

Oscillation and nonoscillation of perturbed half-linear Euler differential equations

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Abstract. Using general results for (non)oscillation of the second order half-linear differential equation

$$(r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \quad \Phi(x) := |x|^{p-2}x, \quad p > 1, \quad (*)$$

we establish new oscillation and nonoscillation criteria for the perturbed half-linear Euler differential equation

$$(\Phi(x'))' + \left[\frac{\gamma_p}{t^p} + c(t) \right] \Phi(x) = 0, \quad \gamma_p := \left(\frac{p-1}{p} \right)^p.$$

1. Introduction and preliminaries

The aim of this paper is to investigate oscillatory properties of solutions of the half-linear second order differential equation

$$(r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \quad \Phi(x) := |x|^{p-2}x, \quad p > 1, \quad (1)$$

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where r, c are continuous functions, $r(t) > 0$, and its special case, the perturbed half-linear Euler differential equation

$$(\Phi(x'))' + \left[\frac{\gamma_p}{t^p} + c(t) \right] \Phi(x) = 0, \quad \gamma_p := \left(\frac{p-1}{p} \right)^p. \quad (2)$$

Recently, several papers (see [5], [6], [7], [4], [12], [13]) appeared, where equation (1) is viewed as a perturbation of the nonoscillatory equation of the same form

$$(r(t)\Phi(x'))' + \tilde{c}(t)\Phi(x) = 0 \quad (3)$$

and (non)oscillation criteria are formulated in terms of the asymptotic behavior of the integrals

$$\int_t^t [c(s) - \tilde{c}(s)] h^p(s) ds, \quad \text{or} \quad \int_t^\infty [c(s) - \tilde{c}(s)] h^p(s) ds,$$

where h is a solution of (3).

Here we follow a slightly more general idea which is motivated by the fact that the exact solution of (3) is not known in many cases and only its asymptotic estimate is available. A typical example is the Euler equation with the critical coefficient

$$(\Phi(x'))' + \frac{\gamma_p}{t^p} \Phi(x) = 0, \quad (4)$$

whose one solution is $x(t) = t^{\frac{p-1}{p}}$ and any linearly independent solution is known only asymptotically $x(t) \sim t^{\frac{p-1}{p}} \log^{\frac{2}{p}} t$ as $t \rightarrow \infty$. We are particularly motivated by the oscillation criterion given in [13], where it was shown that equation (2) is oscillatory provided

$$\liminf_{t \rightarrow \infty} \frac{1}{\log t} \int_t^t c(s) s^{p-1} \log^2 s ds > 2 \left(\frac{p-1}{p} \right)^{p-1}, \quad (5)$$

and it was conjectured that the constant $2 \left(\frac{p-1}{p} \right)^{p-1}$ in (5) can be replaced by the better constant $\frac{1}{2} \left(\frac{p-1}{p} \right)^{p-1}$. At the same time it was conjectured (based on [5, Theorem 3]) that (2) is nonoscillatory provided

$$\limsup_{t \rightarrow \infty} \frac{1}{\log t} \int_t^t c(s) s^{p-1} \log^2 s ds < \frac{1}{2} \left(\frac{p-1}{p} \right)^{p-1} \quad (6)$$

and

$$\liminf_{t \rightarrow \infty} \frac{1}{\log t} \int_t^t c(s) s^{p-1} \log^2 s ds > -\frac{3}{2} \left(\frac{p-1}{p} \right)^{p-1}. \quad (7)$$

The aim of this paper is to prove both these conjectures. We use first an extension of [5, Theorem 2] to the situation when h is *not* a solution of (3) and then we apply the recently established oscillation criterion for the perturbed Euler–Weber half-linear differential equation

$$(\Phi(x'))' + \left[\frac{\gamma_p}{t^p} + \frac{\mu_p}{t^p \log^2 t} + c(t) \right] \Phi(x) = 0, \quad \mu_p := \frac{1}{2} \left(\frac{p-1}{p} \right)^{p-1} \quad (8)$$

coupled with the “oscillation constant improvement procedure” introduced for higher order linear differential equation in [2].

It is well known that the oscillation theory of half-linear equation (1) is very similar to that of the linear Sturm–Liouville differential equation (which is the special case $p = 2$ in (1))

$$(r(t)x')' + c(t)x = 0$$

even if the additivity of the solution space of (1) is lost and only homogeneity remains. In particular, similarly to the linear case, equation (1) can be classified as *oscillatory* or *nonoscillatory* according to whether every nontrivial solution has/does not have infinitely many zeros on every interval of the form $[T, \infty)$. For general background of the half-linear oscillation theory we refer to [1, Chap. 3], [3] or to [8].

The basic tools of the half-linear oscillation theory are the so-called *variational principle* and *Riccati technique*. The first one consists in the fact that (1) is oscillatory if and only if for every $T \in \mathbb{R}$ there exists a nontrivial function $y \in W_0^{1,p}(T, \infty)$ such that

$$\mathcal{F}(y; T, \infty) = \int_T^\infty [r(t)|y'|^p - c(t)|y|^p] dt \leq 0.$$

The Riccati technique is based on the fact that if $x(t) \neq 0$ in an interval I is a solution of (1) then $w = r\Phi(x'/x)$ solves in I the Riccati type first order differential equation

$$w' + c(t) + (p-1)r^{1-q}(t)|w|^q = 0, \quad (9)$$

where q is the conjugate number of p , i.e. $\frac{1}{p} + \frac{1}{q} = 1$. Actually, according to the Sturm comparison theory for half-linear equations, solvability of (9) can be replaced by the associated inequality as shows the following statement which can be found e.g. in [1, Theorem 3.7.1] or [8, Theorem 2.2.1].

Lemma 1. *Equation (1) is nonoscillatory if and only if there exists a differentiable function w such that*

$$R[w](t) := w'(t) + c(t) + (p-1)r^{1-q}(t)|w(t)|^q \leq 0 \quad (10)$$

for large t .

We finish this section with an oscillation criterion for (8) which is proved in [4] using the variational principle and which we will need in the proof of Theorem 1 of the next section.

Proposition 1. *If*

$$\int_0^\infty c(t)t^{p-1} \log t \, dt = \infty, \quad (11)$$

then equation (8) is oscillatory.

2. Oscillation and nonoscillation criteria

We start with a technical result concerning the function $h(t) = t^{\frac{p-1}{p}} \log^{\frac{2}{p}} t$ which is asymptotically equivalent (in a certain sense) to the so-called nonprincipal solution of Euler equation (4). For more details concerning principal and nonprincipal solutions of (4) and of related equations we refer to [10].

Lemma 2. *Let $h(t) = t^{\frac{p-1}{p}} \log^{\frac{2}{p}} t$, denote $\tilde{R}[w] := w' + \gamma_p/t^p + (p-1)|w|^q$ the Riccati operator associated with (4), and let $w_h = \Phi(h'/h)$. Then*

$$h^p(t)\tilde{R}[w_h](t) = \frac{K}{t \log t} (1 + o(1)), \quad \text{as } t \rightarrow \infty, \quad (12)$$

where K is a real constant.

PROOF. By a direct computation we have

$$h'(t) = \left(\frac{p-1}{p}\right) t^{-\frac{1}{p}} \log^{\frac{2}{p}} t \left(1 + \frac{2}{(p-1) \log t}\right)$$

and hence (using the formula $(1+x)^p = 1 + px + \frac{p(p-1)}{2}x^2 + \frac{p(p-1)(p-2)}{6}x^3 + \dots$)

$$\begin{aligned} |w_h|^q &= \left(\frac{p-1}{p}\right)^p \frac{1}{t^p} \left(1 + \frac{2}{(p-1) \log t}\right)^p \\ &= \left(\frac{p-1}{p}\right)^p \frac{1}{t^p} \left(1 + \frac{2p}{(p-1) \log t} + \frac{2p}{(p-1) \log^2 t} + \frac{4p(p-2)}{3(p-1)^2 \log^3 t} + \dots\right), \end{aligned}$$

$$(p-1)|w_h|^q h^p = (p-1) \left(\frac{p-1}{p} \right)^p \frac{1}{t} \log^2 t \\ \times \left(1 + \frac{2p}{(p-1) \log t} + \frac{2p}{(p-1) \log^2 t} + \frac{4p(p-2)}{3(p-1)^2 \log^3 t} + \dots \right),$$

and

$$h^p w'_h = -p \left(\frac{p-1}{p} \right)^p \frac{1}{t} \log^2 t \left(1 + \frac{2}{(p-1) \log t} \right)^{p-2} \\ \times \left(1 + \frac{2}{(p-1) \log t} + \frac{2}{(p-1) \log^2 t} \right) \\ = -p \left(\frac{p-1}{p} \right)^p \frac{1}{t} \log^2 t \left(1 + \frac{2(p-2)}{(p-1) \log t} + \frac{2(p-2)(p-3)}{(p-1)^2 \log^2 t} \right. \\ \left. + \frac{4(p-2)(p-3)(p-4)}{3(p-1)^3 \log^3 t} + \dots \right) \times \left(1 + \frac{2}{(p-1) \log t} + \frac{2}{(p-1) \log^2 t} \right) \\ = -p \left(\frac{p-1}{p} \right)^p \frac{1}{t} \log^2 t \left(1 + \frac{2}{\log t} + \frac{2}{\log^2 t} + \frac{4p(p-2)}{3(p-1)^2 \log^3 t} + \dots \right).$$

Summarizing the previous computations,

$$h^p \tilde{R}[w_h] = K \frac{1}{t \log t} (1 + o(1)), \quad \text{as } t \rightarrow \infty, \quad K := - \left(\frac{p-1}{p} \right)^p \frac{4p(p-2)}{3(p-1)^2},$$

□

Using the previous lemma and Proposition 1 coupled with the “oscillation constant improvement procedure” introduced in [2], we obtain the following oscillation criterion.

Theorem 1. *If*

$$\liminf_{t \rightarrow \infty} \frac{1}{\log t} \int_T^t c(s) s^{p-1} \log^2 s \, ds > \frac{1}{2} \left(\frac{p-1}{p} \right)^{p-1} \quad (13)$$

for some (and hence every) $T \in \mathbb{R}$ sufficiently large, then equation (2) is oscillatory.

PROOF. We rewrite (2) into the form

$$(\Phi(x'))' + \left[\frac{\gamma_p}{t^p} + \frac{\mu_p}{t^p \log^2 t} + \left(c(t) - \frac{\mu_p}{t^p \log^2 t} \right) \right] \Phi(x) = 0$$

and we regard this equation as a perturbation of the so-called half-linear Euler–Weber differential equation

$$(\Phi(x'))' + \left[\frac{\gamma_p}{t^p} + \frac{\mu_p}{t^p \log^2 t} \right] \Phi(x) = 0. \quad (14)$$

According to Proposition 1, it is sufficient to show that

$$\int_T^\infty \left[c(t) - \frac{\mu_p}{t^p \log^2 t} \right] t^{p-1} \log t \, dt = \infty. \quad (15)$$

To this end, we proceed as follows. According to (13) there exists $\varepsilon > 0$ such that still

$$\liminf_{t \rightarrow \infty} \frac{1}{\log t} \int_T^t c(s) s^{p-1} \log^2 s \, ds > \mu_p + \varepsilon \quad (16)$$

for t sufficiently large, say $t > \tilde{T}$. From (16) we have that

$$\frac{1}{t} \int_T^t c(s) s^{p-1} \log^2 s \, ds > (\mu_p + \varepsilon) \frac{\log t}{t} \quad (17)$$

for $t > \tilde{T}$. At the same time, using integration by parts and (17)

$$\begin{aligned} \int_T^b \left[c(t) - \frac{\mu_p}{t^p \log^2 t} \right] t^{p-1} \log t \, dt &= \int_T^b c(t) t^{p-1} \log t \, dt - \mu_p \int_T^b \frac{1}{t \log t} \, dt \\ &= \left[\frac{1}{\log t} \int_T^t c(s) s^{p-1} \log^2 s \, ds \right]_T^b \\ &\quad + \int_T^b \frac{1}{\log^2 t} \frac{\int_T^t c(s) s^{p-1} \log^2 s \, ds}{t} \, dt - \mu_p \log \left(\frac{\log b}{\log T} \right) \\ &= \left[\frac{1}{\log t} \int_T^t c(s) s^{p-1} \log^2 s \, ds \right]_T^b + \int_T^{\tilde{T}} \frac{1}{\log^2 t} \frac{\int_T^t c(s) s^{p-1} \log^2 s \, ds}{t} \, dt \\ &\quad + \int_{\tilde{T}}^b \frac{1}{\log^2 t} \frac{\int_T^t c(s) s^{p-1} \log^2 s \, ds}{t} \, dt - \mu_p \log \left(\frac{\log b}{\log T} \right) \\ &\geq \frac{1}{\log b} \int_T^b c(s) s^{p-1} \log^2 s \, ds + K + (\mu_p + \varepsilon) \int_T^b \frac{1}{t \log t} \, dt - \mu_p \log \left(\frac{\log b}{\log T} \right) \\ &> (\mu_p + \varepsilon) + K + \varepsilon \log \left(\frac{\log b}{\log T} \right) \rightarrow \infty \end{aligned}$$

as $b \rightarrow \infty$, where $K = \int_T^{\tilde{T}} t^{-1} \log^{-2} t \left(\int_T^t c(s) s^{p-1} \log^2 s \, ds \right) dt$. Consequently, (2) is oscillatory by Proposition 1. $\square$

Next we prove a general nonoscillation criterion for (1) where this equation is viewed as a perturbation of (3). In contrast to [5, Theorem 3], we do not suppose that h is a solution of (3).

Theorem 2. *Let $h \in C^1$ be a positive function such that $h'(t) > 0$ for large t , say $t > T$, $\int^\infty r^{-1}(t)h^{-2}(t)(h'(t))^{2-p} dt < \infty$, and denote*

$$G(t) := \int_t^\infty \frac{ds}{r(s)h^2(s)(h'(s))^{p-2}}. \quad (18)$$

Suppose that

$$\lim_{t \rightarrow \infty} G(t)r(t)h(t)\Phi(h'(t)) = \infty \quad (19)$$

and

$$\lim_{t \rightarrow \infty} G^2(t)r(t)h^3(t)(h'(t))^{p-2} [(r(t)\Phi(h'(t)))' + \tilde{c}(t)\Phi(h(t))] = 0. \quad (20)$$

If

$$\limsup_{t \rightarrow \infty} G(t) \int_T^t [c(s) - \tilde{c}(s)]h^p(s) ds < \frac{1}{2q}, \quad (21)$$

and

$$\liminf_{t \rightarrow \infty} G(t) \int_T^t [c(s) - \tilde{c}(s)]h^p(s) ds > -\frac{3}{2q} \quad (22)$$

for some $T \in \mathbb{R}$ sufficiently large, then (1) is nonoscillatory.

PROOF. Denote

$$v(t) = r(t)h(t)\Phi(h'(t)) - \frac{1}{2qG(t)}, \quad C(t) = \int_T^t [c(s) - \tilde{c}(s)]h^p(s) ds$$

and let $w(t) = h^{-p}(t)[v(t) - C(t)]$. To prove that (1) is nonoscillatory, according to Lemma 1 it suffices to show that w satisfies (10) and this happens if v satisfies the inequality (suppressing the argument t)

$$v' - rh'^p + \tilde{c}h^p + prh'^p Q \left(\frac{v - C}{rh\Phi(h')} \right) \leq 0, \quad Q(s) := \frac{|s|^q}{q} - s + \frac{1}{p}. \quad (23)$$

Indeed, suppose that (23) holds, then

$$\begin{aligned} w' &= h^{-p}(v' - rh'^p + \tilde{c}h^p) - p(v - C)h'h^{-p-1} \\ &\leq h^{-p} \left[rh'^p - ch^p - prh'^p \left(\frac{1}{q} \left| \frac{v - C}{rh\Phi(h')} \right|^q - \frac{v - C}{rh\Phi(h')} + \frac{1}{p} \right) - p \frac{h'(v - C)}{h} \right] \\ &= -c - (p - 1)r^{1-q}|w|^q. \end{aligned}$$

To verify (23) let us first estimate (again suppressing the argument t)

$$\begin{aligned}\frac{v-C}{rh\Phi(h')} &= \frac{rh\Phi(h') - \frac{1}{2qG} - C}{rh\Phi(h')} \\ &= 1 - \frac{1+2qGC}{2qGrh\Phi(h')} \rightarrow 1 \quad \text{as } t \rightarrow \infty\end{aligned}$$

since the numerator of the last fraction is bounded by (21) and (22), while the denominator tends to ∞ by (19). Now, let $\varepsilon > 0$ be such $\limsup$ in (21) is less than $\frac{1}{2q} - 2\varepsilon$ and $\liminf$ in (22) is greater than $-\frac{3}{2q} + 2\varepsilon$. This means that the expression in these limits satisfies

$$-\frac{3}{2q} + \varepsilon < G(t)C(t) < \frac{1}{2q} - \varepsilon \quad \Longleftrightarrow \quad |1 + 2qG(t)C(t)| < 2 - \varepsilon$$

for large t , i.e.,

$$(1 + 2qG(t)C(t))^2 < (2 - \varepsilon)^2 \quad (24)$$

for large t . Now, since $(v - C)/(rh\Phi(h')) \rightarrow 1$ as $t \rightarrow \infty$ and $Q(1) = 0 = Q'(1)$, by the second degree Taylor formula, to $\varepsilon(q-1)/4 > 0$ there exists $\hat{T}$, such that

$$\begin{aligned}Q\left(\frac{v(t)-C(t)}{r(t)h(t)\Phi(h')}\right) &\leq \left(\frac{q-1}{2} + \frac{(q-1)\varepsilon}{4}\right) \left(\frac{v(t)-C(t)}{r(t)h(t)\Phi(h')} - 1\right)^2 \\ &= \frac{q-1}{2} \left(1 + \frac{\varepsilon}{2}\right) \frac{(1+2qG(t)C(t))^2}{4q^2G^2(t)r^2(t)h^2(t)(h'(t))^{2p-2}} \\ &< \frac{q-1}{2} \left(1 + \frac{\varepsilon}{2}\right) \frac{(2-\varepsilon)^2}{4q^2G^2(t)r^2(t)h^2(t)(h'(t))^{2p-2}}\end{aligned}$$

for $t > \hat{T}$. Using these estimate we have

$$\begin{aligned}&v' - rh'^p + \tilde{c}h^p + prh'^pQ\left(\frac{v-C}{rh\Phi(h')}\right) \\ &= (r\Phi(h'))'h + rh'^p + \frac{G'}{2qG^2} - rh'^p + \tilde{c}h^p + prh'^pQ\left(\frac{v-C}{rh\Phi(h')}\right) \\ &\leq (r\Phi(h'))'h + \tilde{c}h^p - \frac{1}{2qG^2rh^2(h')^{p-2}} + \frac{q}{2} \left(1 + \frac{\varepsilon}{2}\right) rh'^p \left(\frac{v-C}{rh\Phi(h')} - 1\right)^2 \\ &\leq h[(r\Phi(h'))' + \tilde{c}\Phi(h)] + \frac{1}{2qG^2rh^2(h')^{p-2}} \left[-1 + \frac{(2-\varepsilon)^2}{4} \left(1 + \frac{\varepsilon}{2}\right)\right] \\ &< \frac{1}{2qG^2rh^2(h')^{p-2}} \left\{2qG^2rh^3(h')^{p-2}[(r\Phi(h'))' + \tilde{c}\Phi(h)] - \frac{\varepsilon}{2}\right\} < 0\end{aligned}$$

for large t since the first term in braces tends to zero according to (20) and $-1 + \frac{(2-\varepsilon)^2}{4} \left(1 + \frac{\varepsilon}{2}\right) < -\frac{\varepsilon}{2}$, if $\varepsilon > 0$ is sufficiently small. $\square$

The previous theorem and Lemma 2 applied to (2) prove the conjecture mentioned at the beginning of the paper.

Corollary 1. *Suppose that (6) and (7) hold, then (2) is nonoscillatory.*

PROOF. We will use the previous theorem where equation (4) plays the role of (3). We take $h(t) = t^{\frac{p-1}{p}} \log^{\frac{2}{p}} t$. Then

$$G(t) = \int_t^\infty \frac{ds}{h^2(s)(h'(s))^{p-2}} \sim \left(\frac{p}{p-1}\right)^{p-2} \int_t^\infty \frac{ds}{s \log^2 s} = \left(\frac{p}{p-1}\right)^{p-2} \frac{1}{\log t},$$

$$G(t)h(t)\Phi(h'(t)) \sim \frac{p-1}{p} \log t \rightarrow \infty \quad \text{as } t \rightarrow \infty.$$

and using the the fact that $h^p(t)\tilde{R}[w_h](t) = h(t)[(\Phi(h'(t)))' + \frac{\gamma_p}{t^p}\Phi(h(t))]$, from Lemma 2 we have

$$G^2(t)h^3(t)(h'(t))^{p-2} \left[(\Phi(h'(t)))' + \frac{\gamma_p}{t^p}\Phi(h(t)) \right] \sim \frac{\text{const}}{t} \rightarrow 0, \quad \text{as } t \rightarrow \infty.$$

Consequently, all assumptions of Theorem 2 are satisfied and we have the required statement. $\square$

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