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Title: On the diophantine equation $x^2 + 2^\alpha 3^\beta 5^\gamma 7^\delta = y^n$

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Let $S = \{p_1, \ldots, p_s\}$ be a set of distinct primes and denote by $\mathbf{S}$ the set of non-zero integers composed only of primes from S. Further, denote by Q the product of the primes from S. Let $f \in \mathbb{Z}[X]$ be a monic quadratic polynomial with negative discriminant D_f contained in $\mathbf{S}$. Consider equation $f(x) = y^n$ (2) in integer unknowns x, y, n with $n \geq 3$ prime and $y > 1$. It follows from a general result of [**?**] that in (2) n can be bounded from above by an effectively computable constant depending only on Q. This bound is, however, large and is not given explicitly. Using some results of Bugeaud and Shorey [**?**] we derive, apart from certain exceptions, a good and completely explicit upper bound for n in (2) (see Theorems 1 and 2). Further, combining our Theorem 2 with some deep results of Cohn [**?**] and de Weger [**?**] we give all non-exceptional (see Section 1) solutions of equation $x^2 + 2^\alpha 3^\beta 5^\gamma 7^\delta = y^n$ (6), where x, y, n, α, β, γ, δ are unknown non-negative integers with $x \geq 1$, $\gcd(x, y) = 1$ and $n \geq 3$ (cf. Theorem 3). When, in (6), $\alpha \geq 1$ is also assumed then our Theorem 3 is a generalization of a result of Luca [**?**]. In this case all the solutions of equation (6) are listed.

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