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Statistical approximation in the space of locally integrable functions

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Abstract. In this paper, using A-statistical convergence, we prove a Korovkin type approximation theorem which deals with the problem of approximating a function f by a sequence $\{T_n(f;x)\}$ of positive linear operators over the weighted space of locally integrable functions.

1. Introduction

It is known that the classical approximation operators tend to converge to the value of the function being approximated. However, at points of discontinuity, they often converge to the average of the left and right limits of the function. There are, however, some exceptions such as the interpolation operator of Hermite–Fejer [4] that do not converge at points of simple discontinuity. In this case, the matrix summability methods of Cesáro type are applicable to correct the lack of convergence [5].

Statistical convergence which is a regular non-matrix summability method is also effective in "summing" non-convergent sequences [10], [12], [13]. Recently, their use in approximation theory has been considered in [8], [16]. The purpose of the present paper is to study a Korovkin type

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approximation theorem via A-statistical convergence in the space of locally integrable functions.

Approximation theory has important applications in the theory of polynomial approximation in various areas of functional analysis, numerical solutions of differential and integral equations [1], [23].

Before proceeding further we recall some notation on statistical convergence. Let $A = (a_{nk})$ be an infinite summability matrix. For a given sequence $x := (x_k)$, the A-transform of x, denoted by $Ax := ((Ax)_n)$, is given by $(Ax)_n = \sum_{k=1}^{\infty} a_{nk}x_k$, provided the series converges for each n. We say that A is regular if $\lim_n (Ax)_n = L$ whenever $\lim_n x_n = L$ [18]. Assume now that A is a non-negative regular summability matrix and K is a subset of $\mathbb{N}$, the set of all natural numbers. The A-density of K is defined by $\delta_A(K) := \lim_n \sum_{k=1}^{\infty} a_{nk}\chi_K(k)$ provided the limit exists, where χ_K is the characteristic function of K. The sequence $x := (x_k)$ is A-statistically convergent to the number L if, for every $\varepsilon > 0$, $\delta_A\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\} = 0$ [6], [11], [22], [24]. We denote this limit by $\mathrm{st}_A\text{-}\lim x = L$. The case in which $A = C_1$, the Cesáro matrix, A-statistical convergence reduces to statistical convergence [10], [12], [13]. We note that if $A = (a_{nk})$ is a non-negative regular summability matrix for which $\lim_n \max_k\{a_{nk}\} = 0$, then A-statistical convergence is stronger than convergence [22].

It should be noted that the concept of A-statistical convergence may also be given in normed spaces: Assume $(X, \|\,.\,\|)$ is a normed space and $u = (u_k)$ is a $X-$valued sequence. Then (u_k) is said to be A-statistically convergent to $u_0 \in X$ if, for every $\varepsilon > 0$, $\delta_A\{k \in \mathbb{N} : \|u_k - u_0\| \geq \varepsilon\} = 0$, [20], [21].

2. A Korovkin type theorem via A-statistical convergence

In this section, using A-statistical convergence, we prove a Korovkin type approximation theorem which deals with the problem of approximating a function f by a sequence $\{T_n(f;x)\}$ of positive linear operators over the weighted space of locally integrable functions.

Let $w(x) = 1 + x^2$, $-\infty < x < +\infty$, and let $h > 0$. By $L_{p,w}(\mathrm{loc})$ we denote the space of all measurable functions f for which

$$\left(\frac{1}{2h}\int_{x-h}^{x+h}|f(t)|^p dt\right)^{\frac{1}{p}} \le M_f\, w(x), \quad -\infty < x < +\infty,$$

where M_f is a positive constant depending on f and $p \ge 1$. It is known [15] that $L_{p,w}(\mathrm{loc})$ is a linear normed space with norm

$$\|f\|_{p,w} = \sup_{-\infty<x<+\infty} \frac{\left(\frac{1}{2h}\int_{x-h}^{x+h}|f(t)|^p\, dt\right)^{\frac{1}{p}}}{w(x)},$$

and $\|f\|_{p,w}$ may also depend on h.

For any real numbers a, b we write

$$\|f; L_p(a,b)\| := \left(\frac{1}{b-a}\int_a^b |f(t)|^p dt\right)^{\frac{1}{p}},$$

$$\|f; L_{p,w}(a,b)\| := \sup_{a\le x\le b} \frac{\|f; L_p(x-h,x+h)\|}{w(x)},$$

$$\|f; L_{p,w}(|xt| \ge a)\| := \sup_{|x|\ge a} \frac{\|f; L_p(x-h,x+h)\|}{w(x)}.$$

With this notation the norm in $L_{p,w}(\mathrm{loc})$ may be written in the form

$$\|f\|_{p,w} = \sup_{-\infty<x<+\infty} \frac{\|f; L_p(x-h,x+h)\|}{w(x)}.$$

Let $L^k_{p,w}(\mathrm{loc})$ be the subspace of functions $f \in L_{p,w}(\mathrm{loc})$ such that there exists a constant k_f with

$$\lim_{|x|\to\infty} \frac{\|f - k_f\, w; L_p(x-h,x+h)\|}{w(x)} = 0.$$

If $k_f = 0$, we then write $L^0_{p,w}(\mathrm{loc})$ instead of $L^k_{p,w}(\mathrm{loc})$.

Some Korovkin type approximation theorems for a sequence of positive linear operators acting in the weighted space of continuous functions have been studied in [14], [17]. Similar type of theorems in $L_p(a,b)$ may be found in [9]. See also [2], [3], [7], [19], [26] for related results.

The set of all positive linear operators acting on $L_{p,w}(\mathrm{loc})$ into itself will be denoted by $(L_{p,w}(\mathrm{loc}) \to L_{p,w}(\mathrm{loc}))^+$. It is shown in [15] that if the sequence of operators $T_n \in (L_{p,w}(\text{loc}) \to L_{p,w}(\text{loc}))^+$ is uniformly bounded and

$$\lim_{n\to\infty} \|T_n(t^m; x) - x^m\|_{p,w} = 0, \quad (m = 0, 1, 2),$$

then $\lim_{n\to\infty} \|T_n f - f\|_{p,w} = 0$ for each function $f \in L^k_{p,w}(\text{loc})$. It is also observed in [15] that the above mentioned result fails if $L^k_{p,w}(\text{loc})$ is replaced by $L_{p,w}(\text{loc})$.

Replacing ordinary limit operator with A-statistical limit operator, we shall consider the analogous problems.

We shall require the following

Lemma 1. *Let $A = (a_{nk})$ be a non-negative regular summability matrix. Assume that the sequence of positive linear operators $T_n : L_{p,w}(\text{loc}) \to L_{p,w}(\text{loc})$ satisfy*

$$\text{st}_A\text{-}\lim_n \|T_n f_v - f_v\|_{p,w} = 0 \tag{1}$$

where $f_v(y) = y^v$, $v = 0, 1, 2$. Then, for any continuous and bounded f on the real axis, we have

$$\text{st}_A\text{-}\lim_n \|T_n f - f; L_{p,w}(a, b)\| = 0.$$

Proof. Since f is uniformly continuous on any closed interval, given $\varepsilon > 0$, there exists a $\delta = \delta(\varepsilon) > 0$ such that $|f(t) - f(x)| < \varepsilon$ whenever $|t - x| < \delta$, $x \in [a, b]$, $t \in \mathbb{R}$. Let $M := \sup_{x\in\mathbb{R}} |f(x)|$. Then $|f(t) - f(x)| \le 2M$ if $|t - x| \ge \delta$, $x \in [a, b]$, $t \in \mathbb{R}$. Hence

$$\begin{aligned}|f(t) - f(x)| &= |f(t) - f(x)|\chi_{[x-\delta,x+\delta]}(t) + |f(t) - f(x)|\chi_{\mathbb{R}\setminus[x-\delta,x+\delta]}(t)\\ &< \varepsilon + 2M\,\frac{(t-x)^2}{\delta^2}.\end{aligned}$$

Therefore

$$\begin{aligned}&\|T_n f - f; L_{p,w}(a, b)\| = \|T_n(f(t); x) - f(x);\ L_{p,w}(a, b)\|\\ &\quad \le \|T_n(|f(t) - f(x)|; x);\ L_{p,w}(a, b)\| + M\|T_n f_0 - f_0; L_{p,w}(a, b)\|\\ &\quad < \varepsilon\|T_n(f_0; x); L_{p,w}(a, b)\| + \frac{2M}{\delta^2}\|T_n((t-x)^2; x); L_{p,w}(a, b)\|\\ &\qquad + M\|T_n f_0 - f_0; L_{p,w}(a, b)\|\\ &\quad \le \varepsilon + (1 + M)\|T_n(f_0; x); L_{p,w}(a, b)\|\end{aligned}$$

$$+\frac{2M}{\delta^2}\{\|T_nf_2-f_2;L_{p,w}(a,b)\|+2C\|T_nf_1-f_1;L_{p,w}(a,b)\| \\ +C^2\|T_nf_0-f_0;L_{p,w}(a,b)\|\}$$

where $C=\max\{|a|,|b|\}$. Now letting

$$H:=\max\left\{1+M,\ \frac{2C^2M}{\delta^2},\ \frac{4CM}{\delta^2}\right\}$$

we get

$$\begin{aligned}\|T_nf-f;L_{p,w}(a,b)\|&<\varepsilon\\ &+H\{\|T_nf_0-f_0;L_{p,w}(a,b)\|\\ &\quad+\|T_nf_1-f_1;L_{p,w}(a,b)\|+\|T_nf_2-f_2;L_{p,w}(a,b)\|\}\end{aligned}\tag{2}$$

for each $n\in\mathbb{N}$. Given $r>0$ choose $\varepsilon>0$ such that $\varepsilon<r$. Define

$$D:=\{n:\|T_nf_0-f_0;L_{p,w}(a,b)\|+\|T_nf_1-f_1;L_{p,w}(a,b)\| \\ +\|T_nf_2-f_2;L_{p,w}(a,b)\|\geq r-\varepsilon\},$$

$$D_1:=\left\{n:\|T_nf_0-f_0;L_{p,w}(a,b)\|\geq\frac{r-\varepsilon}{3H}\right\},$$

$$D_2:=\left\{n:\|T_nf_1-f_1;L_{p,w}(a,b)\|\geq\frac{r-\varepsilon}{3H}\right\},$$

$$D_3:=\left\{n:\|T_nf_2-f_2;L_{p,w}(a,b)\|\geq\frac{r-\varepsilon}{3H}\right\}.$$

Then $D\subseteq D_1\cup D_2\cup D_3$. Hence, by (2)

$$\sum_{k:\|T_kf-f;L_{p,w}(a,b)\|\geq r}a_{nk}\leq\sum_{k\in D}a_{nk}\leq\sum_{k\in D_1}a_{nk}+\sum_{k\in D_2}a_{nk}+\sum_{k\in D_3}a_{nk}.\tag{3}$$

Now taking limit as $n\to\infty$, (1) and (3) yield the result. □

Theorem 2. *Let $A=(a_{nk})$ be a non-negative regular summability matrix. If the sequence of positive linear operators $T_n:L_{p,w}(\mathrm{loc})\to L_{p,w}(\mathrm{loc})$ satisfies the following conditions*

(a) *there exists an $H>0$ such that $\delta_A\{n\in\mathbb{N}:\|T_n\|\geq H\}=0$*
and

(b) $\mathrm{st}_A\text{-}\lim_n \|T_n f_v - f_v\|_{p,w} = 0$ *where* $f_v(t) = t^v$, $v = 0, 1, 2$; *then*

$$\mathrm{st}_A\text{-}\lim_n \|T_n f - f\|_{p,w} = 0 \tag{4}$$

for each $f \in L^k_{p,w}(\mathrm{loc})$.

PROOF. If $f \in L^k_{p,w}(\mathrm{loc})$, then $F := f - k_f\, w \in L^0_{p,w}(\mathrm{loc})$. Considering the inequality

$$\begin{aligned}\|T_n f - f\|_{p,w} &\le \|T_n F - F\|_{p,w} + k_f \|T_n w - w\|_{p,w} \\ &\le \|T_n F - F\|_{p,w} + k_f \|T_n f_0 - f_0\|_{p,w} + k_f \|T_n f_2 - f_2\|_{p,w}\end{aligned}$$

we conclude by (b) that if (4) holds for the function F, then so does for the function f. So, it suffices to prove the theorem for the function $f \in L^0_{p,w}(\mathrm{loc})$. Hence given $\varepsilon > 0$, there exists x_0 such that for all x satisfying the condition $|x| \ge x_0$ we have

$$\left(\frac{1}{2h}\int_{x-h}^{x+h} |f(t)|^p dt\right)^{\frac{1}{p}} < \varepsilon\, w(x). \tag{5}$$

It follows from Lusin's theorem [25] that there exists a continuous function φ on the interval $[-x_0 - h, x_0 + h]$ for which

$$\|f - \varphi; L_p(-x_0 - h, x_0 + h)\| < \varepsilon \tag{6}$$

holds. Now choose a $\delta > 0$ such that

$$\delta < \min\left\{\frac{2h\varepsilon^p}{M^p(x_0)},\, h\right\} \tag{7}$$

where $M(x_0) := \max\{\max_{|x| \le x_0 + h} |\varphi(x)|,\, 1\}$.

Define a continuous function $g : \mathbb{R} \to \mathbb{R}$ by

$$g(x) = \begin{cases} \varphi(x), & |x| \le x_0 + h \\ 0, & |x| \ge x_0 + h + \delta \\ \text{linear}, & \text{otherwise.}\end{cases}$$

Considering (5) and (6) we have

$$\|f - g\|_{p,w} \le \|f - g; L_{p,w}(-x_0, x_0)\| + \|f - g; L_{p,w}(|x| \ge x_0 + h + \delta)\|$$

$$+\left\|f-g;L_{p,w}(x_0,x_0+h+\delta)\right\|+\left\|f-g;L_{p,w}(-x_0-h-\delta,-x_0)\right\|$$
$$\le 2\varepsilon+\left\|f-g;L_p(x_0-h,x_0+2h+\delta)\right\|$$
$$+\left\|f-g;L_p(-x_0-2h-\delta,-x_0+h)\right\|$$
$$\le 2\varepsilon+\left\|f-\varphi;L_p(x_0-h,x_0+h)\right\|+\left\|f;L_p(x_0+h,x_0+2h+\delta)\right\|$$
$$+\left\|g;L_p(x_0+h,x_0+h+\delta)\right\|+\left\|f;L_p(-x_0-2h-\delta,-x_0-h)\right\|$$
$$+\left\|g;L_p(-x_0-h-\delta,-x_0-h)\right\|+\left\|f-\varphi;L_p(-x_0-h,-x_0+h)\right\|.$$

Using the fact that

$$|g(x)|\le M(x_0)\quad \text{for all } x\in\mathbb{R} \tag{8}$$

and considering (5) and (6) again, one can get

$$\left\|f-g\right\|_{p,w}<4\varepsilon+2M(x_0)\left(\frac{\delta}{2h}\right)^{\frac{1}{p}}+\left\|f;L_p(x_0+h,x_0+3h)\right\|$$
$$+\left\|f;L_p(-x_0-3h,-x_0-h)\right\|.$$

Also, by (5) and (7),

$$\left\|f-g\right\|_{p,w}<6\varepsilon+2\varepsilon w(x_0+2h)=C_1\varepsilon \tag{9}$$

where $C_1=6+2w(x_0+2h)$.

Since $w(x)=1+x^2$, we can find a point $x_1>x_0$ such that

$$\frac{M(x_0)}{w(x_1)}<\varepsilon \quad\text{and}\quad g(x)=0 \tag{10}$$

for $|x|>x_1$. Now let $E:=\{n\in\mathbb{N}:\left\|T_n\right\|\le H\}$. Then, by (a), $\delta_A(E)=1$. Hence, given $\varepsilon>0$, by (9) and (10) we get, for any $n\in E$, that

$$\begin{aligned}\left\|T_nf-f\right\|_{p,w}&\le\left\|T_n(f-g)\right\|_{p,w}+\left\|T_ng-g\right\|_{p,w}+\left\|f-g\right\|_{p,w}\\&\le\left\|T_n\right\|\,\left\|f-g\right\|_{p,w}+\left\|T_ng-g\right\|_{p,w}+\left\|f-g\right\|_{p,w}\\&<(H+1)C_1\varepsilon+\left\|T_ng-g;L_{p,w}(-x_1,x_1)\right\|\\&\quad+\left\|T_ng-g;L_{p,w}(|x|\ge x_1)\right\|.\end{aligned}$$

By (8) and (10),

$$\begin{aligned}\left\|T_nf-f\right\|_{p,w}&<(H+1)C_1\varepsilon+\left\|T_ng-g;L_{p,w}(-x_1,x_1)\right\|+\frac{M(x_0)}{w(x_1)}\\&<K\varepsilon+\left\|T_ng-g;L_{p,w}(-x_1,x_1)\right\|\end{aligned} \tag{11}$$

for any $n \in E$, where $K = (H+1)C_1 + 1$. Now given $r > 0$ choose $\varepsilon > 0$ such that $K\varepsilon < r$. Hence

$$\sum_{k\in E:\, \|T_k f - f\|_{p,w} \geq r} a_{nk} \leq \sum_{k\in E:\, \|T_k g - g; L_{p,w}(-x_1,x_1)\| \geq r - K\varepsilon} a_{nk}.$$

Now, it follows from Lemma 1, for any $f \in L^0_{p,w}(\mathrm{loc})$, that

$$\mathrm{st}_A\text{-}\lim_n \|T_n f - f\|_{p,w} = 0$$

whence the result. $\square$

The preceding theorem enables us to establish a statistical approximation result for all functions in $L_{p,w}(\mathrm{loc})$. We give it formally as follows.

Theorem 3. *Let $A = (a_{nk})$ be a non-negative regular summability matrix and let the sequence of positive linear operators $T_n : L_{p,w}(\mathrm{loc}) \to L_{p,w}(\mathrm{loc})$ satisfy the conditions (a) and (b) in Theorem 2. Then, for any function $f \in L_{p,w}(\mathrm{loc})$, we have*

$$\mathrm{st}_A\text{-}\lim_n \left(\sup_{x\in\mathbb{R}} \frac{\|T_n f - f; L_p(x-h, x+h)\|}{w^*(x)} \right) = 0,$$

where w^* is a weight function such that $\lim_{|x|\to\infty} \frac{1+x^2}{w^*(x)} = 0$.

Proof. By hypothesis, given $\varepsilon > 0$, there exists x_0 such that for all x with $|x| \geq x_0$ we have

$$\frac{1+x^2}{w^*(x)} < \varepsilon. \tag{12}$$

Let $f \in L_{p,w}(\mathrm{loc})$ and

$$a_n := \|T_n f - f; L_{p,w}(|x| > x_0)\|.$$

By (a) the set $E := \{n \in \mathbb{N} : \|T_n\| \leq H\}$ has A-density one. Hence we have $a_n \leq M$ for all $n \in E$. By Lusin's theorem [25] we can find a continuous function φ on $[-x_0 - h, x_0 + h]$ such that

$$\|f - \varphi; L_p(-x_0 - h, x_0 + h)\| < \varepsilon. \tag{13}$$

Now define the function G by

$$G(x) := \begin{cases} \varphi(-x_0 - h), & x \leq -x_0 - h \\ \varphi(x), & |x| \leq x_0 + h \\ \varphi(x_0 + h), & x \geq x_0 + h. \end{cases}$$

We see that G is continuous and bounded on the whole real axis. Now let $n \in E$ and $f \in L_{p,w}(\text{loc})$. Then by hypothesis and (13) we get

$$\begin{aligned} b_n &:= \|T_n f - f; L_{p,w}(-x_0, x_0)\| \le \|T_n(f - G); L_{p,w}(-x_0, x_0)\| \\ &\quad + \|T_n G - G; L_{p,w}(-x_0, x_0)\| + \|f - G; L_{p,w}(-x_0, x_0)\| \\ &\le (\|T_n\| + 1)\, \|f - \varphi; L_p(-x_0 - h, x_0 + h)\| \\ &\quad + \|T_n G - G; L_{p,w}(-x_0, x_0)\|, \end{aligned}$$

so it is easy to see that

$$b_n < (H+1)\varepsilon + \|T_n G - G; L_{p,w}(-x_0, x_0)\|. \tag{14}$$

On the other hand, a simple calculation shows, for any $n \in E$, that

$$\begin{aligned} u_n &:= \sup_{x \in \mathbb{R}} \frac{\|T_n f - f; L_p(x-h, x+h)\|}{w^*(x)} \\ &\le w(x_0) b_n + a_n \sup_{|x| \ge x_0} \frac{1+x^2}{w^*(x)}. \end{aligned} \tag{15}$$

It follows from (12), (14) and (15) that

$$\begin{aligned} u_n &< (H+1) w(x_0)\varepsilon + w(x_0)\, \|T_n G - G; L_{p,w}(-x_0, x_0)\| + H\varepsilon \\ &= K\varepsilon + w(x_0)\, \|T_n G - G; L_{p,w}(-x_0, x_0)\| \end{aligned}$$

where $n \in E$, $f \in L_{p,w}(\text{loc})$, and $K := (H+1)w(x_0) + H$. Given $r > 0$ choose $\varepsilon > 0$ such that $K\varepsilon < r$. Hence one can get

$$\sum_{k \in E\,:\, u_k \ge r} a_{nk} \le \sum_{k \in E\,:\, \|T_k G - G; L_{p,w}(-x_0, x_0)\| \ge \frac{r - K\varepsilon}{w(x_0)}} a_{nk}.$$

Now Lemma 1 implies $\mathrm{st}_A\text{-}\lim u_n = 0$. This completes the proof. □

3. Concluding remarks

In this section we exhibit an example of a sequence of positive linear operators for which the Korovkin theorem does not work but our A-statistical theorem works.

Following [15] we first define $P_n : L_{p,w}(\text{loc}) \to L_{p,w}(\text{loc})$ by

$$P_n(f;x) = \begin{cases} \dfrac{x^2}{(x+h)^2} f(x+h), & (2n-2)h \le x \le (2n+1)h \\ f(x), & \text{otherwise} \end{cases} \tag{16}$$

where $h > 0$. It is noted in [15] that (P_n) is sequence of positive linear operators such that (P_n) is uniformly bounded and

$$\lim_{n\to\infty} \|P_n f_v - f_v\|_{p,w} = 0$$

where $f_v(y) = y^v$ $(v = 0, 1, 2)$. It is also observed in [15] that there is a function f in $L_{p,w}(\text{loc})$ for which

$$\lim_{n\to\infty} \|P_n f - f\|_{p,w} = 0 \tag{17}$$

does not hold, but (17) holds for every $f \in L^k_{p,w}(\text{loc})$.

Now define $T_n : L_{p,w}(\text{loc}) \to L_{p,w}(\text{loc})$ by $T_n(f;x) = (1+u_n)P_n(f;x)$ where (P_n) is defined by (16) and (u_n) is an A-statistically null sequence but not convergent. We note that if A is non-negative regular matrix such that $\lim_n \max_k\{a_{nk}\} = 0$, then A-statistical convergence is stronger than convergence [22]. So it is possible to construct such an (u_n). Without loss of generality we may assume that (u_n) is a non-negative. Hence (T_n) satisfies all conditions of our Theorem 2. So we have, for every $f \in L^k_{p,w}(\text{loc})$, that

$$\text{st}_A\text{-}\lim_n \|T_n f - f\|_{p,w} = 0$$

but $\{\|T_n f - f\|_{p,w}\}$ does not tend to zero.

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