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Title: Generalized convex functions and a solution of a problem of Zs. Páles

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Following an idea of Beckenbach, given a real function α defined on a convex subset U of a linear space and $t \in (0;1)$, we define: convexity, t-convexity, Jensen convexity, affinity, t-affinity and Jensen affinity of a function $f : U \to \mathbb{R}$ with respect to α. Some generalizations of Berstein–Doetsch and Sierpiński theorems are proved. Natural generalizations of Jensen and Cauchy functional equations are considered. A three variable functional equation on α which is a necessary condition for the existence of discontinuous Jensen affine functions with respect to α is presented. In one-dimensional case the explicit form of all Jensen affine functions with respect to α, involving the homographic functions, are determined. Applying this result we obtain a complete solution of a problem posed by Zs. Páles. Moreover, without any regularity assumptions, some functional equations are solved.

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