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The Diophantine equation $\alpha\binom{x}{m}+\beta\binom{y}{n}=\gamma$

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Abstract. The number of solutions of the Diophantine equation $\alpha\binom{x}{m}+\beta\binom{y}{n}=\gamma$ with $\alpha,\beta,\gamma\in\mathbb{Q}$ and $m,n\in\mathbb{N}$, $m\neq n$ in rational integers (x,y) is investigated. We apply the general Bilu–Tichy-criterion [5] for polynomial Diophantine equations $f(x)=g(y)$ in order to obtain ineffective finiteness of solutions (x,y) in the case $m,n\geq 3$. Simplicity and two-interval-monotonicity of the local extrema of $\binom{x}{m}$ also guarantee finiteness in the case $\min(m,n)=2$.

1. A short account on history

W. Sierpiński [14] considered the problem to determine all triangular numbers which are also tetrahedral. This means to determine the solutions (k,x,y) of the equation

$$k=\binom{x}{3}=\binom{y}{2}\quad\text{with}\quad k,x,y\in\mathbb{N}.\tag{1.1}$$

In 1966, Avanesov [1] gave the complete solution, proving that $k=1,10,120,1540,7140$ are the only numbers k which satisfy (1.1). Twenty years later, Kiss [10] was able to show finiteness for the generalized equation

$$\binom{x}{p}=\binom{y}{2}\qquad\text{with } p \text{ being an arbitrary fixed odd prime.}\tag{1.2}$$

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His proof heavily relies on Baker's theorem of finiteness of integer solutions of hyperelliptic equations [2], so Kiss' result is effective. He also uses the simple fact that the polynomial $f(x) = \binom{x}{p} + \frac{1}{8}$ only has simple roots. Furthermore, p has to be prime since Eisenstein's irreducibility criterion is applied. However, in 1993, Barja, Molinelli and Blanco Ferro [3] observed that this assumption is not necessary for $f(x)$ in order to have just simple zeros, so that they obtained finiteness of the number of positive integer solutions (x, y) of the Diophantine equation

$$\binom{x}{m} = \binom{y}{2} \quad \text{for arbitrary fixed } m \geq 3. \tag{1.3}$$

A special problem appearing in Section D3 of Guy's book [9] of unsolved problems in number theory is the following: Are $(4, 2)$, $(6, 6)$ and $(10, 21)$ the only positive integer pairs (x, y) with $x \geq 4$ and $y \geq 2$ such that the equation

$$\binom{x}{4} = \binom{y}{2}$$

holds? Pintér [11] and de Weger [6] gave the affirmative answer. The tools are once more lower bounds of linear forms in logarithms of algebraic numbers. Other special Diophantine equations involving binomial coefficient polynomials have been treated by these authors in [12], [7].

Recent work in a very similar subject area has been done by Bilu, Brindza, Kirschenhofer, Pintér and Tichy in [4] where they could show, that for $m \neq 2$, $n \geq 1$ and $(m, n) \neq (2, 1)$, the equation

$$x(x+1)\dots(x+m-1) = 1^n + 2^n + \dots + (y-1)^n$$

has at most finitely many solutions in rational integers x, y.

In the present paper we deal with a more general form of Diophantine equation involving two binomial coefficients of the above type with arbitrary multiplicative rational factors. However, as we use Bilu–Tichy's criterion for $m, n \geq 3$ (see Section 3, Theorem 3.3) we loose control over effective bounds of solutions, so that our result is ineffective. For the case $n = 2$, Baker's effective theorem about elliptic and hyperelliptic equations applies (see for instance [13]).

Theorem 1.1 (Baker, 1975)**.** *We consider the Diophantine equation* $f(x)=by^2$ *in* $(x,y)\in\mathbb{Q}^2$ *with* $f(x)\in\mathbb{Q}[x]$ *and* $b\in\mathbb{Q}\setminus\{0\}$*. If* $f(x)$ *has at least three simple roots then all solutions satisfy* $\max(|x|,|y|)\le C_1$ *for some computable* C_1 *depending only on* b *and* f*.*

2. Main theorem

Theorem 2.1.

1. *The Diophantine equation*

$$\alpha\binom{x}{m}+\beta\binom{y}{n}=\gamma \quad \text{with} \quad m>n\ge 3,\ \alpha,\beta,\gamma\in\mathbb{Q},\ \alpha>0,\ \beta\ne 0$$

   has at most finitely many solutions in rational integers x, y*.*

2. *The same holds for the case* $n=2$ *and* $m\ge 3$ *with exception of*

$$m=4,\qquad \frac{8\gamma+\beta}{\alpha}\in\left\{-\frac{1}{3},\frac{3}{16}\right\}.$$

   Moreover, it holds $\max(|x|,|y|)<C_1$*, where* C_1 *is an effectively computable constant depending only on* α,β,γ *and* m*.*

Note that (1.3) is obtained by the choice of $(\alpha,\beta,\gamma)=(1,-1,0)$ and $n=2$ in the second statement.

For the proof we need some information about monotonicity of extrema of $\left|\binom{x}{m}\right|$. For convenience, we call a real polynomial $f(x)$ *two-interval-monotone*, if there exist two intervals I_1 and I_2 (one possibly empty) with $I_1\cup I_2=(-\infty,\infty)$ such that the local extrema of $|f(x)|$ are strictly decreasing on I_1 and strictly increasing on I_2.

Lemma 2.2. $p_m(x)=\binom{x}{m}$ *with* $m\ge 2$ *is two-interval-monotone.*

Proof. Assume by induction that $|p_m(x)|$ has this property (obviously it holds for $m=2$). Since $|p_m(x)|$ is symmetric with respect to the line $x=\frac{m-1}{2}$ we just have to show that the local maxima of $|p_m(x)|$ are decreasing on $[0,(m-1)/2]$ for all m. If m is even then there is an extremum at $x=\frac{m-1}{2}$ and by symmetry that one with minimal value. Then, multiplication with $\left|\frac{x-m}{m+1}\right|$ does not change the decreasing property of the extrema of $|p_m(x)|$, but now this property holds on the larger interval $[0, m/2]$. On the other hand, if m is odd, there are exactly two extrema of minimal value. Again, $\left|\frac{x-m}{m+1}\right| |p_m(x)|$ has the decreasing property on the larger interval $[0, m/2]$ because $|x-m|$ is strictly decreasing on $[0, m]$, so in particular on $[(m-2)/2, m/2]$. Now we are finished because of $p_{m+1}(x) = \frac{x-m}{m+1} p_m(x)$.

In order to apply Theorem 1.1 we have to examine the zero distribution of the polynomial $\binom{x}{m} + \delta$ for general $\delta \in \mathbb{Q}$. Lemma 2.2 enables us to give a simple proof of the following property, which originally has been stated by Yuan Ping-Zhi [15, Theorem 2].

Corollary 2.3. *The polynomial* $\binom{x}{m} + \delta$ *with* $\delta \in \mathbb{Q}$ *and* $m \geq 3$ *has at least three simple roots with exception of* $(m, \delta) \in \{(4, -3/128), (4, 1/24)\}$.

Proof. As $\binom{x}{m}$ has exactly $m-1$ simple real stationary points, all non-real roots of $\binom{x}{m} + \delta$ are simple. Moreover, the only multiple roots of $\binom{x}{m} + \delta$ are real roots of order two. By Lemma 2.2 there are at most two extrema of the same value, which means that the number of roots of order two of $\binom{x}{m} + \delta$ is at most two. Consequently, there are always three simple roots in case of $m \geq 7$. If $m = 3$ then $\binom{x}{3} + \delta$ has exactly three simple roots except in the case where $|\delta| = \sqrt{3}/27$, which is a contradiction to $\delta \in \mathbb{Q}$. Assume now $m = 4$. If $\delta \neq 1/24$ and $\delta \neq -3/128$ there are always four simple roots (i.e. for $\delta \in]-3/128, 1/24[$ four simple real roots, for $\delta > 1/24$ four simple non-real roots, for $\delta < -3/128$ two simple real and two simple non-real roots. Further we observe that $\binom{x}{4} - 3/128$ has two simple real roots and a real root of order two; on the other hand, $\binom{x}{4} + 1/24$ has two real roots of order two. In the case of $m = 5$ three simple roots are always guaranteed. Finally, let $m = 6$. The condition $p_6'(x) = 0$ yields five values, namely $x = \frac{5}{2}, \frac{5}{2} \pm \frac{1}{6}\sqrt{105 \pm 24\sqrt{7}}$. The points $x_i = \frac{5}{2} \pm \frac{1}{6}\sqrt{105 \pm 24\sqrt{7}}$ are crucial and we have $p(x_i) = -\frac{2}{243} \pm \frac{7}{1215}\sqrt{7}$, which is again a contradiction to $\delta \in \mathbb{Q}$. This finishes the proof.

3. Decomposition properties

Let f be a complex polynomial and η an arbitrary complex number. Put $\delta(f;\eta) := \deg\gcd(f-\eta, f')$. The following important decomposition lemma is extracted from [8].

Lemma 3.1. *Let $f \in \mathbb{C}[x]$ be non-constant and let $f = p \circ q$, where $p, q \in \mathbb{C}[x]$. If $\deg p \geq 2$ then there exists $\eta \in \mathbb{C}$ with $\delta(f;\eta) \geq \deg q$.*

Proof. Let ξ be a root of p' (which exists by $\deg p \geq 2$) and put $\eta = p(\xi)$. Then both the polynomials $f - \eta$ and f' are divisible by $q - \xi$, hence $\deg q = \deg(q - \xi) \leq \deg\gcd(f - \eta, f')$.

Suppose $f, p, q \in \mathbb{R}[x]$, $f = p \circ q$ with $\deg p \geq 2$ and f just having simple extremal points. Then $\xi \in \mathbb{R}$ and $\eta \in \mathbb{R}$. In this case, $\delta(f;\eta)$ can easily be interpreted as the number of extremal points of f having value η. If we further claim f to be two-interval-monotone then f has at most two extremal points of the same value, i.e. $\delta(f;\eta) \leq 2$ for all $\eta \in \mathbb{R}$. As a straight-forward consequence we have

Corollary 3.2. *Let $f \in \mathbb{R}[x]$ be non-constant, two-interval-monotone and having just simple extremal points. Further let $\lambda_1 f(\lambda_2 x + \lambda_3) + \lambda_4 = p(q(x))$ with $p(x), q(x) \in \mathbb{R}[x]$, $\deg p \geq 2$ and $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbb{R}$, $\lambda_1\lambda_2 \neq 0$. Then $\deg q \leq 2$.*

Note that the double-shift $x \mapsto \lambda_2 x + \lambda_3$ and $f \mapsto \lambda_1 f + \lambda_4$ does not affect two-interval-monotonicity and simplicity of extremal points. Summing up, the only possible decompositions of such polynomials $f(x)$ are

(1) $f(x) = e_1 h(x) + e_0$ with $\deg h = \deg f$, $e_0, e_1 \in \mathbb{R}$ and

(2) $f(x) = \phi(h(x))$, $h(x)$ being a real polynomial of degree at most two and $\deg \phi \geq 2$.

We recall from [5] a very comfortable criterion in order to decide whether a given polynomial Diophantine equation $f(x) = g(y)$ has infinitely many rational solutions with a bounded denominator or not. In what follows $\theta, \rho \in \mathbb{Q} \setminus \{0\}$, $q, s, t \in \mathbb{Z}_{>0}$, $r \in \mathbb{Z}_{\geq 0}$ and $v(x) \in \mathbb{Q}[x]$ a non-zero polynomial (which may be constant). We will use the following explicit definition of the s-th Dickson polynomial:

$$D_s(x,\theta)=\sum_{i=0}^{\lfloor s/2\rfloor} d_{s,i}x^{s-2i} \qquad \text{with} \qquad d_{s,i}=\frac{s}{s-i}\binom{s-i}{i}(-\theta)^i.$$

$(f_i(x),g_i(x))$ or switched with $1\le i\le 5$ is called a *standard pair* if it can be represented by an explicit form listed in the following table. Depending on i we say (f_i,g_i) is a standard pair of the first, second, third, fourth, fifth kind, respectively.

kind	explicit form of $(f_i(x),g_i(x))$, $1\le i\le 5$
first	$(x^q,\theta x^r v(x)^q)$ with $0\le r<q, (r,q)=1, r+\deg v>0$
second	$(x^2,(\theta x^2+\rho)v(x)^2)$
third	$(D_s(x,\theta^t),D_t(x,\theta^s))$ with $(s,t)=1$
fourth	$\left(\theta^{-s/2}D_s(x,\theta),-\rho^{-t/2}D_t(x,\rho)\right)$, with $(s,t)=2$
fifth	$\left((\theta x^2-1)^3,3x^4-4x^3\right)$

Theorem 3.3 (Bilu–Tichy, 2000). *Let $f(x),g(x)\in\mathbb{Q}[x]$ be non-constant polynomials. Then the following two assertions are equivalent:*

(a) *The equation $f(x)=g(y)$ has infinitely many rational solutions with a bounded denominator.*

(b) *$f\circ\kappa_1=\phi\circ f_i$ and $g\circ\kappa_2=\phi\circ g_i$ where $\kappa_1,\kappa_2\in\mathbb{Q}[x]$ are linear, $\phi(x)\in\mathbb{Q}[x]$, and (f_i,g_i) is a standard pair.*

In the sequel we use the notation $\phi(x)=e_kx^k+e_{k-1}x^{k-1}+\dots+e_0$ and $v(x)=v_lx^l+v_{l-1}x^{l-1}+\dots+v_0$.

Proof of Main Theorem 2.1

1. While introducing new parameters ($\alpha\leftrightarrow-\frac{\beta}{\alpha}$ and $\beta\leftrightarrow\frac{\gamma}{\alpha}$) the original equation becomes

$$p_m(x)=\alpha p_n(y)+\beta.$$

First, let $\deg\phi\ge 2$. Having in mind Corollary 3.2 and $m\ne n$, we observe by looking at the five standard pairs, that we can straightly exclude the standard pairs number two ($v(x)$ must be a constant) and four ($s=t=2$)

as $m \neq n$. Standard pair number five can also be repelled because of multiple roots. There remain the following cases to consider

Standard pair of the first kind:
$$p_m(Ax+B)=\phi(x^2), \qquad \alpha p_n(\tilde{A}x+\tilde{B})+\beta=\phi(\theta v_0^2 x).$$
Standard pair of the first kind:
$$p_m(Ax+B)=\phi(\theta(v_2x^2+v_1x+v_0)), \quad \alpha p_n(\tilde{A}x+\tilde{B})+\beta=\phi(x).$$
Standard pair of the third kind:
$$p_m(Ax+B)=\phi(x^2-2\theta), \qquad \alpha p_n(\tilde{A}x+\tilde{B})+\beta=\phi(x).$$

We note that necessarily $m=2n$ since otherwise no representation can exist. Just by linear transformation of the argument we get an polynomial identity just in terms of the polynomials under consideration, i.e.

$$p_m(x)=\alpha p_n(v_2x^2+v_1x+v_0)+\beta \tag{3.1}$$

for some probably new parameters v_2, v_1, v_0. The impossibility of such a representation would imply $\deg\phi=1$. Thereafter, the standard pair of the first kind with $q\geq 3$ and the standard pair of the fifth kind are of no use since $\phi(x^q)$, $\phi(3x^4-4x^3)$ respectively, have multiple extremal points. Keeping in mind that $m, n \geq 3$ we just have to deal with the representations involving Dickson polynomials. However, Dickson polynomials $D_s(x,\theta)$ are known to take exactly two different extremal values for $s\geq 3$. Hence, polynomials of the form $\lambda_1 D_s(\lambda_2 x+\lambda_3,\theta)+\lambda_4$ have the same property, so they have not got the two-interval-monotonicity property. Finally, we just have to treat (3.1), i.e.

$$\binom{x}{m}=\alpha\binom{v_2x^2+v_1x+v_0}{n}+\beta.$$

The impossibility of such an equation would fully settle the case $n\geq 3$. We first list explicitly the upper most coefficients of the polynomial $n!\binom{x}{n}=x^n+k_{n-1}^{(n)}x^{n-1}+k_{n-2}^{(n)}x^{n-2}+\cdots+k_0^{(n)}$ which can be calculated via multiple sums and Maple

$$k_{n-i}^{(n)}=\sum_{0\leq j_1<j_2<\cdots<j_i\leq n-1}\prod_{k=1}^{i}(-j_k)$$

$$= \sum_{j_i=j_{i-1}+1}^{n-1} \sum_{j_{i-1}=j_{i-2}+1}^{n-2} \cdots \sum_{j_2=j_1+1}^{n-i+1} \sum_{j_1=0}^{n-i} \prod_{k=1}^{i} (-j_k).$$

In particular, for small i,

$$k_{n-1}^{(n)} = -\frac{1}{2}n(n-1)$$

$$k_{n-2}^{(n)} = \frac{1}{24}n(n-1)(n-2)(3n-1)$$

$$k_{n-3}^{(n)} = -\frac{1}{48}n^2(n-1)^2(n-2)(n-3)$$

$$k_{n-4}^{(n)} = \frac{1}{5760}n(n-1)(n-2)(n-3)(n-4)(15n^3-30n^2+5n+2)$$

$$k_{n-5}^{(n)} = -\frac{1}{11520}n^2(n-1)^2(n-2)(n-3)(n-4)(n-5)(3n^2-7n-2).$$

By immediate comparison of coefficients of upper powers of x we derive the following list of equations in the variables α, β, v_2, v_1, v_0, m, n.

Eq. 0 $[x^m]$: $\quad 1 = \alpha v_2^n$

Eq. 1 $[x^{m-1}]$: $\quad k_{m-1}^{(m)} = \alpha\left[nv_2^{n-1}v_1\right]$

Eq. 2 $[x^{m-2}]$: $\quad k_{m-2}^{(m)} = \alpha\left[nv_2^{n-1}v_0 + \frac{n(n-1)}{2}v_2^{n-2}v_1^2 + k_{n-1}^{(n)}v_2^{n-1}\right]$

Eq. 3 $[x^{m-3}]$:

$$k_{m-3}^{(m)} = \alpha\Big[n(n-1)v_2^{n-2}v_1v_0 + \frac{n(n-1)(n-2)}{6}v_2^{n-3}v_1^3 + k_{n-1}^{(n)}(n-1)v_2^{n-2}v_1\Big]$$

Eq. 4 $[x^{m-4}]$:

$$k_{m-4}^{(m)} = \alpha\Big[\frac{n(n-1)}{2}v_2^{n-2}v_0^2 + \frac{n(n-1)(n-2)}{2}v_2^{n-3}v_1^2v_0 + \frac{n(n-1)(n-2)(n-3)}{24}v_2^{n-4}v_1^4 + k_{n-1}^{(n)}(n-1)v_2^{n-2}v_0 + k_{n-1}^{(n)}\frac{(n-1)(n-2)}{2}v_2^{n-3}v_1^2 + k_{n-2}^{(n)}v_2^{n-2}\Big]$$

$$\text{Eq. 5}\quad [x^{m-5}]:\quad k_{m-5}^{(m)}=\alpha\left[\frac{n(n-1)}{2}v_2^{n-3}v_1v_0^2\right.$$
$$+\frac{n(n-1)(n-2)(n-3)}{6}v_2^{n-4}v_1^3v_0$$
$$+\frac{n(n-1)(n-2)(n-3)(n-4)}{120}v_2^{n-5}v_1^5$$
$$+k_{n-1}^{(n)}(n-1)(n-2)v_2^{n-3}v_1v_0$$
$$\left.+k_{n-1}^{(n)}\frac{(n-1)(n-2)(n-3)}{6}v_2^{n-4}v_1^3+k_{n-2}^{(n)}(n-2)v_2^{n-3}v_1\right]$$

$$\text{Eq. 6}\quad [x^{m-6}]:\quad k_{m-6}^{(m)}=\alpha\left[\frac{n(n-1)(n-2)}{6}v_2^{n-3}v_0^3\right.$$
$$+\frac{n(n-1)(n-2)(n-3)(n-4)}{24}v_2^{n-5}v_1^4v_0$$
$$+\frac{n(n-1)(n-2)(n-3)}{4}v_2^{n-4}v_1^2v_0^2$$
$$+\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{720}v_2^{n-6}v_1^6$$
$$+k_{n-1}^{(n)}\frac{(n-1)(n-2)}{2}v_2^{n-3}v_0^2$$
$$+k_{n-1}^{(n)}\frac{(n-1)(n-2)(n-3)}{2}v_2^{n-4}v_1^2v_0$$
$$+k_{n-1}^{(n)}\frac{(n-1)(n-2)(n-3)(n-4)}{24}v_2^{n-5}v_1^4$$
$$+k_{n-2}^{(n)}(n-2)v_2^{n-3}v_0$$
$$\left.+k_{n-2}^{(n)}\frac{(n-2)(n-3)}{2}v_2^{n-4}v_1^2+k_{n-3}^{(n)}v_2^{n-3}\right].$$

Note that the use of Eq. 4 and 5 requires $n\geq 3$ respectively $n\geq 4$ for Eq. 6. By solving the system Eq. 0–4 we obtain $v_2^2=15/(16n^2-4)$. Obviously, a necessary condition consists in $v_2\in\mathbb{Q}$, or equivalently, there must be a solution to the equation

$$\frac{15}{4n^2-1}=\frac{l_1^2}{l_2^2}$$

with $(l_1, l_2) = 1$. One sees, that the Pellian equation $4n^2 - 15l_2^2 = 1$ has to be satisfied. The smallest solutions in terms of n are $n = 2$ and $n = 122$. Hence, we can use Eq. 6 without any further restriction on n. This gives a single equation basically in terms of n, namely $n(n^2-1)(n^2-4)(4n^2-1)=0$ which has no solution for $n \geq 3$, a contradiction.

2. In case of $n = 2$ we rewrite the original equation in the following way:

$$\alpha\binom{x}{m} + \beta\binom{y}{n} = \gamma, \text{ and equivalently, } \binom{x}{m} - \frac{8\gamma+\beta}{8\alpha} = -\frac{\beta}{8\alpha}(2y-1)^2.$$

By Corollary 2.3 the polynomial $\binom{x}{m} - \frac{8\gamma+\beta}{8\alpha}$ has at least three simple roots with exception of the cases $m = 4, \frac{8\gamma+\beta}{\alpha} = \frac{3}{16}$ and $m = 4, \frac{8\gamma+\beta}{\alpha} = -\frac{1}{3}$. Applying Theorem 1.1 to $\binom{x}{m} - \frac{8\gamma+\beta}{8\alpha}$ we immediately get the statement. $\square$

Remark. We thank the referee for informing us about the paper of Yuan Ping-Zhi [15] in order to improve Theorem 2.1. Csaba Rakaczki, in a recent work, attacks the equation $x(x-1)\cdots(x-m+1) = \lambda y(y-1)\cdots(y-n+1)$ by another approach to obtain general ineffective results.

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