

Title: Gröbner bases for complete ℓ -wide families

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Let $n > 0$, k, ℓ be integers with $0 \leq \ell - 1 \leq k \leq n$, and consider the complete ℓ -wide family

$$\mathcal{F}^{k,\ell} = \{F \subseteq [n] : k - \ell < |F| \leq k\}.$$

We describe (reduced) Gröbner bases of the ideal of polynomials, over an arbitrary field $\mathbb{F}$, which vanish on the characteristic vectors of the elements of $\mathcal{F}^{k,\ell}$. As an application, we obtain results on certain inclusion matrices related to $\mathcal{F}^{k,\ell}$. We show that if $0 \leq m \leq \min(k, n - k + \ell - 1)$ then

$$\text{rank}_{\mathbb{F}} I \left(\mathcal{F}^{k,\ell}, \binom{[n]}{\leq m} \right) = \sum_{i=\max(0, m-\ell+1)}^m \binom{n}{i}, \quad (1)$$

where $\mathbb{F}$ is an arbitrary field. We prove also a special case of a conjecture of Frankl related to the determination of the maximum number of subsets of $[n]$ with no shattered set of size t and with no chain of size $\ell + 1$. The paper extends the results obtained for the case of uniform families (the case $\ell = 1$) in [?].

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