the Normal Moore Space Conjecture was consistently

[2] X is *metalindelöf* (resp., *metacompact*) if every open cover has a point-countable (resp., point-finite) open refinement.

[3] A space X is *collectionwise normal* if every closed discrete collection of closed sets can be separated by a pairwise-disjoint collection of open sets.

false. Assuming the existence of sufficiently large cardinals, the Normal Moore Space Conjecture was finally shown to be consistently true, and hence independent, first by P. Nyikos and K. Kunen, and a bit later using a more flexible technique, by A. Dow, Tall, and W. Weiss. The analogous problem for locally compact spaces was formulated by Watson, and worked on over a period of years by Tall, who obtained a number of positive partial results in which typically the character of the space was bounded by some cardinal (e.g., $\aleph_\omega$). It is Tall's work on this problem that apparently prompted Zoli in his paper to refer to it as the "Toronto project". Here's the result:

Greatest Hit # 1: *It is consistent (modulo sufficiently large cardinals) that all locally compact normal spaces are collectionwise normal* [Ba91a].

In fact, Zoli obtained a more general result which has the Normal Moore Space Conjecture as a corollary. In his review of the paper, Watson calls this result "one of the finest results of the last few years in general topology."

5. Base-multiplicity

Zoli's early work included, in particular, results about point-countable bases. He retained an interest in such "base-multiplicity" topics throughout his professional life. Probably his most interesting work in this area are the results (with S. Davis, W. Just, S. Shelah, and P. Szeptycki) in [BDJSS00]. The primary motivation for the results in this paper is an old (circa 1976) question of R. Heath and W. Lindgren: Does every first-countable space with a weakly uniform base have a (possibly different) point-countable base? Recall that a base $\mathcal{B}$ is *weakly uniform* if the intersection of any infinite subcollection of $\mathcal{B}$ is either empty or a singleton.

Old partial results of Davis, G. M. Reed, and M. Wage say that there is a counterexample under $\mathsf{MA} + \mathfrak{c} > \aleph_2$, but the answer is positive in ZFC if there are not more than $\aleph_1$-many isolated points. These results already suggest that some interesting and difficult combinatorics are at the heart of this problem. Much later, Arhangel'skiĭ, W. Just, E. Reznichenko and Szeptycki showed that, under CH, every first-countable space with a weakly uniform base and no more than $\aleph_\omega$-many isolated points has a point-countable base.

In [BDJSS00], the authors finish off the problem, obtaining a consistent positive answer to the Heath–Lindgren question, with no restriction on the number of isolated points. The deep set-theoretic combinatorial results they develop to prove this are sure to have many other applications.

For many other interesting results of Zoli on the base-multiplicity theme, see the papers [BG01], [BGri03], [Ba02a], and [Ba03a].

6. Dowker spaces

A classical homotopy extension theorem of K. Borsuk in 1937 had as part of the hypothesis that $X \times [0,1]$ is normal. But it was not known at the time if normality of X was sufficient to imply normality of $X \times [0,1]$. In 1951 C. H. Dowker characterized those normal spaces X whose product with the unit interval $[0,1]$ is not normal as precisely those normal spaces which are not countably paracompact. He asked if such spaces, soon to be called *Dowker spaces*, exist. In 1971, M. E. Rudin constructed a Dowker space. But this was far from the end of the matter, because it turned out the Dowker pathology was present in many natural topological problems. Thus it was important to search for "nice" Dowker spaces. Rudin's example failed to be nice in many ways. In particular, it was not "small" in the sense of cardinality or weight (which were $\aleph_\omega^\omega$), or character (which was $\aleph_\omega$). Many Dowker spaces that were small, and/or "nice" in other ways, were constructed, but only assuming various axioms beyond ZFC. E.g., Rudin herself constructed a Dowker manifold (non-metrizable of course) assuming CH. But for decades the only known ZFC Dowker space was still Rudin's 1971 example.

So Zoli's 1996 example of an entirely new ZFC Dowker space was very exciting and certainly deserves the "greatest hit" label.

Greatest Hit # 2: *There is a σ-discrete Dowker space of cardinality $\mathfrak{c}$ in* ZFC [Ba96].

J. E. Vaughan, in his review of Zoli's paper, calls it "a milestone in set-theoretic topology." Indeed it was, not only for it's properties stated above, or just that it was the first new ZFC Dowker space in a quarter century, but even more for the technique, which he subsequently applied, in highly non-trivial fashion, to obtain solutions of long-standing problems of Nagami and K. Morita, which we also are calling greatest hits.

Greatest Hit # 3 : *Solution to Nagami's problem: there is a normal screenable non-paracompact space* [Ba98b].

R. H. Bing defined a space to be *screenable* if every open cover has a σ-disjoint open refinement. In 1955, K. Nagami explicitly asked the natural question whether normal screenable spaces are paracompact. It is easily seen that normal, countably paracompact, screenable spaces are paracompact, so a counterexample if it exists must be a Dowker space. In 1983, M. E. Rudin obtained an example under $\diamondsuit^{++}$, a powerful combinatorial axiom that holds under $\mathsf{V}=\mathsf{L}$. In 1998, Zoli finally settled the problem with his ZFC example.

Greatest Hit #4 : *Morita conjectures established: X is metrizable iff its product with every Morita P-space is normal* [Ba01b].

In 1976, K. Morita stated three basic conjectures about normality in products. The first one was solved by Rudin in 1978, and the second implies the third, so we only discuss the second. Morita studied the class of P-spaces, i.e., normal spaces whose product with every metrizable space is normal. It is well-known that not every normal space is in this class (e.g., any Dowker space). Now, if X is metrizable, then trivially $X \times Y$ is normal for every Morita P-space Y. Morita's second conjecture is that the reverse also holds. K. Chiba, T. Przymusinski, and Rudin showed that the second conjecture (and hence all three) is true if, for each uncountable cardinal κ, there is a Morita P-space X which has a well-ordered increasing open cover in type ω_1, but there is no refinement of this open cover by at most κ-many closed sets. Such examples were constructed by A. Beslagic and Rudin in 1985 under $\mathsf{V}=\mathsf{L}$. But there was no ZFC solution to the problem until Zoli, using another version of his Dowker space technique, constructed spaces in ZFC having the same properties as those constructed by Beslagic and Rudin under $\mathsf{V}=\mathsf{L}$. This was an outstanding achievement which finally settled the three conjectures of Morita in the affirmative in ZFC.

7. Miscellaneous

The name of this category does not imply a value judgement of any sort. In fact, we begin this section by discussing two more of his greatest hits!

Greatest Hit #5: *Solution to the Moore–Mrowka problem: the Proper Forcing Axiom implies that compact countably tight spaces are sequential* [Ba89].

In an AMS Notices article in 1964, R. C. Moore and S. Mrowka asked if every countably tight compact Hausdorff space is sequential[4]. In other words, if the topology of a compact Hausdorff space X is determined by its countable subsets, must the topology of X in fact be determined by its convergent sequences?

This natural and important problem received quite a bit of attention. Nyikos called it "Classic Problem VI" in his 1977 list of major open problems in set-theoretic topology. Arhangel'skiĭ gives a thorough discussion of it in a 1978 survey paper, where he puts it as Number 1 in an extensive list of open problems, and indicates that he believes there should be a ZFC counterexample. In 1976, Ostaszweski and Fedorchuk had each constructed counterexamples under the axiom $\diamondsuit$. But since then not very much happened until the power of Shelah's technique of "proper forcing" became widely known. In 1986, D. Fremlin and Nyikos had obtained some related results using Fremlin's write-up of a proper forcing method due to S. Todorčević. Nyikos also showed that $\mathsf{MA} + \neg\mathsf{CH}$ is not sufficient to solve the Moore–Mrowka problem, and that the stronger Proper Forcing Axiom (PFA) implied a positive answer for hereditarily normal spaces. Then Zoli completed the solution, showing that the answer is positive under PFA. The result followed as a corollary to a more general statement which had some of the results of Fremlin and Nyikos as other corollaries. D. B. Shakhmatov, in his 1991 survey of results on the structure of compact spaces over the previous 8-10 years, called Zoli's result "the main advance in the theory of compact spaces during the covered period."

Greatest Hit # 6: *Every open cover of a monotonically normal space*

[4] X is *countably tight* (resp., *sequential*) if for any non-closed subset A of X, there exists $x \in \overline{A}\backslash A$ and a countable $A' = \{a_n\}_{n\in\omega} \subset A$ with $x \in \overline{A'}$ (resp., $\{a_n\}_{n\in\omega}$ converges to x).

X has a σ-disjoint (partial) refinement $\mathcal{V}$ by open sets such that $X \setminus \bigcup \mathcal{V}$ is the union of a discrete family of closed subspaces each homeomorphic to some stationary subset of a regular uncountable cardinal (the cardinal may vary with the subspaces) [BR92].

The class of monotonically normal spaces was introduced by Heath, D. J. Lutzer and P. Zenor in 1973 as a common generalization of ordered spaces and metrizable (or more generally, "stratifiable") spaces. Balogh and Rudin proved the deep and powerful result stated above, a very important corollary of which is that R. Engelking and Lutzer's theorem, which says that an ordered space is paracompact if and only if it does not contain a closed copy of a stationary subset of a regular uncountable cardinal, extends to the class of monotonically normal spaces. The Balogh–Rudin result answered almost every question in the literature having to do with covering properties of monotonically normal spaces. The proof is lengthy and complicated – just what one would expect, given these authors!

Zoli obtained many more significant results but there is no space here to mention them all. We content ourselves with discussing just one more very nice paper. An interesting problem of M. Katetov, which dates back to 1951, is whether every normal T_2-space X in which the Baire and Borel algebras in X coincide must be perfectly normal. (If a space is perfectly normal, they must coincide.) In [Ba88a], Zoli obtained several examples giving a negative answer to the problem under CH, and another one based on a consistent construction due to A. Miller of a subset M of the real line in which every subset is Baire but not every subset is G_δ. It is not known if a counterexample to Katetov's question exists in ZFC. In the same paper, Zoli solves in ZFC a 1965 problem of K. A. Ross and K. Stromberg by constructing a normal locally compact space in which there exists a closed Baire set which is not a zero-set.

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