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Locally conformal manifolds endowed with a skew-symmetric vector field

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Abstract. Geometrical and structural properties are proved for locally conformal almost cosymplectic manifolds which are structured by the presence of a skew-symmetric conformal vector field.

1. Introduction

Let $M(\Omega, \phi, \eta, \xi, g)$ be a $(2m+1)$-dimensional generalised Kenmotsu manifold (or K-manifold) [15]. A general discussion of the geometrical structures which appear here and in the sequel can be found in the standard references [17] and [22] which also contain more background information; for more specific reading and additional references, see in particular also [12]. For the convenience of the reader, we remind in an appendix the complete list of axioms for the generalised Kenmotsu manifolds we consider, together with some properties which will be called upon throughout the paper.

In the case under consideration, the tensor fields satisfy the following relations:

$$\phi^2 = -\operatorname{Id} + \eta \otimes \xi, \quad \phi\xi = 0, \quad \eta \circ \phi = 0, \quad \eta(\xi) = 1. \tag{1}$$

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We also assume that the Reeb vector field ξ satisfies the equations:

$$\nabla\xi = -dp + \eta \otimes \xi, \qquad \nabla^2\xi = \eta \wedge dp, \tag{2}$$

where dp stands for the soldering form $dp = \sum_{A=0}^{2m} \omega^A \otimes e_A$, which is the canonical vector valued 1-form of M. Moreover, we also note the following relation

$$g(\phi Z, \phi Z') = g(Z, Z') - \eta(Z)\eta(Z'), \qquad Z, Z' \in \Xi(M). \tag{3}$$

By (2) one has that

$$\nabla_Z\xi = -Z + \eta(Z)\xi, \tag{4}$$

and one finds that

$$\operatorname{div}\, \xi = -2m = \text{const.} \tag{5}$$

In general, if $\mathcal{O} = \operatorname{vect}\{\, \xi,\ e_A \mid A = 1, \ldots, 2m\}$ is an adapted vectorial frame over M and $\mathcal{O}^* = \operatorname{covect}\{\, \eta,\ \omega^A \mid A = 1, \ldots, 2m\}$ its associated coframe, we recall the general definition

$$\operatorname{div}\, Z = \operatorname{Tr}(\nabla Z) = \sum_{A=1}^{2m} \omega^A(\nabla_{e_A} Z) + \eta(\nabla_\xi Z).$$

Hence, by (5) one may say that the Reeb vector field ξ defines a homothetie. Next, denote by θ_B^A $(A, B \in \{0, 1, \ldots, 2m\})$ the connection forms. One finds

$$\omega^A = \theta_A^0, \qquad (1 \leq A \leq 2m), \tag{6}$$

and, consequently,

$$d\eta = 0, \qquad \nabla_\xi\xi = 0, \qquad \mathcal{L}_\xi\eta = 0. \tag{7}$$

This shows that M is endowed with an almost contact structure and from the third equation of (7), it follows according to [10] that η is a Pfaffian transformation [6]. Moreover, one further finds that

$$\nabla^2\xi = \eta \wedge dp \quad \Longrightarrow \quad -\frac{1}{2m}\operatorname{Ric}(\xi) = 1, \tag{8}$$

and

$$\mathcal{L}_\xi\nabla\xi = 0. \tag{9}$$

This proves that ξ is both an exterior concurrent vector field and an affine vector field. Since by virtue of (4) one also has that

$$\langle \nabla_Z \xi, Z' \rangle = \langle \nabla_{Z'} \xi, Z \rangle, \quad Z, Z' \in \Xi(M),$$

one sees that ξ is an Okumura gradient [14].

It is well known that various special types of vector fields play a distinguished role in differential geometry, as they often induce remarkable geometric properties on the base manifold. The skew symmetric conformal vector fields [19] are one kind of such peculiar vector fields. Skew symmetric conformal vector fields have been studied and their geometric consequences investigated for several special manifolds, e.g. on some almost cosymplectic, para Sasakian, and para co-Kaehlerian manifolds, amongst others. In the present paper we continue this line of investigation and study the geometrical and structural consequences of the presence of a skew symmetric conformal vector field on a generalised Kenmotsu manifold.

For a comprehensive survey of related results in this context, and a summary of the general theory of skew symmetric conformal vector fields, we refer to [12]. In particular, [12] also contains a discussion of the existence theory for skew symmetric conformal vector fields, where it is argued that the existence of such a vector field is determined by an exterior differential system in involution, depending on 2 arbitrary functions of one argument.

Thus, in Section 3 we consider a skew symmetric conformal vector field C having $-\xi$ as generative vector field, therefore satisfying

$$\nabla C = f dp + \xi \wedge C, \qquad f \in \Lambda^0 M. \tag{10}$$

In the sequel we agree to call C and f the principal vector field and the principal scalar respectively. We show that C, ϕC, and ξ define an almost commutative triple. It is proved that C is a 2-exterior concurrent vector and C, ϕC, and ξ define a 3-foliation. Various properties involving the Lie derivatives of the fundamental 2-form Ω and of C, ϕC, and ξ are discussed. Finally we also derive some properties of the hypersurface M_ξ normal to ξ.

2. Preliminaries

Let (M, g) be a Riemannian C^∞-manifold and let ∇ be the covariant differential operator defined with respect to the metric tensor g. We assume in the sequel that M is oriented and that ∇ is the Levi–Civita connection. Let $\Gamma TM = \Xi(M)$ be the set of sections of the tangent bundle TM, and

$$\flat : TM \xrightarrow{\flat} T^*M \quad \text{and} \quad \sharp : TM \xleftarrow{\sharp} T^*M$$

the classical isomorphisms defined by g (i.e. $^\flat$ is the index lowering operator, and $^\sharp$ is the index raising operator).

Following [17], we denote by

$$A^q(M, TM) = \Gamma\,\mathrm{Hom}(\Lambda^q TM, TM),$$

the set of vector valued q-forms ($q < \dim M$), and we write for the covariant derivative operator with respect to ∇

$$d^\nabla : A^q(M, TM) \to A^{q+1}(M, TM). \tag{11}$$

It should be noticed that in general $d^{\nabla^2} = d^\nabla \circ d^\nabla \neq 0$, unlike $d^2 = d \circ d = 0$. We also note that througout the paper the term $\nabla^2 Z$ is defined as $d^\nabla(\nabla Z)$, for any vector field $Z \in \Xi(M)$, thus in particular also for the vector fields C and ξ. We denote by $dp \in A^1(M, TM)$ the canonical vector valued 1-form of M, which is also called the soldering form of M [6]. Since ∇ is symmetric one has that $d^\nabla(dp) = 0$.

Let $\mathcal{O} = \mathrm{vect}\{e_A \mid A = 0, \ldots, 2m\}$ be a local field of adapted vectorial frames over M and let $\mathcal{O}^* = \mathrm{covect}\{\omega^A \mid A = 0, \ldots, 2m\}$ be its associated coframe. If we put $e_0 = \xi$ and $\omega^0 = \eta$, then the soldering form dp is expressed by

$$dp = \sum_{A=0}^{2m} \omega^A \otimes e_A = \eta \otimes \xi + \sum_{A=1}^{2m} \omega^A \otimes e_A,$$

and we recall that E. Cartan's structure equations can be written as

$$\nabla e_A = \sum_{B=0}^{2m} \theta_A^B \otimes e_B = \omega^A \otimes \xi + \sum_{B=1}^{2m} \theta_A^B \otimes e_B, \tag{12}$$

$$d\omega^A = -\sum_{B=0}^{2m} \theta_B^A \wedge \omega^B = -\eta \wedge \omega^A - \sum_{B=1}^{2m} \theta_B^A \wedge \omega^B, \tag{13}$$

$$d\theta_B^A = -\sum_{C=0}^{2m} \theta_B^C \wedge \theta_C^A + \Theta_B^A. \tag{14}$$

In the above equations θ (respectively Θ) are the local connection forms in the tangent bundle TM (respectively the curvature 2-forms on M).

3. The principal vector field

First, one calculates from (10), and making use of (2) and (12)–(14), that

$$\begin{cases} dC^A + \displaystyle\sum_{B=1}^{2m} C^B \theta_B^A = (f + C^0)\omega^A - C^A \eta, \\ \qquad\qquad dC^0 = (f + C^0)\eta - C^0 \eta. \end{cases} \tag{15}$$

Consequently, one has that

$$dC^0 = f\eta = \mathcal{L}_C \eta, \tag{16}$$

Now, in general, a vector field V satisfying

$$\nabla V = \lambda dp - v \otimes V, \tag{17}$$

for some $\lambda \in C^\infty M$, and with $v = V^\flat$, is called a torse forming vactor field [21]. Under these conditions, a vector field $\mathcal{T}$ whose covariant derivative satisfies

$$\nabla \mathcal{T} = f dp + \mathcal{T} \wedge V, \tag{18}$$

for some $f \in C^\infty M$, is called a skew symmetric conformal Killing vector field [12].

In the case under consideration, we observe that the first equation of (2) realizes (17) for $V = -\xi$ and $\lambda = -1$, and equation (10) agrees with (18) for $\mathcal{T} = C$ and $V = -\xi$.

In general, having a torse forming vector field V and a skew symmetric conformal Killing vector field $\mathcal{T}$ with generative V, it can be calculated [12] that

$$\nabla^2 V = d^{\nabla}(\nabla V) = (d\lambda + \lambda v) \wedge dp, \tag{19}$$

$$\nabla^2 \mathcal{T} = d^{\nabla}(\nabla \mathcal{T}) = (df - fv + \lambda\alpha) \wedge dp, \tag{20}$$

where $\alpha = \mathcal{T}^{\flat}$. This shows that both $\mathcal{T}$ and V are exterior concurrent vector fields [16]. In the general theory of exterior recurrent vector fields, it is shown [16] that the relations (19) and (20) imply that

$$d\lambda + \lambda v = cv, \tag{21}$$

$$df - fv + \lambda\alpha = c\alpha, \tag{22}$$

for some constant c.

In the present case, where $\lambda = -1$, it follows from (21) that $c = \lambda$. Consequently, (22) reduces to

$$df - fv = 0. \tag{23}$$

For $v = (-\xi)^{\flat} = -\eta$, (23) turns into

$$df + f\eta = 0, \tag{24}$$

where η is exact.

Setting $\|C\|^2 := 2l$, one derives from (15) and (24) that

$$dl = (f + C^0)C^{\flat} - 2l\eta, \tag{25}$$

and by exterior differentiation, one gets also that

$$dC^{\flat} = -2\eta \wedge C^{\flat}. \tag{26}$$

This shows that the Pfaffian $C^{\flat}$ is exterior recurrent by exterior differentiation, or also $d^{2\eta}C^{\flat} = 0$ (see [4]).

Next, one finds by (1) that

$$\nabla \phi C = (f + C^0)\phi dp - \xi \wedge \phi C, \tag{27}$$

and by the Lie bracket one gets by (2), (10) and (27) that

$$[C, \xi] = -f\xi, \qquad [\phi C, \xi] = 0, \qquad [\phi C, C] = 0. \tag{28}$$

It is well known [3] that the first commutator proves that ξ admits an infinitesimal transformation with generator C. Since moreover ϕC commutes with C and ξ, the 3 expressions in (28) together may be abbreviated by saying that ξ, C, and ϕC define an almost commutative triple (see e.g. [5] [11] [20]). Denoting by $\mathcal{D}_C = \{\xi, C, \phi C\}$ the 3-distribution defined by ξ, C, and ϕC, it follows from the above that, if the vector fields are linearly independent, $\mathcal{D}_C$ defines a 3-foliation.

Remark 3.1. By (16) and (24) one has that

$$\mathcal{L}_C \eta = d(i_C \eta) = dC^0 = f\eta = -df,$$

which shows that η is a relatively integral invariant of C [1].

By (10) one now calculates that

$$\langle \nabla_Z C, Z' \rangle + \langle \nabla_{Z'} C, Z \rangle = 2f \langle Z, Z' \rangle, \tag{29}$$

which expresses that C is a conformal vector field. Operating now on (10) by the exterior covariant operator d^∇, and taking into account (26) and (24), one gets

$$\nabla^2 C = C^\flat \wedge dp. \tag{30}$$

This shows that C is an exterior concurrent vector field [18] (see also [16]). Accordingly one has

$$\nabla^2 C = d^\nabla(\nabla C) = -\frac{1}{2m} \operatorname{Ric} C^\flat \wedge dp \quad \Longrightarrow \quad \operatorname{Ric}(C) = -2m. \tag{31}$$

Next we agree to call $C_f = fC$ the associated vector field of C. Operating successively by the operator d^∇ and taking into account (24) and (26), yields

$$\nabla^3 C_f = (\eta \wedge C^\flat) \wedge dp. \tag{32}$$

Then by reference to [16] one may say that the vector field C_f is 2-exterior concurrent.

We have thus proved the following

Theorem 3.1. *Let $M(\Omega, \phi, \eta, \xi, g)$ be a $(2m+1)$-dimensional generalised Kenmotsu manifold with Reeb vector field ξ (resp. covector $\xi^\flat = \eta$) and let C be the principal vector field on M. One then has the following properties:*

(i) *η is a relatively integral invariant of C;*

(ii) *ξ, C, and ϕC define an almost commutative triple;*

(iii) *the Pfaffian $C^\flat$ is exterior recurrent and has -2η as recurrence form;*

(iv) *the associated vector field C_f is 2-exterior concurrent and satisfies*

$$\nabla^3 C_f = (\eta \wedge C^\flat) \wedge dp.$$

4. The fundamental 2-form

In this section we discuss some properties of the local cosymplectic form Ω, therefore using the Lie derivative $\mathcal{L}_Z = i_Z d + d i_Z$, $Z \in \Xi(M)$.

The fundamental 2-form Ω on M can be expressed as,

$$\Omega = \sum_{i=1}^{m} \omega^i \wedge \omega^{i^*}, \qquad i^* = i + m. \tag{33}$$

Exterior derivation of (33), and replacing $d\omega^i$ end $d\omega^{i^*}$ using (13), yields

$$\begin{aligned} d\Omega = &\sum_{i=1}^{m} \Big(-\eta \wedge \omega^i - \sum_{B=1}^{2m} \theta^i_B \wedge \omega^B \Big) \wedge \omega^{i^*} \\ &- \sum_{i=1}^{m} \omega^i \wedge \Big(-\eta \wedge \omega^{i^*} - \sum_{B=1}^{2m} \theta^{i^*}_B \wedge \omega^B \Big). \end{aligned} \tag{34}$$

Taking into account the Kähler relations

$$\theta^i_j = \theta^{i^*}_{j^*}, \qquad \theta^{i^*}_j = \theta^{j^*}_i, \tag{35}$$

it follows from (34) that

$$d\Omega = -2\eta \wedge \Omega. \tag{36}$$

This shows that the pairing (Ω, η) defines a locally conformal cosymplectic structure.

Taking the Lie derivative of (33) w.r.t. ξ gives

$$\begin{aligned}\mathcal{L}_\xi \Omega = \sum_{i=1}^{m} \Big(-\omega^i - \sum_{B=1}^{2m} \theta_B^i(\xi)\omega^B \Big) \wedge \omega^{i^*} \\ + \sum_{i=1}^{m} \omega^i \wedge \Big(-\omega^{i^*} - \sum_{B=1}^{2m} \theta_B^{i^*}(\xi)\omega^B \Big).\end{aligned} \tag{37}$$

Invoking again de Kähler relations (35), one finds from (37) that

$$\mathcal{L}_\xi \Omega = -2\Omega, \tag{38}$$

which shows that ξ defines an infinitesimal conformal transformation of Ω.

On the other hand, for $(\phi C)^\flat$ one finds after some calculation

$$d(\phi C)^\flat = (f + C^0)\Omega - 2\eta \wedge (\phi C)^\flat, \tag{39}$$

and by (39) one gets that

$$\mathcal{L}_C \Omega = (f - C^0)\Omega. \tag{40}$$

Hence, similar as ξ, the principal vector field C also defines an infinitesimal conformal transformation of Ω.

Finally, one derives by (26) that

$$\mathcal{L}_{\phi C} \Omega = 0, \tag{41}$$

which proves that Ω is an invariant of ϕC (see also [9] and [1]).

Moreover, denote by $\mathbb{L}$ Weyl's operator of type (1.1) on forms (see e.g. [7])

$$\mathbb{L}u = u \wedge \Omega, \qquad u \in \Lambda^1 M.$$

Then, setting

$$(C^\flat)_q = C^\flat \wedge \Omega^q,$$

a calculation on basis of (36) and (26) yields

$$d(C^\flat)_q = 2(1+q)\eta \wedge (C^\flat)_q.$$

Summarizing, we can formulate the following

Theorem 4.1. *Regarding the cosymplectic structure defined by Ω we have the following properties:*

(i) *ξ defines an infinitesimal conformal transformation of Ω, i.e.*

$$\mathcal{L}_\xi \Omega = -2\Omega;$$

(ii) *C defines an infinitesimal conformal transformation of Ω, i.e.*

$$\mathcal{L}_C \Omega = (f - C^0)\Omega;$$

(iii) *Ω is an invariant of ϕC, i.e.*

$$\mathcal{L}_{\phi C} \Omega = 0.$$

Moreover, in the case under consideration, also the following expression holds

$$d(C^\flat)_q = 2(1+q)\eta \wedge (C^\flat)_q.$$

5. Normal hypersurfaces

In view of (7) and (2), the 1-codimensional foliation M_ξ (perpendicular to ξ) is totally umbilical (with principal curvature 1) and the 1-dimensional foliation in the direction of ξ is totally geodesic. As the mean curvature vector ξ of the foliation M_ξ is parallel automatically, the two foliations induce locally a warped product structure on M. Since

$$d\eta = 0, \tag{42}$$

we discuss in the present section some properties of the hypersurface $M^\top = M_\xi$ defined by (42).

Recall that the Weingarten map

$$A : T_p(M_\xi) \to T_p(M_\xi), \qquad (\forall p \in M_\xi),$$

is a linear and self-adjoint application. Then, if $Z^\top$ is any horizontal vector field, one gets by (42) that

$$AZ^\top = \nabla_{Z^\top} \xi = -Z^\top. \tag{43}$$

This shows that $Z^\top$ is a principal vector field of M_ξ.

Recall that $\mathrm{II} = \langle dp, \nabla\xi \rangle$ and $\mathrm{III} = \langle \nabla\xi, \nabla\xi \rangle$ denote the second and the third fundamental forms associated with the immersion

$$x : M_\xi \to M.$$

By (2) one finds that

$$\mathrm{II} = g^\top, \quad \text{and} \quad \mathrm{III} = g^\top, \tag{44}$$

where $g^\top$ means the horizontal component of g. Hence we may conclude that the immersion $x : M_\xi \to M$ is horizontally umbilical and has $2m$ principal curvatures equal to 1.

By reference to [13] the expression for III proves that the Gauss map is conformal, and it can also be seen that M_ξ is Einsteinian. On the other hand, since obviously the mean curvature field ξ is nowhere zero, then by reference to [2], it follows that the product submanifold $M_\xi \times M_\xi$ in $M \times M$ is a $\mathcal{U}$-submanifold, which is a submanifold on which the allied mean curvature vanishes.

We may summarize the above in the following

Theorem 5.1. *Let $x : M_\xi \to M$ be the immersion of the hypersurface normal to ξ, then*

(i) *the Gauss map associated with the immersion $x : M_\xi \to M$ is conformal,*

(ii) *the product submanifold $M_\xi \times M_\xi$ in $M \times M$ is a $\mathcal{U}$-submanifold.*

Appendix: Kenmotsu manifolds

In the present paper $(2m+1)$-dimensional generalised Kenmotsu manifolds $M(\phi, \Omega, \eta, \xi, g)$ or K-manifolds [8] [15] are considered, for which the quintuple of structure tensor fields satisfy the following set of axioms:

$$\begin{cases} \phi^2 = -\operatorname{Id} + \eta \otimes \xi,\ \eta(\xi) = 1,\ \phi\xi = 0,\ \eta \circ \phi = 0, \\ g(\phi Z, \phi Z') = g(Z, Z') - \eta(Z)\eta(Z'), \\ \eta(Z) = g(\xi, Z),\ Z, Z' \in \Xi(M), \\ \Omega(Z, Z') = g(\phi Z, Z'), \quad \text{with} \quad i_\xi \Omega = 0, \quad \text{and} \quad \Omega^m \wedge \eta \neq 0, \\ (\nabla\phi)Z = \eta(Z)\phi dp + (\phi Z)^\flat \otimes \xi, \iff \\ (\nabla_{Z'}\phi)Z = \eta(Z)\phi Z' + g(\phi Z, Z')\xi, \\ \nabla\xi = -dp + \eta \otimes \xi, \quad \iff \quad \nabla_Z \xi = -Z + \eta(Z)\xi, \end{cases} \tag{45}$$

together with

$$d\eta = 0, \quad \text{and} \quad d\Omega = -2\eta \wedge \Omega. \tag{46}$$

One may remark that the equations (46) show that the pairing (η, Ω) of the covector of Reeb η and the structure 2-form Ω of rank $2m$ defines a conformal cosymplectic structure on M. We also observe that the structure tensor field ϕ is skew-symmetric and provides a field of endomorphisms of the tangent spaces of M.

We recall that it has been proved [16] that the Reeb vector field ξ is always exterior concurrent with $+1$ as conformal scalar, and therefore satisfies

$$\nabla^2\xi = d^\nabla(\nabla\xi) = \eta \wedge dp, \tag{47}$$

which implies that

$$\mathcal{R}(\xi, Z) = -2m\eta(Z), \implies \operatorname{Ric}(\xi) = -2m. \tag{48}$$

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