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Controllability of semilinear stochastic evolution equations with time delays

By P. BALASUBRAMANIAM (Gandhigram) and J. P. DAUER (Chattanooga)

Abstract. Controllability of semilinear stochastic evolution equations with time delays is studied by using Caratheodory successive approximate solutions.

1. Introduction

Fixed point techniques are widely used for analyzing the controllability of nonlinear systems in finite and infinite dimensional Banach spaces. Anichini [2], Dauer [7] and Dauer *et al.* [9] studied the controllability of classical nonlinear systems by means of Schaefer's theorem, Fan's theorem, and Leray–Schauder's theorem, respectively. Several authors have used semigroup theory to extend classical finite dimensional controllability results to infinite dimensional spaces for evolution equations with bounded and unbounded operators in Banach spaces (see [4], [8]).

Semigroup theory gives a unified treatment of a wide class of stochastic parabolic, hyperbolic and functional differential equations, and much effort has been devoted to the study of the controllability of such evolution equations (see, [16]). Stochastic control theory is a stochastic generalization of classical control theory, and controllability of linear stochastic

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systems has been one of the well-known problems discussed in the literatures [3], [18], [19]. Recently the controllability of linear stochastic systems has been extended to semilinear stochastic evolution equations in Hilbert space where the semilinear term depends on the probability distribution $\mu(t)$ (see, [5]). If the nonlinear term does not depend on $\mu(t)$, then the process determined is a standard Markov process and there are numerous papers in the literature discussing the stability of such stochastic equations in Hilbert spaces (for details see [1], [10], [12]).

As an example of such a control system, consider a stochastic model for drug distribution in a closed biological system with a simplified heart, one organ or capillary bed, and recirculation of the blood with a constant rate of flow. Here the heart is considered as a mixing chamber of constant volume as described in [17]. Drug concentration in the plasma in given areas of the system is assumed to be a random function of time. Assume that for $t \geq 0, x_1(s,t;w)$ is the concentration at time t in moles per unit volume at points in the capillary $w \in \Omega$. Here Ω is the supporting set of a complete probability measure space (Ω, A, P) with A being the σ-algebra and P is the probability measure.

The heart is considered to be a mixing chamber of constant volume V given by

$$V = \frac{V_e}{\ln(1 + V_e/V_r)},$$

where V_r is the residual volume in the heart and V_e is the injection volume. It is assumed that an initial injection is given at the entrance of the heart resulting in a concentration $x(t)$, $0 \leq t \leq t_1$, of the drug in plasma entering the heart, where t_1 is the duration of injection.

Let the time required for the blood to flow from the heart exit to the entrance of the organ be $t > 0$, and also let t be the time required for blood to flow from the exit of the organ to the heart entrance. Drug concentration in the plasma leaving the heart $x(t;w)$ satisfies the integral equation (see [6])

$$x(t;w) = G(t) + \int_0^t K(s, x(s;w); w)ds,$$

where

$$G(t) = \int_0^{T(t)} \frac{C}{V} x(s)ds, \quad T(t) = \begin{cases} t, & 0 \le t \le t_1, \\ t_1, & t \ge t_1, \end{cases}$$

$$K(s, x(s; w); w) = \frac{-C}{V} x(s; w) - x_1(l, s - t; w),$$

and $x_1(l, s; w) = 0$ if $s < 0$. Here C is the constant volume flow rate of plasma in the capillary bed and $x_1(l, s; w)$ is the drug concentration in the plasma leaving the organ at time s. The mild solutions are in the form of stochastic integral equations.

The objective of this paper is to derive the controllability conditions of the semilinear stochastic evolution equation with dealys in Hilbert space. The Caratheodory successive approximate solution is employed to get the suitable controllability conditions. The considered system is an abstract formulation of a stochastic partial differential equation (see [14]).

2. Preliminaries

Consider a class of delay stochastic evolution equations

$$\begin{aligned} dx(t) = {} & \left[Ax(t) + (Bu)(t) + f(x(t), x(t - \tau(t)))\right] dt \\ & + g(x(t), x(t - \tau(t)))dw(t), \quad t \in J = [0, T], \end{aligned} \tag{2.1}$$

$$x(t) = \phi(t), \quad -r \le t \le 0, \tag{2.2}$$

where A generally unbounded, generates a strongly continuous semigroup $\{ S(t),\ t \ge 0 \}$ of bounded linear operators over a separable (real) Hilbert space H with inner product $\langle \cdot\, , \cdot \rangle$ and norm $\| \cdot \|$. The state $x(\cdot)$ takes the values in the Hilbert space H, the control function $u(\cdot)$ is the stochastic process defined in a Hilbert space U of admissible control functions. B is a bounded linear operator from U into H. The functions $f(x, y)$ and $g(x, y)$ are bounded nonlinear functions satisfying certain given Lipschitz conditions and linear growth conditions and $w(\cdot)$ is a Hilbert space valued Q-Wiener process. Let K be another separable (real) Hilbert space and $w(t)$, $t \ge 0$, be a K-valued Wiener process with mean zero and covariance operator Q, with $\operatorname{tr} Q < \infty$ (tr denotes the trace of the operator), defined by

$$E\langle w(t), g\rangle \langle w(s), h\rangle = (t \wedge s)\langle Qg, h\rangle,$$

for every $g, h \in K$. Here $\langle \cdot\,,\,\cdot\rangle$ denotes the inner product on the space K. Let $K_Q \subset K$ be the closure of $Q^{1/2}K$ with respect to the norm $\|\cdot\|_Q$ defined by

$$\|Q^{\frac{1}{2}}k\|_Q^2 = \langle Q^{\frac{1}{2}}k, Q^{\frac{1}{2}}k\rangle_Q = \langle k, k\rangle, \quad k \in K.$$

Assume $V \subset H$ be a densely imbedding Banach subspace, and suppose that $A : V \to V^*$, the dual of V, is bounded.

For the existence of mild solution of (2.1)–(2.2) assume the following three conditions on f, g, A, and μ:

i) $f : H \times H \to H$ and $g : H \times H \to L(K_Q, H)$, the space of all linear bounded operators from K_Q into H, are two measurable mappings and there exists a positive constant L such that

$$\|f(x,y)\| \vee \|g(x,y)\| \le L(1 + \|x\| + \|y\|), \qquad \text{for every } x, y \in H, \text{ and}$$

$$\|f(x,y) - f(x^*,y^*)\| \vee \|g(x,y) - g(x^*,y^*)\| \le L(\|x - x^*\| + \|y - y^*\|), \qquad \text{for all } x, y, x*, y* \in H.$$

ii) $A : V \to V^*$ is coercive such that it generates an analytic semigroup $\{S_t,\, t \ge 0\}$ on H.

iii) For arbitrarily given $T > 0$, there exist constants $\theta = \theta(T) > 0$ and $K(T) > 0$ such that for any sufficiently large positive integer n

$$\mu\Big\{t : 0 < \tau(t) < \frac{1}{n},\ 0 \le t \le T\Big\} \le \frac{K(T)}{n^\theta},$$

where μ is the Lebesgue measure on $\mathbb{R}^+$.

Let $\tau(\cdot)$ be a continuous non-negative function on $\mathbb{R}^+$ and define

$$r = \sup\{\tau(t) - t : t \ge 0\} < \infty.$$

Let $M^2([-r,0], H)$ denote the family of all continuous H-valued stochastic processes $\phi(t), -r \le t \le 0$ are all $\Im_0$ measurable and

$$\sup_{-r \le t \le 0} \{E\|\phi(t)\|^2,\ -r \le t \le 0\} < \infty.$$

Then for any $T > 0$, an H-valued stochastic process $x(t)$, $t \in [-r, T]$, defined on some given probability space $(\Omega, \Im, \Im_t, \mathbf{P})$ has the following mild solution (see, Liu [13]), where $x(\cdot)$ adapted to $\Im_t$, is measurable and almost surely $\int_0^T \|x(s)\|^2 ds < \infty$,

$$\begin{aligned} x(t) &= S_t\phi(0) + \int_0^t S_{t-\eta}[(Bu)(\eta) + f(x(\eta), (\eta - \tau(\eta)))]d\eta \\ &\quad + \int_0^t S_{t-\eta} g(x(\eta), x(\eta - \tau(\eta)))dw(\eta), \quad t \in J \text{ a. e.}, \\ x(t) &= \phi(t), \quad -r \le t \le 0. \end{aligned}$$

Definition 2.1. The stochastic evolution equation (2.1)–(2.2) is said to be *controllable* on the interval J, if for every continuous initial H-valued stochastic process ϕ defined on $[-r, 0]$, there exists a stochatic control process $u \in U$ which is adapted to the filtration $\{\Im_t\}_{t \ge 0}$ such that the solution $x(\cdot)$ of (2.1)–(2.2) satisfies $x(T) = x_1$, where x_1 and T are preassigned terminal state and time, respectively.

3. Main result

Theorem 3.1. *Suppose that the conditions (i)–(iii) hold and the linear operator W from U into H defined by*

$$Wu = \int_0^T S_{T-s}Bu(s)ds$$

has an invertible operator W^{-1} defined on $H \setminus \operatorname{Ker} W$ (see [11]). In addition, suppose there exist positive constants N_1, N_2 such that

$$\|B\|^2 \le N_1 \quad \text{and} \quad \|W^{-1}\|^2 \le N_2.$$

Then the system (2.1)–(2.2) is controllable on J.

Proof. Using the above hypothesis define the control

$$u(t) = W^{-1}\Big\{x_1 - S_T\phi(0) - \int_0^T S_{T-\eta} f(x(\eta), x(\eta - \tau(\eta)))d\eta$$

$$- \int_0^T S_{T-\eta} g(x(\eta), x(\eta - \tau(\eta))) dw(\eta) \Big\} (t).$$

Now it is shown that, when using this control, the operator defined by

$$\begin{aligned}
(\Phi x)(t) = S_t \phi(0) &+ \int_0^t S_{t-\theta} B W^{-1} \Big\{ x_1 - S_T \phi(0) \\
&- \int_0^T S_{T-\eta} f(x(\eta), x(\eta - \tau(\eta))) d\eta \\
&- \int_0^T S_{T-\eta} g(x(\eta), x(\eta - \tau(\eta))) dw(\eta) \Big\} (\theta) d\theta \\
&+ \int_0^t S_{t-\eta} f(x(\eta), x(\eta - \tau(\eta))) d\eta \\
&+ \int_0^t S_{t-\eta} g(x(\eta), x(\eta - \tau(\eta))) dw(\eta), \quad \text{for } t \in J,
\end{aligned}$$

has a convergent solution (see [13]). Clearly $(\Phi x)(0) = \phi(0)$, which means that the control $u(\cdot)$ steers the semilinear evolution equation from the initial state ϕ to x_1 in time T provided a convergent solution of the nonlinear operator Φ can be obtained.

It is enough to prove the existence of the solution by using the Caratheodory approximate solution which is defined as follows. Fix $T > 0$, for arbitrary, $\nu \geq 1$, $\phi(.) \in C([-r, T], H)$ and all $n \geq \frac{2}{r}$, define

$$\begin{aligned}
\Phi x^n(t) &= \phi(t), \quad -r \leq t \leq 0, \\
\Phi x^n(t) &= S_t^n \phi(0) + \int_0^t 1_{D_n^c}(\theta) S_{t-\theta}^n B W^{-1} \Big\{ x_1 - S_T^n \phi(0) \\
&\quad - \int_0^T S_{T-\eta}^n f\left(x^n \left(\eta - \frac{1}{n^\nu} \right), x^n(\eta - \tau(\eta)) \right) d\eta \\
&\quad - \int_0^T S_{T-\eta}^n g\left(x^n \left(\eta - \frac{1}{n^\nu} \right), x^n(\eta - \tau(\eta)) \right) dw(\eta) \Big\} (\theta) d\theta \\
&\quad + \int_0^t 1_{D_n}(\theta) S_{t-\theta}^n B W^{-1} \Big\{ x_1 - S_T^n \phi(0) \\
&\quad - \int_0^T S_{T-\eta}^n f\left(x^n \left(\eta - \frac{1}{n^\nu} \right), x^n \left(\eta - \tau(\eta) - \frac{1}{n^\nu} \right) \right) d\eta
\end{aligned}$$

$$- \int_0^T S^n_{T-\eta} g\left(x^n\left(\eta - \frac{1}{n^\nu}\right), x^n\left(\eta - \tau(\eta) - \frac{1}{n^\nu}\right)\right) dw(\eta)\Big\}(\theta) d\theta$$
$$+ \int_0^t 1_{D_n^c}(\eta) S^n_{t-\eta} f\left(x^n\left(\eta - \frac{1}{n^\nu}\right), x^n(\eta - \tau(\eta))\right) d\eta$$
$$+ \int_0^t 1_{D_n^c}(\eta) S^n_{t-\eta} g\left(x^n\left(\eta - \frac{1}{n^\nu}\right), x^n(\eta - \tau(\eta))\right) dw(\eta)$$
$$+ \int_0^t 1_{D_n}(\eta) S^n_{t-\eta} f\left(x^n\left(\eta - \frac{1}{n^\nu}\right), x^n\left(\eta - \tau(\eta) - \frac{1}{n^\nu}\right)\right) d\eta$$
$$+ \int_0^t 1_{D_n}(\eta) S^n_{t-\eta} g\left(x^n(\eta - \frac{1}{n^\nu}), x^n\left(\eta - \tau(\eta) - \frac{1}{n^\nu}\right)\right) dw(\eta),$$

where $S^n_{t-\eta} = S_{t-\eta+\frac{1}{n^\nu}}$, $D_n = \{t : \tau(t) < \frac{1}{n^\nu}, \text{ for } 0 \le t \le T\}$ and $D_n^c = [0,T] - D_n$. Here 1_{D_n} denotes the indicator function on the set $D_n \subset R^+$. Here $\Phi x^n(t)$ can be determined by stepwise iterated Ito integrals over the intervals $[0, \frac{1}{n^\nu}], [\frac{1}{n^\nu}, \frac{2}{n^\nu}], \ldots,$ etc. Let $C(J;H)$ denote the space of H-valued continuous functions on J with the usual supremum norm.

It will be shown the sequence $\{\Phi x^n(t)\}$ of approximate solutions converges a.s. in the space $C(J;H)$ to the mild solution $x(\cdot)$ of equation (2.1)–(2.2). Under the assumption of priori boundedness of $E(\sup_{0\le s\le \eta} \|x^n(s)\|^2) < C(T)$, it follows that

$$E\|\Phi x^n(t)\|^2 = \|\phi(t)\|^2, \quad -r \le t \le 0,$$

$$E\|\Phi x^n(t)\|^2 \le \|S^n_t \phi(0)\|^2 + \int_0^t \Big\| 1_{D_n^c}(\theta) S^n_{t-\theta} B W^{-1}\Big\{x_1 - S^n_T \phi(0)$$
$$- \int_0^T S^n_{T-\eta} f\left(x^n\left(\eta - \frac{1}{n^\nu}\right), x^n(\eta - \tau(\eta))\right) d\eta$$
$$- \int_0^T S^n_{T-\eta} g\left(x^n\left(\eta - \frac{1}{n^\nu}\right), x^n(\eta - \tau(\eta))\right) dw(\eta)\Big\}(\theta)\Big\|^2 d\theta$$
$$+ \int_0^t \Big\| 1_{D_n}(\theta) S^n_{t-\theta} B W^{-1}\Big\{x_1 - S^n_T \phi(0)$$
$$- \int_0^T S^n_{T-\eta} f\left(x^n\left(\eta - \frac{1}{n^\nu}\right), x^n\left(\eta - \tau(\eta) - \frac{1}{n^\nu}\right)\right) d\eta$$
$$- \int_0^T S^n_{T-\eta} g\left(x^n\left(\eta - \frac{1}{n^\nu}\right), x^n(\eta - \tau(\eta) - \frac{1}{n^\nu})\right) dw(\eta)\Big\}(\theta)\Big\|^2 d\theta$$

$$
\begin{aligned}
&+ \int_0^t \left\| 1_{D_n^c}(\eta) S_{t-\eta}^n f\left(x^n\left(\eta - \frac{1}{n^\nu}\right), x^n\left(\eta - \tau(\eta)\right)\right)\right\|^2 d\eta \\
&+ \int_0^t \left\| 1_{D_n^c}\left(\eta\right) S_{t-\eta}^n g\left(x^n\left(\eta - \frac{1}{n^\nu}\right), x^n(\eta - \tau(\eta))\right) dw(\eta)\right\|^2 \\
&+ \int_0^t \left\| 1_{D_n}(\eta) S_{t-\eta}^n f\left(x^n\left(\eta - \frac{1}{n^\nu}\right), x^n\left(\eta - \tau(\eta) - \frac{1}{n^\nu}\right)\right)\right\|^2 d\eta \\
&+ \int_0^t \left\| 1_{D_n}(\eta) S_{t-\eta}^n g\left(x^n\left(\eta - \frac{1}{n^\nu}\right), x^n\left(\eta - \tau(\eta) - \frac{1}{n^\nu}\right)\right) dw(\eta)\right\|^2 \\
\le{}& C_1'(T) + N_1 N_2 C_2'(T) \int_0^t 1_{D_n^c}(\theta) \Big\{ \|x_1\| + C_1'(T) \\
&+ 2C_2'(T)T\left[1 + E\left\| x^n\left(\eta - \frac{1}{n^\nu}\right)\right\|^2 + E\left\| x^n(\eta - \tau(\eta))\right\|^2\right]\Big\}(\theta) d\theta \\
&+ N_1 N_2 C_2'(T) \int_0^t 1_{D_n}(\theta) \Big\{ \|x_1\| + C_1'(T) \\
&+ 2C_2'(T)T\left[1 + E\left\| x^n\left(\eta - \frac{1}{n^\nu}\right)\right\|^2 + E\left\| x^n\left(\eta - \tau(\eta) - \frac{1}{n^\nu}\right)\right\|^2\right]\Big\}(\theta) d\theta \\
&+ C_2'(T) \int_0^t 1_{D_n^c}(\eta) \left\{ 1 + E\left\| x^n\left(\eta - \frac{1}{n^\nu}\right)\right\|^2 + E\left\| x^n(\eta - \tau(\eta))\right\|^2\right\} d\eta \\
&+ C_2'(T) \int_0^t 1_{D_n}(\eta) \left\{ 1 + E\left\| x^n\left(\eta - \frac{1}{n^\nu}\right)\right\|^2 + E\left\| x^n\left(\eta - \tau(\eta) - \frac{1}{n^\nu}\right)\right\|^2\right\} d\eta \\
\le{}& C_1'(T) + 2C_2'(T)T + C_2''(T) \int_0^t E\left\| x^n\left(\eta - \frac{1}{n^\nu}\right)\right\|^2 d\eta \\
&+ C_2''(T) \int_0^t 1_{D_n^c}(\eta) E\left\| x^n(\eta - \tau(\eta))\right\|^2 d\eta \\
&+ C_2'(T) \int_0^t 1_{D_n^c}(\eta) \left\{ 1 + E\left\| x^n\left(\eta - \frac{1}{n^\nu}\right)\right\|^2 + E\left\| x^n(\eta - \tau(\eta))\right\|^2\right\} d\eta \\
&+ C_2''(T) \int_0^t 1_{D_n}(\eta) E\left\| x^n(\eta - \tau(\eta))\right\|^2 d\eta \\
&+ C_2'(T) \int_0^t 1_{D_n}(\eta) \left\{ 1 + E\left\| x^n\left(\eta - \frac{1}{n^\nu}\right)\right\|^2 + E\left\| x^n\left(\eta - \tau(\eta) - \frac{1}{n^\nu}\right)\right\|^2\right\} d\eta
\end{aligned}
$$

$$
\begin{aligned}
&+ C_2''(T)\int_0^t 1_{D_n}(\eta) E\left\| x^n\left(\eta - \tau(\eta) - \frac{1}{n^\nu}\right)\right\|^2 d\eta \\
&\le C_3'(T) + C_4'(T)\int_0^t E\left\| x^n(\eta)\right\|^2 d\eta \\
&\quad + C_5'(T)\left\{\mu(D_n^c) + \int_0^t 1_{D_n^c}(\eta)\sup_{0\le s\le \eta} E(\|x^n(s)\|^2) d\eta\right\} \\
&\quad + C_6'(T)\left\{\mu(D_n) + \int_0^t 1_{D_n}(\eta)\sup_{0\le s\le \eta} E\big(\|x^n(s)\|^2\big) d\eta\right\} \\
&\le C_1'''(T) + C_2'''(T)\int_0^t E\Big(\sup_{0\le s\le \eta}\|x^n(s)\|^2\Big) d\eta \\
&\le C_1(T).
\end{aligned}
$$

Further, a known estimate (see Pazy [15, p. 74]) implies for any $0 < \alpha < 1$,

$$
\begin{gathered}
E\|\Phi x^n(t) - \Phi x^n(s)\|^2 \le C_2(\alpha, T)(t-s)^\alpha + C_3(T)(t-s), \\
0 \le s \le t \le T.
\end{gathered}
\tag{3.1}
$$

Next it is shown that $\Phi x^n(t)$ converges to a limit in $L^2(\Omega, H)$ for each $t \in J$. To do so let $m > n \ge 2/r$ and note that $\|S(t)\| \le M\exp(wT)$ for all $t \in J$, then conditions (i)–(iii) imply that there exist positive constants $M_1(T), M_2'(T), M_2(T), \ldots, M_{10}(T)$ such that

$$
\begin{aligned}
&E\Big(\sup_{0\le\eta\le T}\|\Phi x^m(s) - \Phi x^n(s)\|^2\Big) \\
&\le 4M_1(T)\mu(D_n - D_m) + 4M_2'(T)\int_0^t E\|x^m(\eta) - x^n(\eta)\|^2 d\eta \\
&\quad + 4\left\{M_5(T)T\left(\frac{1}{n^\nu} - \frac{1}{m^\nu}\right) + M_6(\alpha, T)T\left(\frac{1}{n^\nu} - \frac{1}{m^\nu}\right)^\alpha\right\} \\
&\quad + 4M_2(T)\int_0^t \sup_{0\le s\le T} E\|x^m(s) - x^n(s)\|^2 d\eta \\
&\quad + 4M_3(T)\int_0^t E\|x^m(\eta) - x^n(\eta)\|^2 d\eta \\
&\quad + 4\left\{M_5(T)T\left(\frac{1}{n^\nu} - \frac{1}{m^\nu}\right) + M_6(\alpha, T)T\left(\frac{1}{n^\nu} - \frac{1}{m^\nu}\right)^\alpha\right\}
\end{aligned}
$$

$$\leq \Big\{ M_1(T)\mu(D_n - D_m) + M_8(T)T\left(\frac{1}{n^\nu} - \frac{1}{m^\nu}\right)$$
$$+ M_9(\alpha, T)T\left(\frac{1}{n^\nu} - \frac{1}{m^\nu}\right)^\alpha \Big\}$$
$$\times M_{10}(T)\int_0^T \sup_{0\leq s\leq T} E\|x^m(s) - x^n(s)\|^2 d\eta. \tag{3.2}$$

Noting that $\mu(D_n - D_m) \to 0$ as $n, m \to \infty$, it follows that $\{\Phi x^n(t)\}$ is Cauchy sequence in $L^2(\Omega, C(J; H))$. Denote this limit in $L^2(\Omega, C(J; H))$ by $x(\cdot)$. A Borel–Cantelli argument easily gives that there exists a subsequence, say $\{\Phi x^{m_i}(t)\}$, which converges to $x(t)$ uniformly in $t \in J$ almost surely. Therefore, $x(\cdot)$ is a $\{\Im_t\}$-adapted continuous H-valued process. Moreover, letting $n \to \infty$ in (3.2) it follows that

$$E\Big(\sup_{0\leq s\leq T} \|x(s) - \Phi x^n(s)\|^2\Big)$$
$$\leq C_4(T)\left\{\left[\frac{1}{n^\nu} + \frac{1}{n^{\alpha\nu}}\right] + \mu\left[t : 0 < \tau(t) < \frac{1}{n^\nu},\ t \in J\right]\right\} \to 0.$$

Now letting $n \to \infty$ in (3.1), the conclusion is immediately obtained. □

4. Example

Consider a stochastic Burgers-type equation with constant time delay (that is $\delta(t) = 2h > 0$).

$$\frac{dY_t(\xi)}{dt} = \nu\frac{\partial^2 Y_t(\xi)}{\partial\xi^2} + \frac{1}{2}\frac{\partial Y_t^2(\xi)}{\partial\xi} + Y_{t-2h}(\xi) + (Bu)(t)$$
$$+ 2t^3 e^{-\eta\lambda_0 t}\frac{dw_t(\xi)}{dt}, \quad t \geq 0,\ \xi \in [0, 1], \tag{4.1}$$

$$Y_t(0) = Y_t(1) = 0, \quad t > 0, \tag{4.2}$$

$$Y_t(\xi) = \phi(t, \xi), \quad t \in\ [-2h, 0],\ \xi \in [0, 1]. \tag{4.3}$$

Here $\nu > 0$ and $\phi(t, \xi) : [-2h, 0] \times \Omega \to X = L^2[0, 1]$ is a suitable $\Im_0$-measurable process. Assume the following three conditions:

(1) Let $\operatorname{dom} A = H^2(0,1) \cap H_0^1(0,1)$ and $(A\psi)\xi = \nu \frac{\partial^2 Y_t(\xi)}{\partial \xi^2}$, $\psi \in \operatorname{dom} A$, and let B be a bounded linear operator from the control space $U = L^2(0,1)$ into a Hilbert space H satisfying the hypothesis stated in the Theorem 3.1.

(2) Define the functions

$$f(Y_t(\xi), Y_{t-2h}(\xi)) = \frac{1}{2} \frac{\partial Y_t^2(\xi)}{\partial \xi} + Y_{t-2h}(\xi),$$

$$g(Y_t(\xi), Y_{t-2h}(\xi)) = 2t^3 e^{-\eta \lambda_0 t}.$$

(3) Let $w_t(\xi)$ is a Wiener process with a bounded, continuous covariance $q(\xi, \zeta)$, namely, there exists a constant $c > 0$ such that $|q(\xi, \zeta)| \leq c$, and further denote

$$\lambda_0 = \inf_{y \in D(A)} \frac{|\nabla y(\xi)|^2}{|y(\xi)|^2}.$$

Then system (4.1)–(4.3) has an abstract formulation as the following semilinear delay stochastic equation in a Hilbert space H

$$dx(t) = \big[Ax(t) + (Bu)(t) + f(x(t), x(t - \tau(t)))\big]\, dt + g(x(t), x(t - \tau(t))) \frac{dw(t)}{dt}, \quad t \in J = [0, T], \tag{4.4}$$

$$x(t) = \phi(t), \quad -2h \leq t \leq 0, \tag{4.5}$$

where the linear operator A is the infinitesimal generator of a strongly continuous semigroup e^{At}, $t \geq 0$, in H. Thus (4.4)–(4.5) has a unique solution (see [14]). All the conditions of Theorem 3.1 are satisfied, and it follows that system (4.1)–(4.3) is completely controllable on J.

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P. BALASUBRAMANIAM
DEPARTMENT OF MATHEMATICS
GANDHIGRAM RURAL INSTITUTE
DEEMED UNIVERSITY
GANDHIGRAM – 624 302, TAMIL NADU
INDIA

E-mail: pbalgri@rediffmail.com

J. P. DAUER
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF TENNESSEE AT CHATTANOOGA
615 MCCALLIE AVENUE
CHATTANOOGA, TN 37403-2598
U.S.A.

E-mail: Jeral-Dauer@utc.edu

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